SHORT TIME DETERMINATION OF THE HEAT DYNAMICS OF BUILDINGS

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PREFACE

This report is the final report from the research project "Short Time Investigation of the heat consumption of houses" that has been funded through the Danish Energy Agency by the 1986 Research Programme of the Danish Ministry of Energy (EFP-86), research project journal No. 603-01-01.

The project has been carried out by the Thermal Insulation Laboratory (TIL) in collaboration with the Institute of Mathematical Statistics and Operation Research (IMSOR).

Bjarne Saxhof
Project Leader

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SUMMARY

For a number of years, it has been normal practise in Denmark (and in some cases mandatory) to calculate the energy demand for heating of buildings at the design stage, either by a simple hand calculation or by computer simulations. When the building is completed no one knows if the building actually has a satisfactory energy performance, at or below the theoretically calculated level. Furthermore, the heat dynamics of the building such as the heat accumulation in walls, floor and ceiling (and the indoor air), which play an important role in usability of solar heat gains and control of the heating system, have almost never been determined or only superficially simulated.

Short time determination of the heat dynamics of buildings is a useful tool to identify the characteristic thermal parameters such as heat loss coefficient and heat accumulation. Also one or several time constants useful in designing an efficient control of the heating system can be determined.

The aim of the present project is a further development and optimisation of the work carried out in a previous project "Regression Models for Energy Consumption in Buildings" funded by the Danish Energy Agency. A new method based on estimation in continuous time of linear differential equations has been used. The equations are generated from a lumped parameter model of the building. The method and the computer program used for the identification are developed at the Institute of Mathematical Statistics and Operation Research at the Technical University of Denmark. Furthermore, use of optimised statistical control of the heat input to the building is introduced in the project through use of Pseudo Random Binary Sequences (PRBS) as well as a so-called Principal Component Analysis (PCA) for creating the most representative air temperature from measurements at several different positions.

The experiments were carried out in the Danish PASSYS test cells, which have the advantage that they have a simple geometry, they are highly insulated and extremely airtight, and they are supplied with a large number of air and surface temperature sensors. PASSYS is a project for investigation of PASsive solar SYStems funded by the European Community.

The PRBS-signal controlling the heat input consists of two different signals designed for each of the expected time constants of the test cell: A short time constant related to the heat capacity of the indoor air, and a long time constant related to the walls, floor and ceiling. The total length of the experiment is only 16 days.

The model describing the test cell is a second order model with two resistances and two capacities. The identification of these four parameters is carried out

by comparison of the "measured" indoor air temperature and the indoor air temperature calculated by the model when supplied with the measured climatic data and the heat input. The former is calculated by use of the PCA as that linear combination of the 7 measured air temperatures which describes most of the variations between the single sensors. In the performed measurements an equal weight was put on each of the sensors by the PCA, indicating that none of the sensors were malfunctioning or in an unfortunate position.

The program used for the identification is called CTLSM (Continuous Time Linear Stochastic Modelling). The advantage of the continuous time formulation is that the identified values of the parameters in the model can be physically interpreted. In this way, the building experts can be directly involved in both formulation of the model and evaluation of the identified results.

A comparison between the output of the identified model and the measured air temperature shows only very small deviations and the standard deviation on the identified parameters is very small.

It has been proven in this project that determination of heat dynamics of buildings by use of well designed PRBS-signals, use of principal component analysis, and model identification in continuous time can be carried out with a high degree of accuracy, with only a few weeks of measurements. The results of this project indicate that an even shorter test period can be achieved. Future development in computer performance and of the computer program would make it possible to automatize the process, e.g. let one computer take care of both input control, data acquisition and on-line identification and automatically stop the experiment, when the requested accuracy has been reached.

INTRODUCTION

Short time determination of heat dynamics of buildings is a way to "measure" the effective thermal behaviour of real buildings. The results of the experiments are some key values which can be used for simulation of yearly energy consumption, comparison with theoretically determined heat loss coefficients, or investigation of aging effects by regular repetition of the measurements.

This project is based on the previous work carried out for the Danish Energy Agency in the EFP-83 project "Regression Models for Energy Consumption in Buildings" where simple mathematical models describing the thermal behaviour of buildings were formulated and used for identification of the thermal parameters of the experimental low-energy building at the Thermal Insulation Laboratory. The experiments carried out in the test building showed the advantage of using a time varying input signal which decor relates the climatic influence from the heat input to the building. Also a method of choosing the most representative indoor air temperature from measurements at several different locations was considered.

In the present project the experience from the previous work carried out is concentrated into a method for short time determination of heat dynamics of buildings. The aim is to optimize the input signal (frequency, power level and duration) and to develop sufficient mathematical models for accurate determination of the heat consumption and heat capacity of buildings.

The data from the old experiments for the regression model project did not contain all the information needed for the new analysis methods, so a new set of experiments was designed. The experimental work was performed in the PASSYS test cells at the Thermal Insulation Laboratory (PASSYS is a project funded by the European Community for testing of PAssive Solar SYStems). The test cells were preferred to the experimental low-energy building because of a simpler geometry, a simpler window arrangement, an even higher insulation level, and a very well defined construction with respect to the used materials and their thermal properties. Furthermore, the south wall in the test cells can easily be exchanged with a different type of wall construction leading to a different mathematical model for estimation. Besides, a comprehensive set of sensors for measurement of air and surface temperatures as well as climatic data are available, which ensures that even more complicated mathematical models can be identified.

With this report we intend to give a brief description of some results from an identification method - or test method - developed at the Technical University of Denmark. The main objective is to propose a statistical method for short time determination of models describing the heat dynamics of buildings. There are several benefits of considering a continuous time model: The continuous time formulation ensures that the parameters are easily interpreted as equivalent thermal parameters, and the methods allow for changes in the sampling time, which ensures that a stiff system like a house, with both short and long time constants, can be identified.

Usually it is a very difficult task to determine the most appropriate locations for sensors (for instance for measuring the indoor air temperature). In case of several sensors it may be difficult to determine the signal(s) expected to be the most representative for the room air temperature (surface temperature, etc.). The present report also illustrates a method for finding the most reasonable signal as a linear combination of all the measured signals.

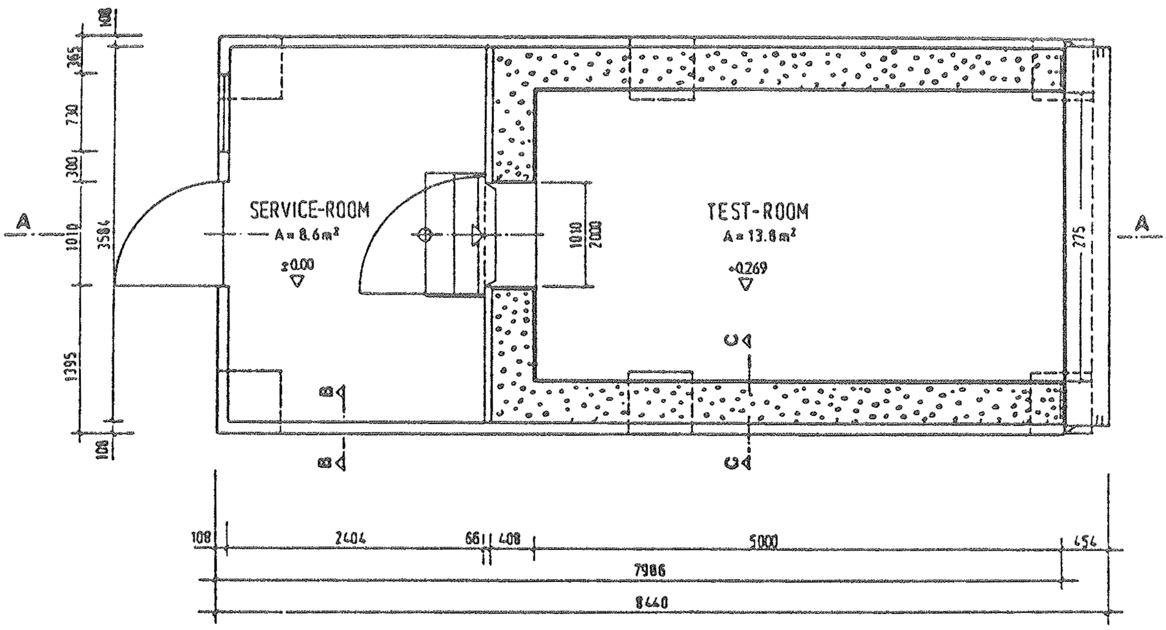
A main objective of the project is to demonstrate that it is possible to identify the main heat dynamical characteristics of a building within a reasonable short time. Actually only 384 hours (16 days) were spent on the main experiment described in this report.

In the discussion most attention is paid to the results - not to the mathematics. However, the key part of the identification method - a maximum likelihood method for estimating linear stochastic differential equations - is outlined in Appendix B.

1 EXPERIMENTAL DESIGN

The experimental design is a very important part of an experiment. Furthermore, it is well known that the design procedure is partly iterative, since results from any experiment can be used for an improved design of future experiments.

The first design of the experiment is based on the knowledge of the physical properties of the test building. The PASSYS test cell consists of a heavily insulated test room and an adjacent service room holding measuring equipment and a cooling system. The two rooms are separated by a well insulated door. The wall, roof and floor are made of a rigid steel frame insulated with mineral wool - the outside is covered with sheets of stainless steel. On the inside 400 mm of polystyrene is glued to a chipboard screwed to the steel frame. Thus the construction has no thermal bridges. On the inside, the polystyrene is covered with a layer of chipboard to which the final cover of 2 mm galvanized steel plates is screwed. The large insulation thickness and



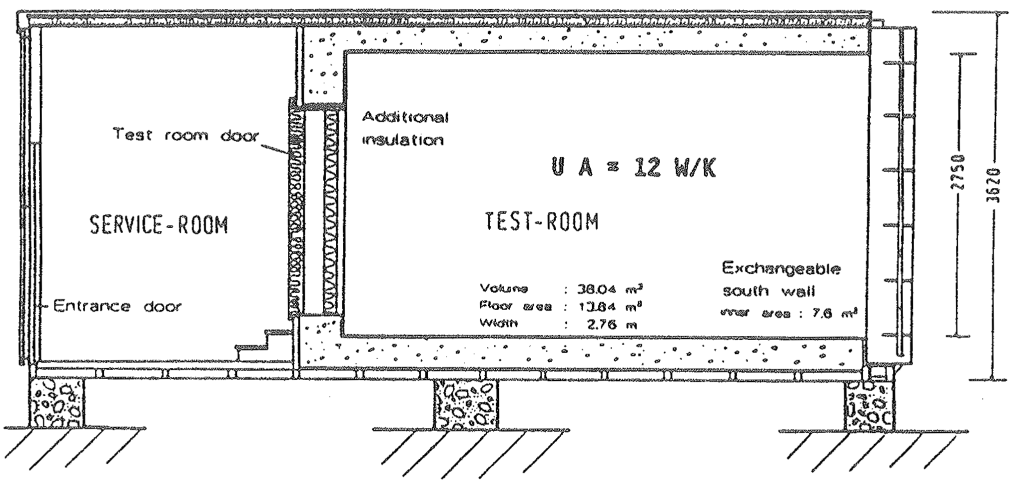


Figure 1: Groundplan and sectional view A-A.

cells relatively large time constants. The overall dimensions are shown in Figure 1.

As a goal for the present test experiment it was chosen to try to estimate simultaneously both the short time and the long time dynamics of the test cell. As a starting point we expected a short time constant around 10 minutes, and a long time constant in the interval 38–100 hours.

In order to ensure a reasonable information for an identification of the dynamics, the system has to be excited in both the short time and the long time part of the frequency scale of variations. This is ensured by controlling the heat input by a Pseudo Random Binary Sequence (PRBS-signal), which can be chosen to excite the system in desired intervals of the frequency scale of variations.

The PRBS-signal is a deterministic signal shifting between two constant levels. The signal may switch from one value to the other only at certain intervals of time, t = 0, T, 2T,..., nT. The levels are used to control the heat supply (on - off). This signal contains some very attractive properties, e.g. the signal is uncorrelated with other external signals (meteorological data), and it is possible by selecting the time period, T, and the order of the signal, n, to excite the system in the areas of the scale of variations, where interesting parameters are expected to be located. See [Godfrey, 1980] for further information.

The time period, T, and the order of the PRBS-signal, n, are determined by the expected time constants in the system. If only one PRBS-signal is used, the period T is of an order of magnitude as the smallest time constant, and n may be selected such that nT is of the order of magnitude as the largest time constant.

To excite the system in each part of the frequency scale of variations, two different PRBS-signals are used in a single experiment. In order to search the short time constant a PRBS-signal with T=20 minutes and n=6 has been selected. The PRBS-signal is periodic with a period of $(2^n - 1)T = 21$ hours. In our experiment this PRBS-signal has been used in two periods, i.e. 42 hours. This procedure yields good possibilities to estimate time constants between 5 minutes and 4 hours.

In order to search for the long time constant a PRBS-signal with T=20 hours and n=4 was used. This corresponds to a test period of 300 hours. This PRBS-signal forms a god basis for estimating time constants between 10 hours and 160 hours.

The total experiment consists of an entrance period of 6 periods using the PRBS-signal corresponding to the short time constant - (T,n) = (20 min., 6). This period contains the transient part of the experiment, and ensures variations around stationary values for the rest of the experiment. Then follows a period of 42 hours using the same PRBS-signal. In this period the relevant data are measured with a sampling time of 5 minutes. The PRBS-signal is then changed to (T,n) = (20 hours, 4). The sampling time is still 5 minutes. After a single period of this signal (300 hours), the PRBS-signal is changed to the first one, (T,n) = (20 min., 6), for 42 hours. Hence, data are collected with a sampling time of 5 minutes in a total period of (42+300+42) hours. In Fig. 3 the total experiment is illustrated by the PRBS-signals.

The heating system in the test room consists of four 75 W electric bulbs. The total energy consumption in the bulbs is measured with an electricity meter. The accuracy is about 0.05 kWh. The electric power, when the bulbs are turned on, is found as a mean value over the total experiment by dividing the total consumption by the total number of hours the bulbs have been turned on. In the service room we use three 500 W electric heaters, that can be controlled by the Data Acquisition System in the low-energy experimental house. This means that we can control the temperature in the service room within 0.5°C. The heating equipment is indicated on Figure 2.

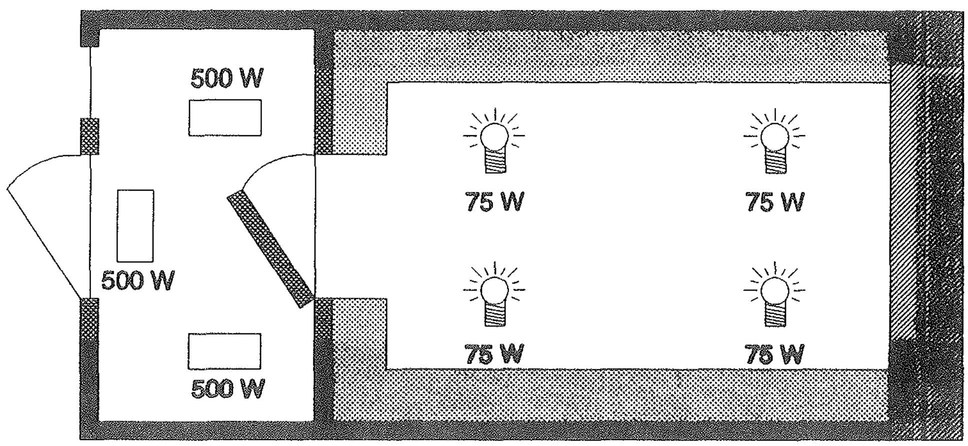


Figure 2: Plan of test cell. The heating system used in the calibration of the cells is shown.

Several experiments have been carried out. However, the results shown in this report only originate from a single experiment, where the heat loss through the partitioning wall has been eliminated by ensuring that the temperature in the service room is equal (within 0.5°C) to the temperature inside the test cell.

In each test cell 7 sensors for measuring the air temperature and 16 sensors for measuring the surface temperature have been used. The location of the sensors, as well as a further description of the measurements are found in Appendix A.

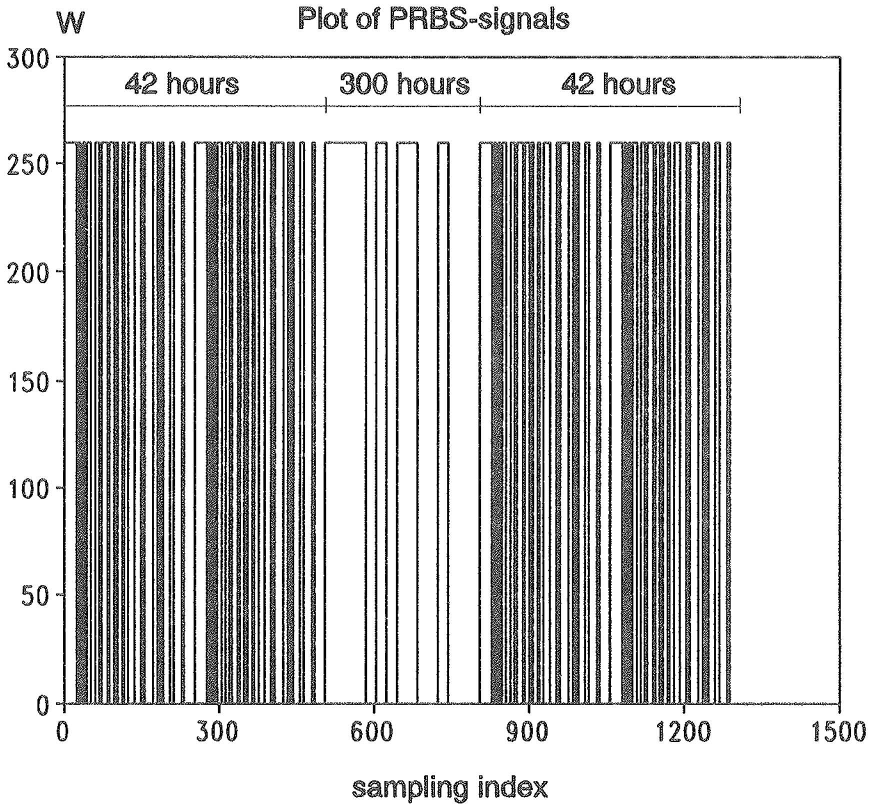


Figure 3: The PRBS's of the total experiment illustrated by the PRBS-signals. (Note, that the sampling index corresponds to 5 minutes at the two periods of 42 hours, and to 1 hour at the long period of 300 hours.)

2 PRINCIPAL COMPONENTS

This section shows how the relevant information from all the sensors is concentrated in so-called principal components. The principal components form the basis for estimating the model in later sections. By using this method we are able to find the most reasonable linear combination of all the measurements for representing the indoor air temperature or the surface temperature. If, for instance, a single sensor is placed unsuccessfully for measuring the indoor air temperature, the principal component will pick up this measurement as non-representative for the indoor air temperature.

In this report the principal components for the air temperature will be considered for illustration purposes only. The principal components correspond to an eigenvalue analysis of the variance matrix for the vector containing the measurements of the indoor air temperature.

Consider the stochastic vector

$$X_t \;=\; (X_{1t}, X_{2t}, \cdots, X_{7t}) \tag{1}$$

which contains the seven measurements at time t of the indoor air temperature. Based on measurements of the indoor air temperature, the mean value vector and the variance matrix Σ, associated with this stochastic vector, are readily calculated.

The eigenvalues of Σ is then calculated and ordered in decreasing order

$$\lambda_1 \geq \lambda_2 \geq \;\cdots\; \geq \lambda_7 \tag{2}$$

and the associated eigenvectors are

$$p_1, p_2, \;\cdots\;, p_7 \tag{3}$$

The i'th principal component is then defined as

$$Y_{it} \;=\; p_i' X_t \tag{4}$$

The first eigenvector then determines that linear combination of the measurements, which accounts for most of the variation of the measurements of indoor air temperature. How much of the total variation, which is described by the first principal component, is determined by the first eigenvalue.

For an ordinary and well planned experiment, the first principal component is representative for the indoor air temperature, and it contains information from all (in this case) seven measurements. So apart from the fact that the analysis will pick up unsuccessful measurements it will also reduce the measurement error, since information from several sensors is contained in the first principal component.

Based on the estimated variance matrix for the indoor air temperature we found the following values of the p_1, p_2 and p_3

$$p_1 = (0.3781, 0.3785, 0.3784, 0.3787, 0.3780, 0.3771, 0.3758)' \quad (5)$$
$$p_2 = (-0.547, 0.266, -0.105, -0.232, 0.011, -0.130, 0.740)' \quad (6)$$
$$p_3 = (-0.583, -0.377, 0.391, 0.037, 0.073, 0.586, -0.126)' \quad (7)$$

The associated eigenvalues explain 99.9948 %, 0.0042 % and 0.0003 %, respectively, of the variations of the indoor air temperature. The first principal component, determined by p_1 and defined though (5), is seen to put equal weight on all seven measurements, and this component will be the best representation for the indoor air temperature. Corresponding to a single measurement the measurement error for this component is approximately $1/\sqrt{7}$ times the original measurement error. A plot of the first principal component is shown in Appendix D.

The second principal component is seen to be approximately the difference between X_7 and X_1. X_7 is the measurement near the wall to the service room, which is heated in such a way that no heat loss takes place through this wall. X_1 is the measurement near the floor of the test cell. Hence, the second principal component measures a difference between the temperature near the wall to the service room and the temperature near the floor (which is the coldest). A plot of the second principal component is shown in Appendix D, and it is seen that this component behaves very much like the PRBS-signal! Hence, it is reasonable to conclude that when the heating system is turned on, there are differences between measurements, which are not present when the heating system is turned off. This agrees very well with the fact that the electric bulbs positioned on the floor were shielded with cylinders of aluminium foil with openings in the top and bottom. When the heat was on (i.e. the bulbs are turned on) the stack effect of the cylinders will force a warm air stream towards the ceiling of the test cell. In case of no heating a more uniform temperature distribution in the test room will occur.

Also the third principal component is interesting. A further analysis has shown that it measures some transient behaviour of the temperatures. The third principal component happens to be large just after the heating system is turned on and small just after it is turned off - see Appendix D. For the higher order principal components no interesting behaviour is found.

For the surface temperature a similar principal component analysis was carried out. Also in this case the first principal component happens to be the best representative for the surface temperature.

3 FORMULATION OF A STOCHASTIC DIFFERENTIAL EQUATION MODEL

An adequate description of the heat dynamics of the test cell requires at least two time constants - one time constant describing the long time variations and another time constant describing the short time variations. A description of the short time constant is essential for modelling the variations of the room air temperature. Unfortunately, it is very difficult to achieve a reasonable description of the short time dynamics when the traditional (deductive) approach is used. But, by using the statistical (inductive) approach it is possible to describe the variations on the whole time scale covered by the experimental data. In the [Madsen, Nielsen and Saxhof, 1992] some proposals for dynamic models with more than one time constant are given, which then contain the capability of describing both long term and short term variations.

The "two time constant" model shown in Figure 4 may be adequate if the dominant heat capacities are located in the outer walls. This is expected to be the case for the test cell. However, it is possible by a statistical test to verify whether the model gives a reasonable description.

The states of the model are the temperature T_m of the large heat accumulating medium with the heat capacity c_m, and the temperature T_i of the room air - possibly including the inner part of the wall - with the capacity c_i. The resistance against heat transfer to the ambient air with temperature T_a is denoted r_a, while r_i denotes the resistance against heat transfer between the room air and the large heat accumulating medium. The heat input to the system is from the electric resistance heaters, Φ_h, and from the sun, Φ_s.

It is readily seen that the equations for the heat transfer become

$$c_m \frac{dT_m}{dt} = \frac{1}{r_a}(T_a - T_m) + \frac{1}{r_i}(T_i - T_m) \tag{8}$$

$$c_i \frac{dT_i}{dt} = \frac{1}{r_i}(T_m - T_i) + \Phi_s + \Phi_h \tag{9}$$

Additional heat inputs are added on the right hand side of (9). It can be shown that the model can be identified (this is further discussed in [Madsen, Nielsen and Saxhof, 1992] even if T_m is not measured.

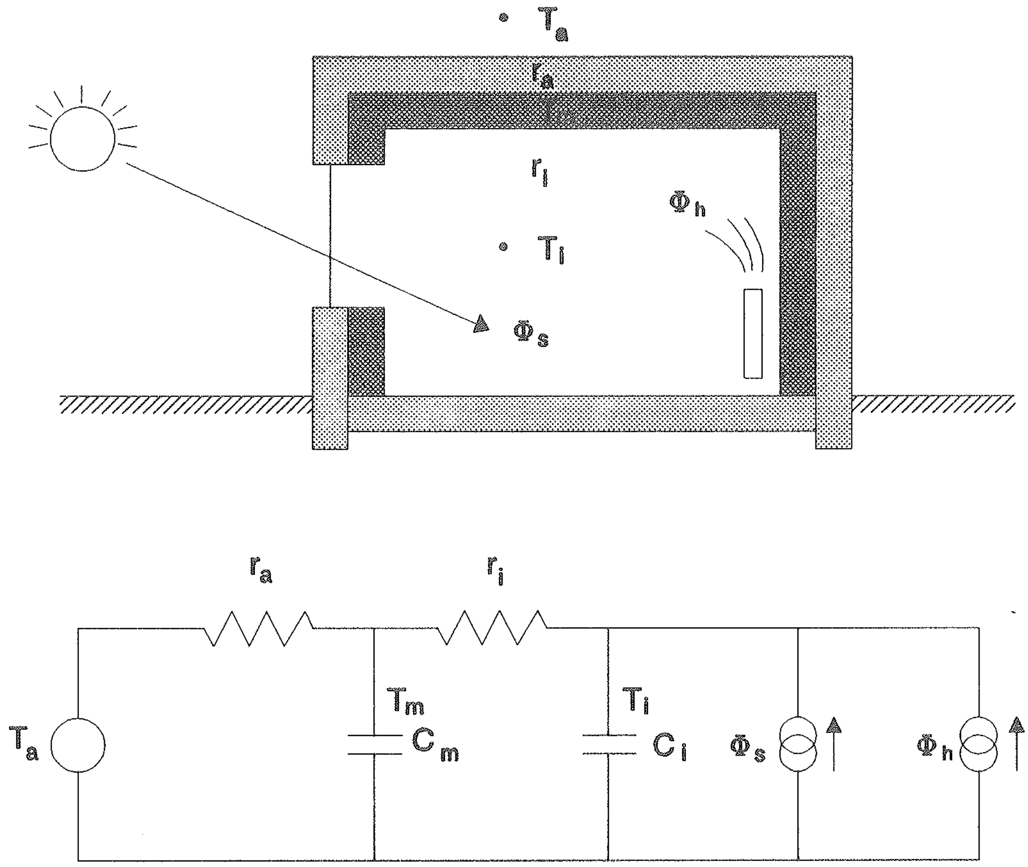


Figure 4: A "two time constant" model with the dominant heat capacities in the outer wall.

In general it is convenient to describe the differential equations in the matrix form

$$\begin{bmatrix} dT_i \\ dT_m \end{bmatrix} = \begin{bmatrix} -\frac{1}{r_i c_i} & \frac{1}{r_i c_i} \\ \frac{1}{r_i c_m} & -\left[\frac{1}{r_a c_m} + \frac{1}{r_i c_m}\right] \end{bmatrix} \begin{bmatrix} T_i \\ T_m \end{bmatrix} dt$$

$$+ \begin{bmatrix} 0 & 1/c_i & 1/c_i \\ 1/(r_a c_m) & 0 & 0 \end{bmatrix} \begin{bmatrix} T_a \\ \Phi_h \\ \Phi_s \end{bmatrix} dt + \begin{bmatrix} dw_i(t) \\ dw_m(t) \end{bmatrix} \qquad (10)$$

where the states are the temperature T_i of the room air, and the temperature T_m of the large heat accumulating medium. The constants c_m, c_i, r_a, and r_i are the equivalent thermal parameters, which describe the dynamic behaviour of the test cell.

The last vector, dw(t), on the right hand side of (10) describes the stochastic part of the model. The stochastic part is introduced in order to describe the deviation between (8)-(9) and the true variations of the states of the system.

In a more compact form the model for the heat dynamics is described by the stochastic differential equation

$$dT = AT dt + BU dt + dw(t) \qquad (11)$$

where the structure of the matrices A and B in the present case is as described in (10). w(t) is a two-dimensional stochastic process, and in the present context we will further restrict w(t) to be a Wiener process with incremental covariance

$$R_1 = \begin{bmatrix} \sigma^2_{1,11} & 0 \\ 0 & \sigma^2_{1,22} \end{bmatrix} \qquad (12)$$

It can be shown than the diagonal structure in (12) is necessary in order to provide identifiability of the system.

Since the temperature of the heat accumulating medium, T_m, is not measured, but only the air temperature, T_i, we can express our measurement as

$$T_r = [1 \;\; 0] \begin{bmatrix} T_i(t) \\ T_m(t) \end{bmatrix} + e(t) \qquad (13)$$

where e(t) is the measurement error, which is assumed to be normally distributed with zero mean and variance σ_2^2 .

A further discussion about the formulation of the stochastic state space model (11)-(13) is given in Appendix B.

4 RESULTS AND VERIFICATION

The used method for parameter estimation is described in Appendix B. In this section only the results are discussed.

At the estimation a sample time of 5 minutes is used in the case of the PRBS-signal with (T,n) = (20 min., 6), whereas a sample time of 1 hour is used in the period where a PRBS-signal with (T,n) = (20 hours, 4) is used.

The results shown below are based on only one experiment with one of the test cells. Furthermore, some test results have shown that the influence from the solar radiation is insignificant, so the system considered is as described by (10), except that Φ_s is not included.

The following (maximum likelihood) estimates are found (the number in brackets are the associated standard error of the estimates):

$$\hat{c}_i = 0.1172 \text{ kWh/°C} \quad (0.0013)$$

$$\hat{c}_m = 0.5004 \text{ kWh/°C} \quad (0.0149)$$

$$\hat{r}_i = 3.2968\text{°C/kW} \quad (0.0660)$$

$$\hat{r}_a = 493.45\text{°C/kW} \quad (16.7)$$

The estimates belonging to the stochastic part of the model are

$$\hat{\sigma}_{1,11}^2 = 0.00152(°C)^2 \quad (0.00017)$$

$$\hat{\sigma}_{1,22}^2 = 0.01177(°C)^2 \quad (0.00096)$$

$$\hat{\sigma}_2^2 = 0.00021(°C)^2 \quad (0.00003)$$

First, it is noted that no large heat capacity is found for the test cell. The resistance against heat transfer to the ambient air, r_a, is found to be very large, whereas the resistance against heat transfer between the heat accumulating medium and the air is more moderate. The estimated standard deviation of the measurement error is $\sqrt{0.00021} = 0.015°C$ - this small measurement error is in fact the 'measurement' standard error of the first principal component, which is found to be representative for the indoor air temperature (the estimated real measurement error is approx. $\sqrt{7} \times 0.015°C$).

An analysis has shown that if we only consider for instance the PRBS-signal corresponding to the high frequency variation (T,n) = (20 min., 6), then it is impossible to give a good estimate of r_a, but the remaining estimates are nearly the same. It is concluded that it is important to use at least two PRBS-signals in order to estimate both the short time and long time dynamics of a house. Furthermore, it is essential to use an estimation method where the sampling time can be changed from period to period.

5 THE PERFORMANCE OF THE MODEL FOR FORECASTING

Based on the above estimates it is possible to estimate the performance of the model for forecasting the indoor air temperature. The standard error of the forecast error in the case of a horizon of 5 minutes is 0.024° C, and if a horizon of one hour is considered the standard error is 0.079° C. Figure 5 shows the observed and predicted air temperature for a very small segment of the total series. If a larger segment is chosen it is hardly possible to see the difference between the observed and the predicted temperature.

The largest differences are observed just after the heating system has been turned off or on. The reasons why are that the dynamic of the heating system (in this case most likely the heat capacity of the electric bulb) has to be included in the model. An analysis of the residuals has shown that the model, except for the fast dynamics of the heating system discussed above, is able to describe all the variations. These include tests in the autocorrelation functions as well as tests in the frequency domain.

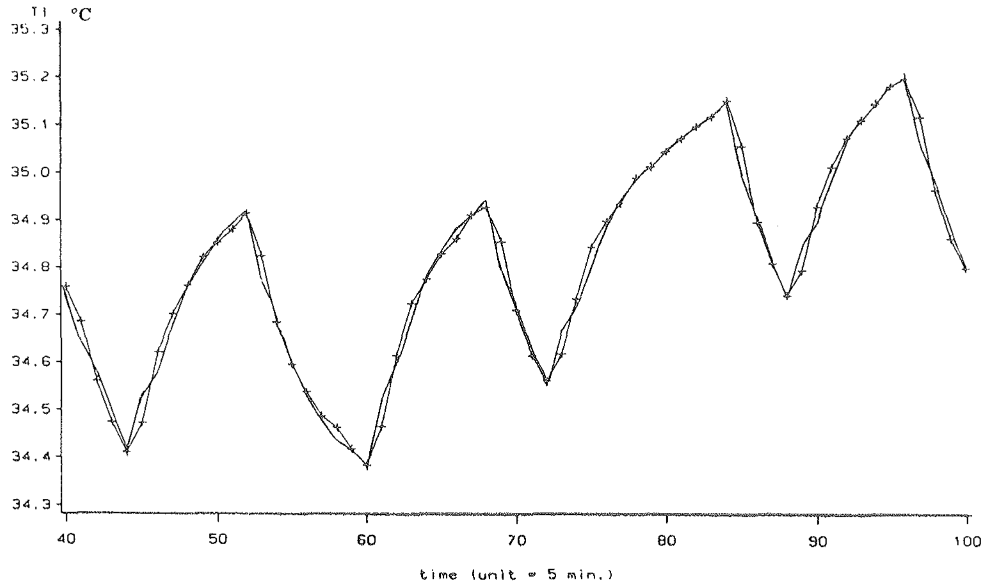


Figure 5: Observed (line with cross) and predicted air temperature (Only a small part of the observations is shown).

6 CONCLUSION

Short time determination of heat dynamics of buildings is a way to "measure" the effective thermal behaviour of real buildings. The results of the experiments are some key values which can be used for simulation of yearly energy consumption, comparison with theoretically determined heat loss coefficients, or investigation of aging effects by regular repetition of the measurements.

Design of an experiment for short time determination of heat dynamics of buildings will always be an iterative process to achieve the specific input signal that leads to the most accurate estimation of the thermal parameters. However, the results in this project show that with a rough estimation of the thermal behaviour based on the construction and some knowledge of the physical properties of the used materials, a suitable input signal can be designed. Especially the use of Pseudo Random Binary Sequence (PRBS) signals has proved to be useful, as one design of the PRBS signal can cope with a large range of time constants. If the building is characterised by both having very short and very long time constants the test signal can be composed of two or more different PRBS signals, designed for each of the expected time constants. Furthermore, the PRBS signal is decor related with the climatic input variables as solar radiation and outdoor air temperature making the influence of these parameters identifiable.

The indoor air temperature has been used for control of the identified model by comparison between the calculated and the measured temperature. The "measured" temperature used in the comparison is created from measurements of the air temperature at 7 different locations in the test room. For this purpose a so-called principal component analysis (PCA) was used for calculation of that linear combination of the measured values (the 1st principal component) which describes most of the variations between the single sensors. The effective measurement error is in this case (7 sensors) reduced to the measurement error of a single sensor. Furthermore, the PCA is a very useful tool to detect if one sensor is measuring something else than expected, e.g. if an air temperature sensor is positioned too near to a heating element, or if the sensor has been damaged and is malfunctioning. The remaining principal components can be used for investigation of the performed test, e.g. to find an explanation for the discrepancies between the output from the identified mathematical model and the measured value.

The identification program CTLSM used in this project is developed at the Institute of Mathematical Statistics and Operation Research (IMSOR) and is characterised by identification in continuous time by integration of the differential equations describing the lumped model of the building. The advantage of the continuous time formulation is that the identified values of the parameters in the model can be physically interpreted. In this way, the building experts can be directly involved in both formulation of the model and evaluation of the identified results.

The experiments carried out in the PASSYS test cells have proved the large possibilities in testing of thermal performance of buildings by use of PRBS-signals and advanced statistical identification. The test period is only 384 hours (16 days) and the identification results indicates that a shorter test period could probably be achieved. Both a short and a long time constant is identified related to the heat capacity of the air and the heat capacity of the walls, floor and ceiling respectively.

The principal component analysis showed that an almost equal weight was put on each of the seven air temperatures in creating the effective air temperature. This indicates that each of the seven sensors was measuring the "correct" air temperature. However, the second principal component shows a change in the temperature distribution between periods where the heat was turned on and periods with the heat turned off. The third principal component indicates a systematic variation in the air temperature just after the heat is turned on and off. The remaining principal components show no systematic variations. The principal component analysis is a powerful tool to point out where the experimental part of the test can be improved.

The output from the identified model and the measured air temperature in the test room are almost identical except in the period just after the heat is turned on and off. A reasonable explanation is that the heat capacity of the heating system is not covered by the model and with the third principal component in mind this seems to be a very good explanation.

Short time determination of the heat dynamics of buildings has been carried out in real tests, but an optimisation of the length of the test period is still missing. Future development of the identification program CTLSM and increased computer performance would make it possible to perform an on-line identification, where the program continuously tests if the accuracy of the identified parameters has reached the desired value, and stops the experiment if positive. In this way the computer can be installed in the actual building and perform the control of the input signal, the data acquisition and the model identification. A first step could be a smaller computer installed in the actual building for data acquisition and control, with a link, e.g. through the public telephone network, to a more powerful computer in a central place, running the identification program.

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A Position of surface and air temperature sensors in the Danish test cells

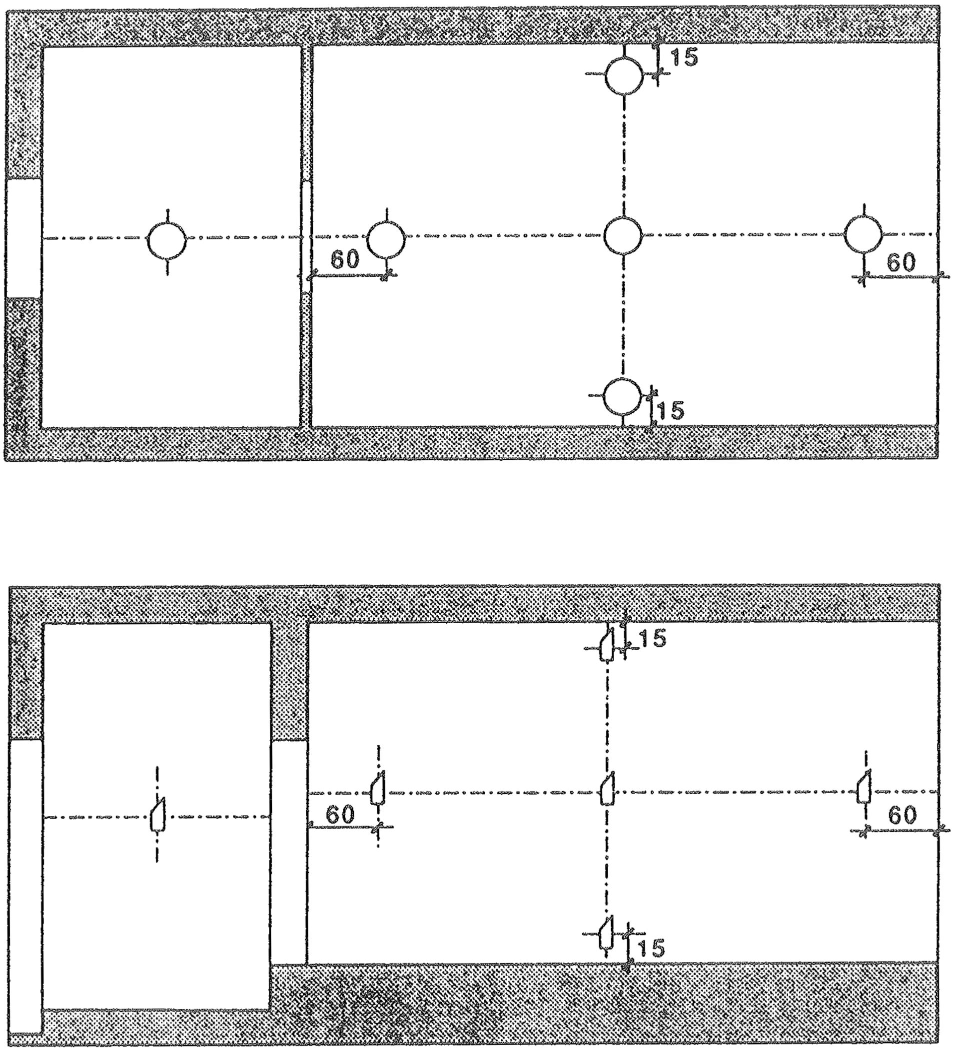


Figure A1. Location of indoor air temperature sensors.

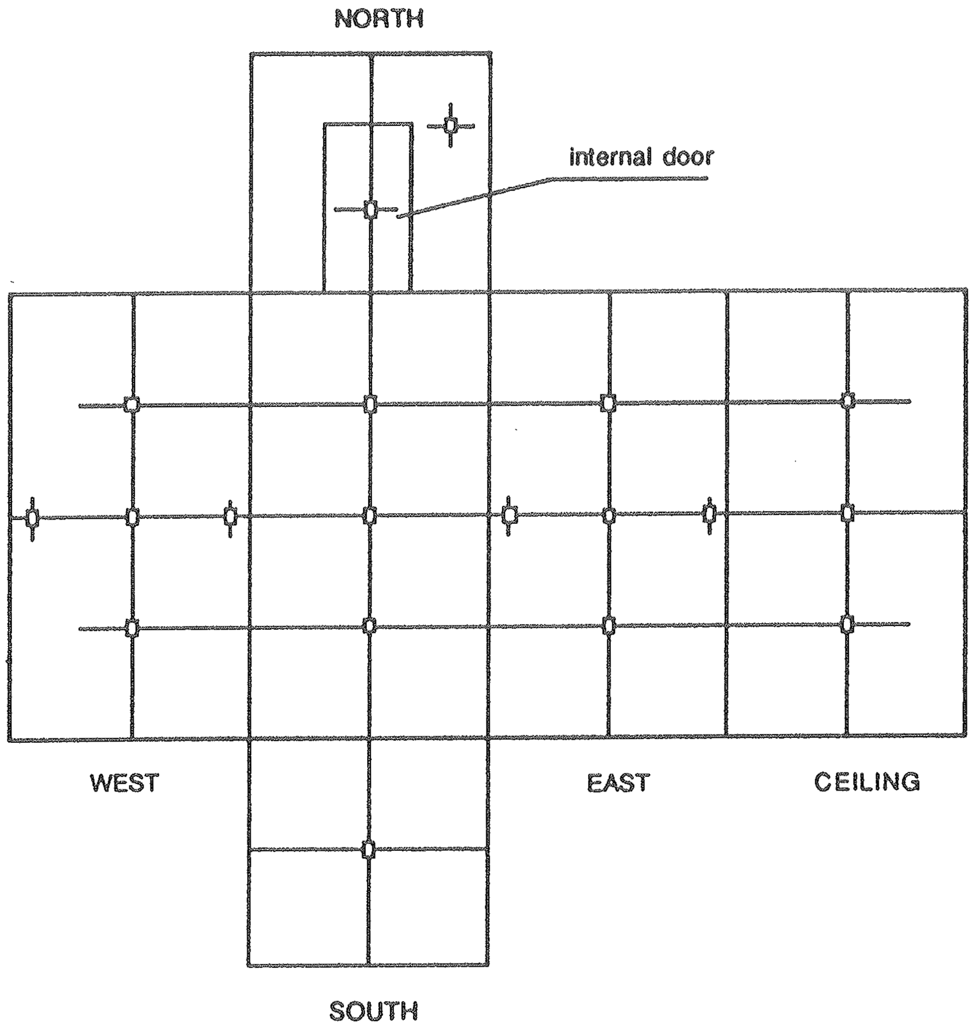


Figure A2. Location of the internal surface temperature sensors (outer sensors at corresponding positions on outer surface).

B A method for estimation of continuous time stochastic differential equation models

The entire estimation for the identification is carried out by a program written in Fortran. Hence it is possible to move the program to other machines (e.g. in term of the source code) because only a Fortran compiler is required. The program is built up in blocks for transforming the differential equations to difference equations, for evaluation of conditional densities (by means of a Kalman filter), for evaluation of the likelihood function and for minimisation.

B.1 Linear stochastic models in state space

In [Madsen, Nielsen and Saxhof, 1992] it is argued that a lumped description of the heat dynamics of buildings can frequently be described by a system of linear differential equations, and in a very useful matrix notation the equations can be parameterised by the linear state space model in continuous time

$$\frac{dT}{dt} = AT + BU \tag{14}$$

where T is the state-vector and U is the input vector.The dynamic behaviour of the system is characterised by the matrix. A and B are a matrices which specify how the input signals (outdoor air temperature, solar radiation, radiator supply, etc) enter the system. For a further discussion of the structure of the matrices and vectors we refer to Equation 10.

Since the description will always be a model of the system, which, of course, is unable to give an exact description of the system some deviations are to be expected. To describe the deviation between (14) and the true variation of the states an additive noise term is introduced. Then the model of the heat dynamics is described by the stochastic differential equation

$$dT = ATdt + BUdt + dw(t) \tag{15}$$

where the m-dimensional stochastic process w(t) is assumed to be a process with independent increments. With the purpose of calculation of the likelihood function we will further restrict w(t) to be a Wiener-process with the incremental covariance $R_1'(t)$.

As an example of a model in the class described by (15) consider the following important example:

$$\begin{bmatrix} dT_i \\ dT_m \end{bmatrix} = \begin{bmatrix} -\frac{1}{r_i c_i} & \frac{1}{r_i c_i} \\ \frac{1}{r_i c_m} & -\left[\frac{1}{r_a c_m} + \frac{1}{r_i c_m}\right] \end{bmatrix} \begin{bmatrix} T_i \\ T_m \end{bmatrix} dt$$

$$+ \begin{bmatrix} 0 & 1/c_i & 1/c_i \\ 1/(r_a c_m) & 0 & 0 \end{bmatrix} \begin{bmatrix} T_a \\ \Phi_h \\ \Phi_s \end{bmatrix} dt + \begin{bmatrix} dw_i(t) \\ dw_m(t) \end{bmatrix} \qquad (16)$$

where the states are the temperature of the T_i of the room air (and the inner part of the walls), and the temperature T_m of the large heat accumulating medium. The constants c_m, c_i, r_a, r_i, A_w and p are the equivalent thermal parameters, which hence describe the dynamic behaviour of the building. The model (16) is further discussed in [Madsen, Nielsen and Saxhof, 1992].

The equation (15) describes the transfer of all the states of the system; and it is most likely that only some of the states are measured. If we for instance consider the state space model in (16) it is reasonable to assume that the temperature of the indoor air is measured; but not the temperature of the large heat accumulation medium (it might also be difficult to find a reasonable temperature to measure in order to represent the temperature of the large heat accumulating medium). In the general case we assume that only a linear combination of the states is measured, and if we introduce T_r to denote the measured or recorded variables we can write

$$T_r(t) = CT(t) + e(t) \qquad (17)$$

where C is a constant matrix, which specifies which linear combination of the states that actually is measured. In practise, however, C most frequently acts only as a matrix which picks out the actual measured states. The term e(t) is the measurement error. It is assumed that e(t) is normal distributed white noise with zero mean and variance R_2. Furthermore, it is assumed that w(t) and e(t) are mutually independent. For the example discussed, where the system is described by (16), and where we assume that only the indoor air temperature is measured, the measurement equation simply becomes

$$T_r(t) = [1 \;\; 0] \begin{bmatrix} T_i(t) \\ T_n(t) \end{bmatrix} + e(t) \qquad (18)$$

where e(t) is the measurement error, which accomplish the measurement of the indoor air temperature.

B.2 Some notes on parameter identifiability

It is a crucial question whether the parameters of a specified state space model can be identified. If a non-identifiable model is specified, the methods for estimation, which we shall discuss later on, will not converge. The problem of identifiability arises from the fact that for a given transfer function model, in general, a whole continuum of possible state space models exists. Therefore, we must introduce a restriction on the structure of the state space model, in order to provide a unique relation between the parameters of the state space model and the transfer function.

Let us illustrate the problem by considering a couple of examples. First we consider a slightly simplified system compared to (16), where the outdoor temperature and the solar radiation are considered to be zero (or constant). Furthermore, only the deterministic part of the model is considered. Hence, (16) can be written as

$$dT = \begin{bmatrix} -a & a \\ b & -(b+c) \end{bmatrix} T dt + \begin{bmatrix} 0 \\ d \end{bmatrix} \Phi_h dt \quad (19)$$

It is readily verified that the parameters of the corresponding original model, i.e. c_i, c_m, r_i and r_m , can be identified (or calculated) if the parameters a, b, c and d of the above model can be identified.

Furthermore, it is assumed that only T_i is measured and the noise e(t) is zero. In general, the identifiability is deduced by investigating the transfer functions belonging to every input to output combination in equations (15) and (17). All these combinations are contained in the (matrix) transfer function G(s), where s is the Laplace operator, and we have

$$T_r(s) = G(s)U(s) \quad (20)$$

where

$$G(s) = C(sI - A)^{-1}B \quad (21)$$

In single-input, single-output cases, as (19), the transfer function G(s) becomes a scalar function. For the considered example, see (19), the transfer function becomes

$$G(s) = \frac{d(s+a)}{s^2 + (a+b+c)s + ac} \tag{22}$$

This has to be compared to what we actually observe, namely

$$G(s) = \frac{b_0 s + b_1}{s^2 + a_1 s + a_2} \tag{23}$$

That is, we observe b_0, b_1, a_1 and a_2. If we compare (22) and (23) it is seen that based upon these values we are able to identify a, b, c and d. Hence the model is identifiable.

As an example of a model which is not identifiable we consider the following slightly modified version of (19), namely

$$dT = \begin{bmatrix} -a & b \\ c & -d \end{bmatrix} T dt + \begin{bmatrix} 0 \\ e \end{bmatrix} \Phi_h dt \tag{24}$$

where the transfer function becomes

$$G'(s) = \frac{e(s+a)}{s^2 + (a+d)s + (ad - cb)} \tag{25}$$

By a comparison with (23) it is seen that both c and b cannot be identified; only the product can. However, by regarding (24) it is not surprising that we got problems, since it contains 5 parameters, and (23) tells us that only 4 can be identified.

B.3 From continuous to discrete time

The previously discussed method uses finite differences for transforming the equations from continuous time to discrete time. For the present method, where the system is assumed to be described by the stochastic differential equation (15) it is possible analytically to perform an integration, which under some assumptions exactly specifies the system equation in discrete time.

For the continuous time model (15) the corresponding discrete time model is obtained by integrating the differential equation through the sample interval [t, t+ τ]. Thus the sampled version of (15) can be written as

$$\begin{aligned} T(t+\tau) &= e^{A(t+\tau-t)}T(t) + \int_t^{t+\tau} e^{A(t+\tau-s)}BU(s)ds + \\ & \int_t^{t+\tau} e^{A(t+\tau-s)}dw(s) \end{aligned} \quad (26)$$

Under the assumption that U(t) is constant in the sample interval the sampled exact version of (15) can be written as the following discrete time model in state space form

$$T(t+\tau) = \phi(\tau)T(t) + \Gamma(\tau)U(t) + v(t;\tau) \quad (27)$$

where

$$\phi(\tau) = e^{A\tau}, \quad \Gamma(\tau) = \int_0^{\tau} e^{AS}Bds \quad (28)$$

$$v(t;\tau) = \int_t^{t+\tau} e^{A(t+\tau-s)}dw(s) \quad (29)$$

On the assumption that w(t) is a Wiener process, $v(t;\tau)$ becomes normal distributed white noise with zero mean and covariance

$$R_1(\tau) = E[v(t;\tau)v(t;\tau)] = \int_0^{\tau} \phi(s)R_1'\phi(s)'ds \quad (30)$$

If the sampling time is constant (equally spaced observations), the stochastic difference equation can be written

$$T(t+1) \;=\; \phi T(t) + \Gamma U(t) + v(t) \tag{31}$$

where the time scale now is transformed in such a way that the sampling time becomes equal to one time unit.

B.4 Maximum likelihood estimates

In the following it is assumed that the observations are obtained at regularly space time intervals, and hence that the time index t belongs to the set {0, 1, 2, ..., N}. N is the number of observations. In order to obtain the likelihood function we further introduce

$$T_r^*(t) \;=\; [T_r(t), T_r(t-1), \cdots, T_r(1), T_r(0)]' \tag{32}$$

i.e. $T_r^*(t)$ is a matrix containing all the observations up to and including time t. Finally, let θ denote a vector of all the unknown parameters - including the unknown variance and covariance parameters in R_1 and R_2.

The likelihood function is the joint probability density of all the observations assuming that the parameters are known, i.e.

$$\begin{aligned} L'(\theta; T_r^*(N)) &= p(T_r^*(N)|\theta) \\ &= p(T_r(N)|T_r^*(N-1),\theta)p(T_r^*(N-1)|\theta) \\ &= \left[\prod_{t=1}^{N} p(T_r(t)|T_r^*(t-1),\theta)\right] p(T_r(0)|\theta) \end{aligned} \tag{33}$$

where successive applications of the rule $P(A \cap B) = P(A|B)P(B)$ are used to express the likelihood function as a product of conditional densities.

Since both v(t) and e(t) are normally distributed the conditional density is also normal. The normal distribution is completely characterised by the mean and the variance. Hence, in order to parameterise the conditional distribution, we introduce the conditional mean and the conditional variance as

$$\hat{T}_r(t|t-1) \;=\; E[T_r(t)|T_r^*(t-1),\theta] \tag{34}$$

and

$$R(t|t-1) \quad = \quad V[T_r(t)|T_r^*(t-1), \theta] \tag{35}$$

respectively. It is noticed that (34) is the one-step prediction and (35) the associated variance. Furthermore, it becomes convenient to introduce the one-step prediction error (or innovation)

$$\epsilon(t) \quad = \quad T_r(t) - \hat{T}_r(t|t-1) \tag{36}$$

Using (33) - (36) the conditional likelihood function (conditioned on $T_r(0)$) becomes

$$\begin{aligned} L(\theta; T_r(N)) \quad = \quad & \prod_{t=1}^{N} \Big[(2\pi)^{-m/2} detR(t|t-1)^{-1/2} \\ & *exp(-1/2\epsilon(t)' R(t|t-1)\epsilon(t))\Big] \end{aligned} \tag{37}$$

where m is the dimension of the T_r vector. Most frequently the logarithm of the conditional likelihood function is considered. It is written

$$\begin{aligned} logL(\theta; T_r(N)) \quad = \quad & -1/2 \sum_{t=1}^{N} [logdetR(t|t-1) \\ & +\epsilon(t)' R(t|t-1)^{-1}\epsilon(t)\Big] + \text{constant} \end{aligned} \tag{38}$$

The conditional mean $\hat{T}_r(t \mid t-1)$ and the conditional variance R(t—t-1) can be calculated recursively by using a Kalman filter (see e.g. [Åström, 1970] or [Madsen, 1989]). The Kalman filter is most easy to understand as formulae for recursively calculating one-step prediction (or estimate) of the state of the system, together with formulae for updating (or reconstructing) this estimate, (see e.g. [Madsen, 1989]). In the present case where the transfer of the states of the system in discrete time is described by (31) and the observations by (17) the equations for updating the estimate of the state T become

$$\hat{T}(t|t) \quad = \quad \hat{T}(t|t-1) + K_t(T_r(t) - C\hat{T}(t|t-1)) \tag{39}$$

$$P(t|t) = P(t|t-1) - K_t R(t|t-1) K_t' \tag{40}$$

where the Kalman gain K_t is

$$K_t = P(t|t-1) C' R(t|t-1)^{-1} \tag{41}$$

The formulae for prediction become

$$\hat{T}(t+1|t) = \phi \hat{T}(t|t) + \Gamma U(t) \tag{42}$$

$$\hat{T}(t+1|t) = C \hat{T}(t+1|t) \tag{43}$$

$$P(t+1|t) = \phi P(t|t) \phi' + R_1 \tag{44}$$

$$R(t+1|t) = C P(t+1|t) C' + R_2 \tag{45}$$

The formulae require some initial values, which describe the prior knowledge about the states of the system in term of the prior mean and variance

$$\hat{T}(1|0) = E[T(1)] = \mu_0 \tag{46}$$

$$P(1|0) = V[T(1)] = V_0 \tag{47}$$

The recursive use of the Kalman filter as it is formulated above can be explained in the following way: Assume that the time is t-1, and we have calculated the prediction and the associated variance for the state at time t. When the next observation ($T_r(t)$) at time t becomes available e.g.. (39) - (41) can be used for updating the estimate of the state. Using the updated values it is then possible to use e.g.. (42) - (45) to calculate the prediction and the associated variance for the state at time t+1. This was one step of the recursive calculations, which constitute the Kalman filter.

The maximum-likelihood estimate (ML-estimate) is the set $\hat{\theta}$ which maximises the likelihood function. For the optimisation of the likelihood function the IMSL-routine ZXMIN (1980) was used.

An estimate of the uncertainty of the parameters is obtained by the fact that the ML-estimator is asymptotically normally distributed with mean and variance

$$D = H^{-1} \tag{48}$$

where the matrix H is given by

$$\{h_{lk}\} = -E\left[\frac{\partial^2}{\partial\theta_1\partial\theta_k} logL(\theta; T_r(N))\right] \tag{49}$$

An estimate of D is obtained by equating the observed value with its expectation and applying

$$\{h_{lk}\} \approx -\left[\frac{\partial^2}{\partial\theta_1\partial\theta_k} logL(\theta; T_r(N))\right]_{|\theta=\hat{\theta}} \tag{50}$$

The above equation is thus used for estimating the variance of the estimates. If an estimated variance is large compared to the actual estimated value for a parameter, this indicates that probably this parameter can be eliminated from the model (the parameter is equal to zero).

C Evaluation of the Model

The described methods for evaluation of the model can be divided into two categories. The first category contains purely statistical methods based on the residuals of the model. These are generally used methods within time series analysis for evaluation of empirically determined dynamic models. The second category contains an illustration of the performance of the model for simulation and forecasting. Furthermore, the estimated parameters are naturally compared with parameters calculated by the traditional approach from the physical characteristics of the building - if this calculation is reasonable and possible.

C.1 Methods based on the residuals

The purpose of all model building is to describe all systematic variation by parameters in a model. In the model building case most frequently several models are considered. And the best model is selected as the smallest model, which is capable to describe all the variation. If all systematic variations are described by a specific model the residuals will be a white noise process, i.e. the residuals are uncorrelated in time. This section describes the methods used for testing the assumption that the residuals are white noise.

Test in the autocorrelation function

Let $\{\epsilon_t\}$ denote the sequence of residuals, and let $\rho_\epsilon(k)$ denote the autocorrelation function for the residuals (Box and Jenkins, 1976). If $\{\epsilon_t\}$ is white noise we have

$$\hat{\rho}_\epsilon(k) \ \ \overset{\in}{approx.} \ \ N(0, 1/N) \tag{51}$$

where $N(\mu, \sigma^2)$ denotes the normal distribution with mean μ and standard deviation σ. Finally, N denotes the number of residuals, which frequently is equal to the number of observations.

By using (51) confidence limits for the autocorrelation function under the hypothesis that $\{\epsilon_t\}$ is white noise is easily calculated. Frequently the 2σ-limits corresponding to approximately 95% confidence limits are used.

Besides the autocorrelation function it is very common to also consider the partial autocorrelation function (see [Box and Jenkins, 1976]). A very useful rule is that if the autocorrelation and the partial correlation are nearly identical, then the model is satisfactory (- or a little more correct: it is not likely that the model can be improved in the considered class of stationary models).

Test in the cumulated periodogram

This test is especially useful for detecting periodical behaviour of the residuals, which is not described by the model. The cumulated periodogram is a test for white noise in the frequency domain.

For the frequencies $f_i = i/N, i = 0, 1, \cdots, [N/2]$ the periodogram for the residuals is calculated as

$$\hat{I}(f_i) \;=\; \frac{1}{N}\left[\left(\sum_{t=t}^{N} \epsilon_t \cos 2\pi f_i t\right)^2 + \left(\sum_{t=1}^{N} \epsilon_t \sin 2\pi f_i t\right)^2\right]$$

The periodogram is a description in the frequency domain of the variations of $\{\epsilon_t\}$, since I(f_i) is that part of the variations of $\{\epsilon_t\}$ that is concentrated at the frequency f_i.

The cumulated periodogram is then

$$\hat{C}(f_i) \;=\; \left(\sum_{j=i}^{i} \hat{I}(f_j)\right) / \left(\sum_{j=1}^{[N/2]} \hat{I}(f_j)\right) \qquad (52)$$

which is a non-decreasing function defined for the frequencies $f_i = i/N, i = 0, 1, \cdots, [N/2]$.

For white noise the variation is equally distributed on all the frequencies - the name 'white noise' originates, in fact, from the analogy with optics. Hence, the theoretical periodogram is constant. In the case of white noise the total variation is N σ_ϵ^2, and, thus, the theoretical periodogram for white noise is

$$I(f_i) \;=\; 2\sigma_\epsilon^2 \qquad (53)$$

Theoretically, the accumulated periodogram is therefore a straight line from the point (0, 0) to (0.5, 1) - see (52). Hence, if the residuals are actually white noise, we will then expect that the estimated accumulated periodogram is 'close' to that line. The closeness can be judged by a drawing e.g. 95% confidence limits on both sides of the theoretical line. The 95% confidence interval is given by

$$[i/r - 1.36/\sqrt{r}, i/r + 1.36/\sqrt{r}] \tag{54}$$

where r = [N/2] and i = 0, 1,··· , [N/2]. The white noise hypothesis is rejected (on a 5% significance level) if the estimated cumulated periodogram for any i falls outside the above defined confidence limit.

C.2 Simulation and forecasting.

Since applications of heat dynamic models for buildings are, in general, simulation or forecasting, it seems appropriate to illustrate the performance of the estimated models for simulation and forecasting.

In the (dynamic) simulation the estimated system equations and the (measured) independent variables are used to govern the dependent variables. A plot of the simulated variables together with the measured independent variables versus time is then used to illustrate the performance of the model, and its ability to reproduce the dynamic characteristics.

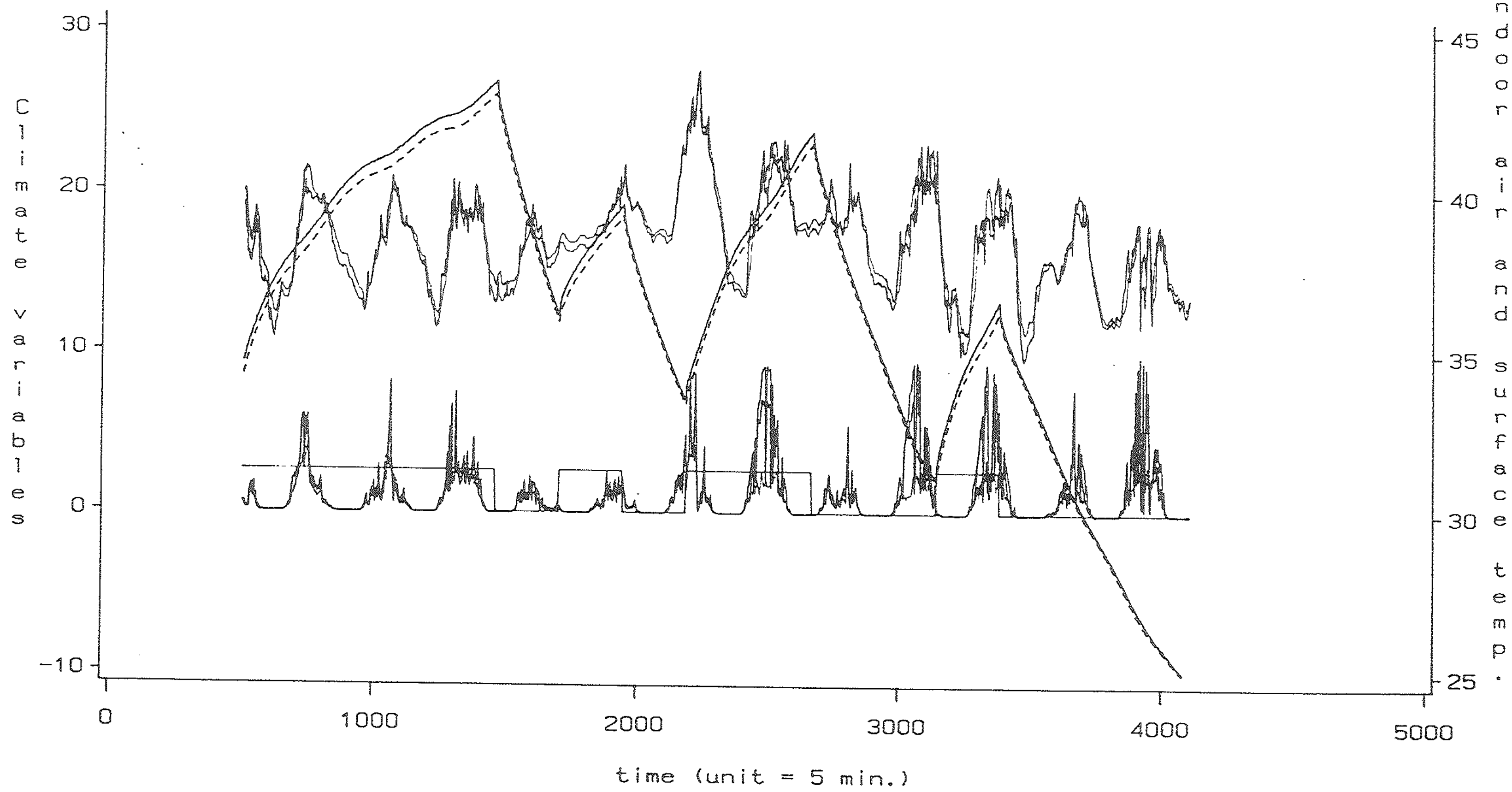
The performance of the model for forecasting, which might be important in control situations, is illustrated by a time plot, which shows the forecasted values (with a given horizon) of the independent variables together with the corresponding measured values. This approach differs from the above described dynamic simulation by the fact that the forecast at time t (of the independent variable at time 't + horizon') utilises the actually measured values of the independent variables up to, and including, time t, whereas the dynamic simulation does not use measured independent variables at all. If the method based on an integration of the differential equations, as described in section B.2, is used, it is possible to select the forecast horizon as wanted, and not necessarily equal to the sample time.

D Some figures

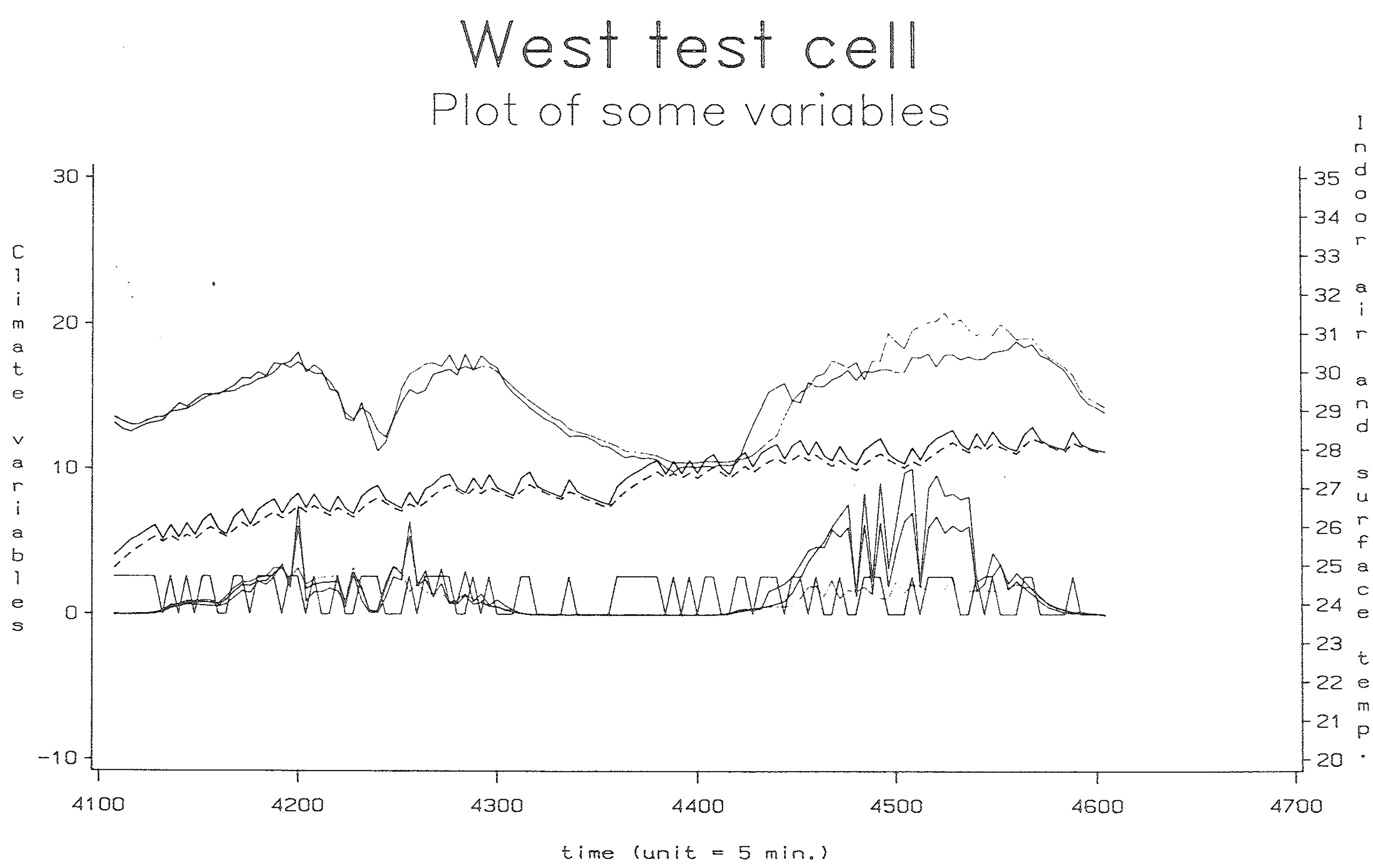
Some figures referred to in the report.

West test cell

Plot of some variables



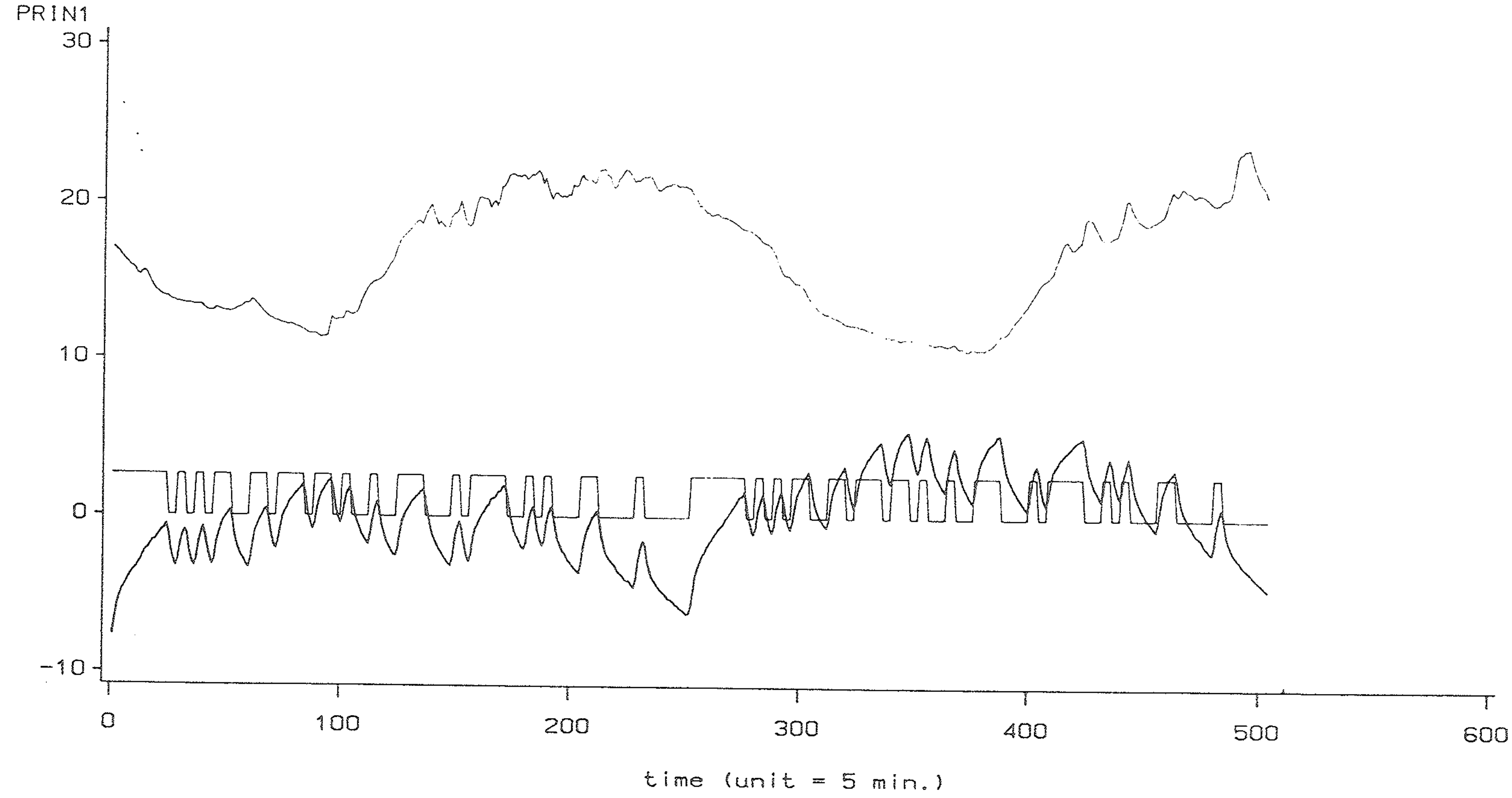
PRBS-signal: T=20 hours and n=4



PRBS-signal: T=20 min and n=6 - second period

West test cell

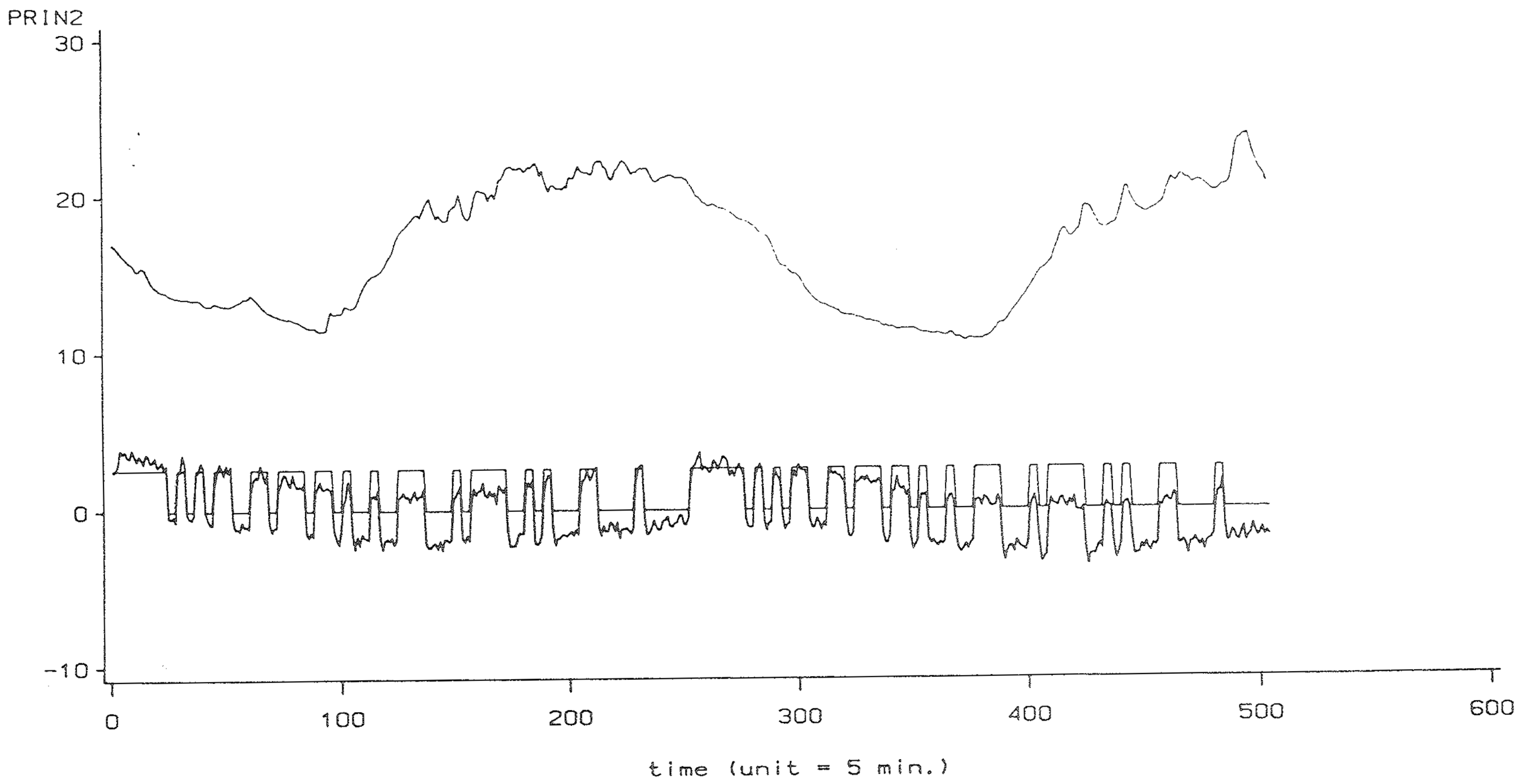
First princip. comp. for air temp.



PRBS-signal: T=20 min and n=6 - first period

West test cell

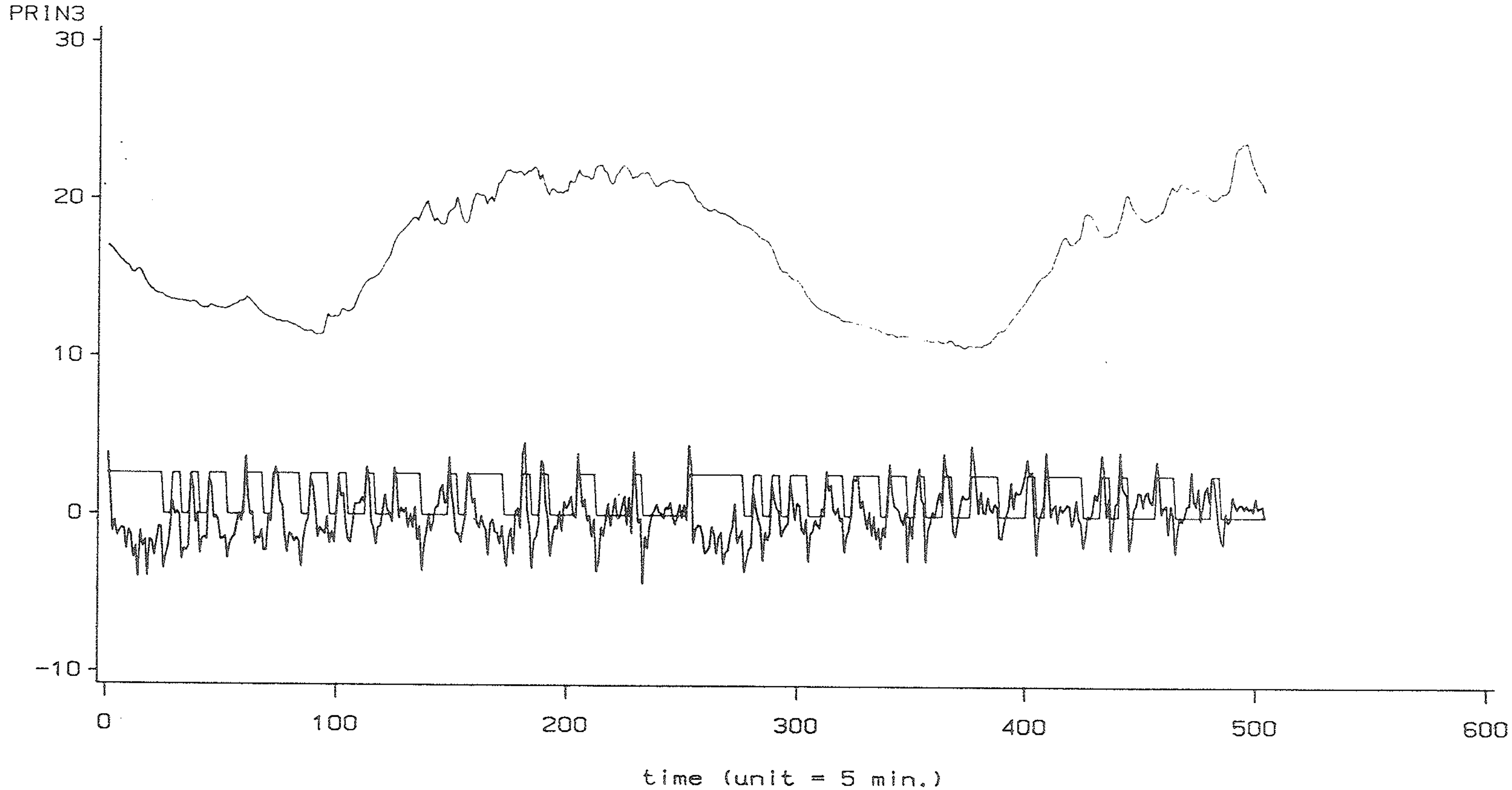
Second princip. comp. for air temp.



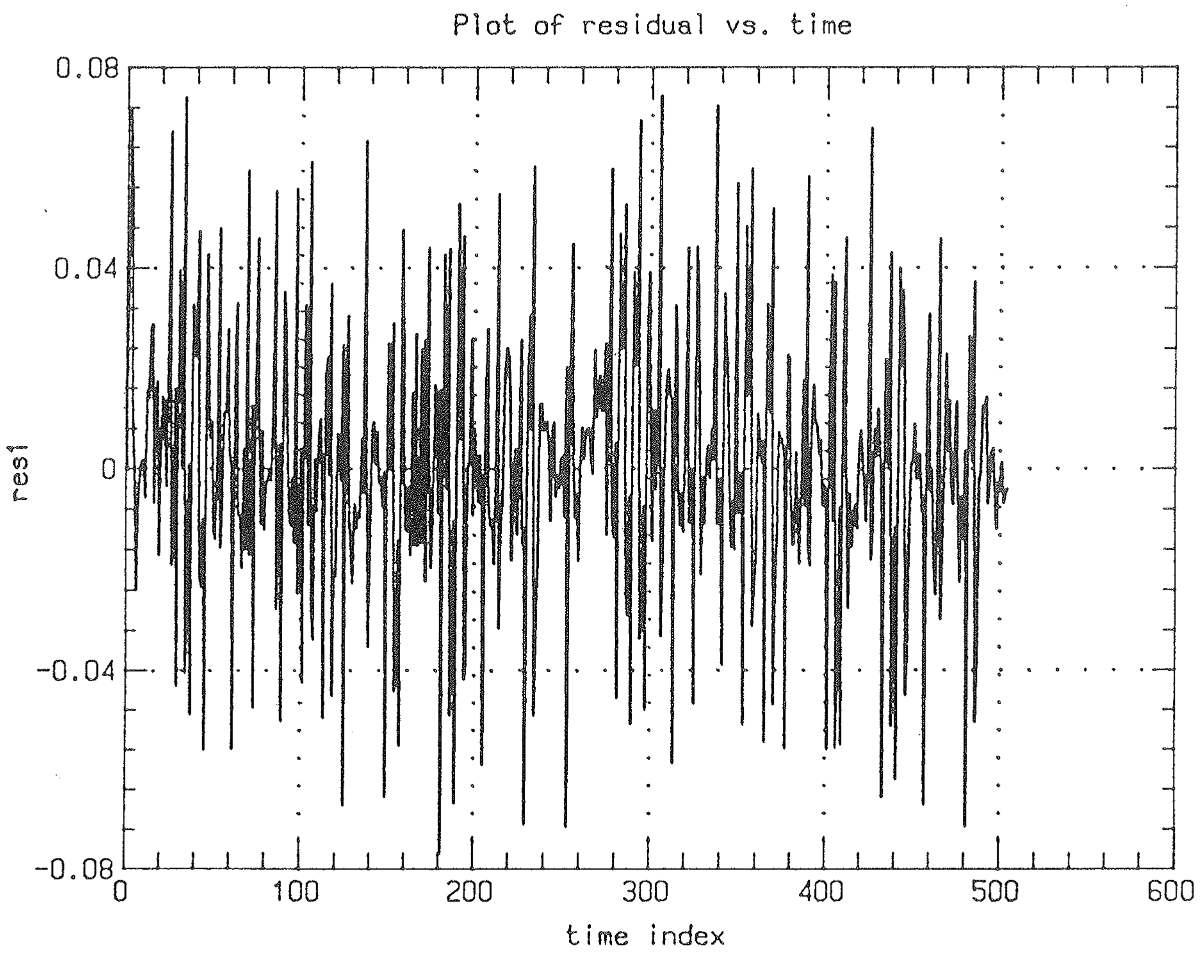
PRBS-signal: T=20 min and n=6 - first period

West test cell

Third princip. comp. for air temp.



PRBS-signal: T=20 min and n=6 - first period


Plot of residual vs. time
0.08
0.04
0
-0.04
-0.08
resl
0
100
200
300
400
500
600
time index

Low-Energy Publications from the Thermal Insulation Laboratory (TIL), Technical University of Denmark

1 Bolet, B., Rasmussen, N.H. & Korsgaard, V.: Ressourcebesparende kassettebyggesystem til lavenergihuse (Resource saving element building system for low-energy houses), in Danish with a summary in English. TIL, Report no. 197, ISSN 0905-1511, December 1988.

2 Byberg, M.R.: Fremtidens lavenergihuse (Low-Energy Houses of the Future), in Danish, XII Nordic VVS Congress in Copenhagen, VVS Vision 82, 2/6-4/6 1982. Off-print. TIL, Report no. 82-26, June 1982.

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