TR-262(1992)

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composite and dynamic aspects

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IUFRO/S5.02 - Timber Engineering Meeting
Bordeaux - France
August 17 - 21, 1992

Building Materials Laboratory
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1. Introduction

Over the last three decades much valuable research has been made on the creep behavior of wood, see recent reviews by Govic (1), and Hoffmeyer (2) for example. Creep of wood as related to orthotropy, dynamic loading, and climatic conditions, however, are still phenomenons full of puzzles. Results are needed very much in these areas in order to provide practice with proper tools for the design of safe wood structures.

The scope of this paper is to put forward an idea which might be worthwhile exploring in creep research to come. The idea is that new understanding on creep behavior of wood does not necessarily mean that we have to drop the great many advantages, subsequently explained, which is offered in normal design practice by the simple Power-Law creep description presented in Equation 1 which is deduced from data obtained in dead-load experiments at equilibrium climatic conditions. Modifications might be introduced considering more real and complicated conditions without changing formally the type of creep description.

A demonstration is made on how to keep the simple Power-Law creep description considering both orthotropy and dynamic loading at equilibrium climatic conditions. Composite theory is used together with ideas and mathematical tools presented in (3). Experimental "impulses" to the work presented come primarily from the empirical evidence subsequently presented in Equation 1 on orthotropic creep and from Pentoney's compilation of data (4) from tests on the damping capacity of wood parallel and perpendicular to grain. Typical results from (4) are shown in Figure 1 indicating that wood has a very fast reacting creep component (relaxation time < 0.01 sec) on top of reactions normally observed.

It is emphasized that the paper is a demonstration which might inspire to more exact analysis. Intentions have been to *explore* possibilities and trends - not to identify mechanisms in details and quantify their responses. In a similar way a subsequent paper (5) deals with the practical ability of the Power-Law creep description to consider climatic variations.

The term "Power-Law" is frequently abbreviated by "PL" in the subsequent text.

2. Wood as a homogeneous PL material

2.1 Power-Law creep

The simple Power-Law creep expression presented in Equation 1 was suggested by the author in (3,6) from experimental data presented in the literature on creep tests at constant load and equilibrium climatic conditions. Time, creep function, and Young's modulus are denoted by t, c = c(t), and E respectively. The so-called relaxation time (or creep doubling time) τ^o considers the joint influence of temperature (T °C), moisture content (U %), and orthotropy. Formally Equation 1 is a rationalized version of the well-known creep expression c(t) = (1 + at^b)/E suggested by Clouser in (7) with constants a and b.

Relaxation time is reduced considerably by climatic variations. Orders of magnitudes are given in (3). Recent experimental evidence is reported in (2,8) on creep versus variable moisture environment. As previously indicated, PL-creep and moisture variations are the subjects of a subsequent paper.

$$c = c(t) = \frac{1}{E}\left(1 + \left(\frac{t}{\tau^o}\right)^b\right) \quad \textit{with} \quad b \approx 1/4 \ \textit{and}$$

$$\tau^o_{15,20}(days) \approx \begin{cases} 10^4 - 10^5 & \textit{tension, compression, bending} \parallel \textit{to grain} \\ 30-300 & \textit{shear} \parallel \textit{to grain} \\ 3-30 & \textit{tension} \perp \textit{to grain} \end{cases} \qquad (1)$$

$$\tau^o = \tau^o_{15,20} * 10^{\frac{15-U}{10} + \frac{20-T}{15}} \qquad (U > 30 = 30)$$

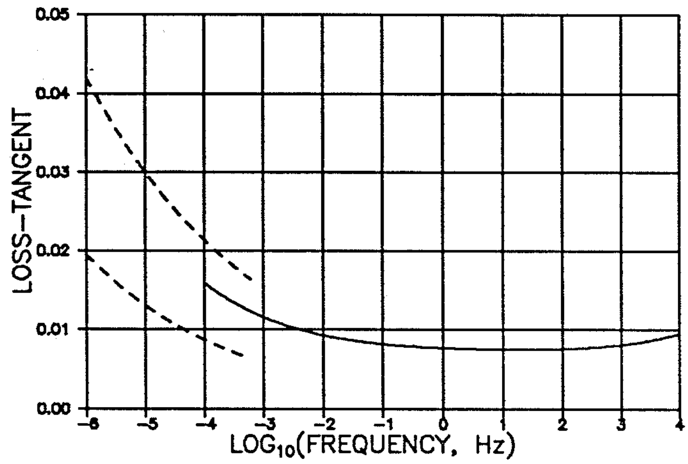

Figure 1. Damping capacity frequency spectrum for wood in bending as presented in (4). Solid line: Average of six domestic woods. Upper and lower dashed lines: Compression wood and normal wood respectively from Hoop Pine wood. (Loss-tangent ≈ log-decrement/π)

2.2 PL-advantages in design

Normally, data from dead-loaded wood at equilibrium climate are very well fitted by the PL-expression in Equation 1. The PL-expression is simple to handle numerically. Time is normalized with respect to relaxation time which means that "master-graph" solutions (general solutions) on wood mechanical behavior can be developed which apply for any equilibrium climatic condition. Specific solutions are obtained from these master-graphs by simple time-scaling with respect to actual relaxation time at given climatic condition. This has recently been explained by the author in (6,9,10).

Such features of a creep description are very much appreciated in practice. It is therefor worthwhile to investigate the possibilities of formally keeping the simple PL-expression for design even if more real and complicated conditions violate the immediate limits of its applicability. This is the prime subject of the present work, as already indicated in Section 1.

2.3 Basic rheological relations

The very basic expressions applying for wood as a homogeneous linear viscoelastic materials are presented in (3) and summarized in Equation 2. The integral expressions relate strain, ε = ε(t) and stress, σ = σ(t), through the creep function, c(t), or the relaxation function, r(t). The Laplace transforms, ε(overlined), σ(overlined), c(overlined), and r(overlined) of strain, stress, creep function, and relaxation function respectively are related through the so-called analogy Young's modulus, E^A (which in the case of Power-Law creep is given in Equation 3). The complex variable is denoted by s. Complex stiffness, E_C, with real part, E_R, and imaginary part, E_I, is derived from the analogy modulus replacing the complex variable with iω where i is imaginary unity and ω is cyclic frequency. The so-called loss tangent, tan(δ) = E_I/E_R, is a measure of energy dissipated when the material is subjected to cyclic load.

The symbol, L^{-1}, means inversion of Laplace transform. The PL-relaxation function presented in Equation 3 is determined analytically in (3). In general, however, creep- and relaxation functions can only be determined numerically. This means by numerical inversion of Laplace transforms or by using the complex stiffness results just obtained in an alternative approach presented in the appendix at the end of the paper.

$$\varepsilon = \int_{t=-\infty}^{t} c(t-\theta)\frac{d\sigma}{d\theta}d\theta \quad or \quad \sigma = \int_{t=-\infty}^{t} r(t-\theta)\frac{d\varepsilon}{d\theta}d\theta \quad or \quad \frac{\overline{\sigma}(s)}{\overline{\varepsilon}(s)} = E^A$$

$$E^A = E^A(s) = \frac{1}{s\overline{c}(s)} = s\overline{r}(s) \qquad \textit{Analogy Young's modulus}$$

$$c(t) = \mathcal{L}^{-1}\left(\frac{1}{sE^A(s)}\right) \; ; \; r(t) = \mathcal{L}^{-1}\left(\frac{E^A(s)}{s}\right) \qquad \textit{Creep- and relax functions} \tag{2}$$

$$E_C = E_C(\omega) = E^A(i\omega) = E_R(\omega) + iE_I(\omega) \qquad \textit{Complex Young's modulus}$$

$$\tan(\delta) = \frac{E_I}{E_R} \qquad \textit{Loss-tangent}$$

$$E^A = \frac{(\tau^o s)^b}{\Gamma(1+b) + (\tau^o s)^b} \qquad \textit{PL-creep} \Rightarrow$$

$$r = r(t) = E\sum_{n=0}^{\infty}\frac{(-X)^n}{\Gamma(1+nb)} \qquad \textit{with} \quad X = \Gamma(1+b)\left(\frac{t}{\tau}\right)^b \tag{3}$$

$$r \approx \frac{1}{c} \qquad \textit{when } b < \frac{1}{3}$$

3. Wood as a PL-composite

Wood is modelled in this paper as a fiber reinforced material. The fibres are parallel and very long. They have a volume concentration of c = 0.9 (vol fibres/total vol). The matrix (lignin mainly) is isotropically viscoelastic and the fibre phase is plane-isotropically viscoelastic. Both components exhibit PL-creep with properties presented in Equation 4 and Table 1. The "static" Young's moduli ($E_{ST} = r(0.001\ \text{day})$) presented in Table 1 are defined as stiffnesses obtained from approximately 1.5 minutes tests.

$$\left.\begin{aligned} c_k &= c_k(t) = \frac{1}{E_k}\left(1 + \left(\frac{t}{\tau_k^o}\right)^{b_k}\right) \\ E_k^A &= E_k^A(s) = \frac{(\tau_k^o s)^{b_k}}{\Gamma(1+b_k) + (\tau_k^o s)^{b_k}} \end{aligned}\right\} \quad k = M,F \tag{4}$$

Table 1. Numbers in parenthesis are $E_{ST} = r(0.001\ day)$.

COMPONENT	VOL-CONC	E∥ MPa	E⊥ MPa	τ days	b
MATRIX (M)	1-c = 0.1	16000 (613)	16000 (613)	10^{-10}	0.20
FIBER (F)	c = 0.9	16000 (15720)	2000 (1965)	10^{4}	0.25

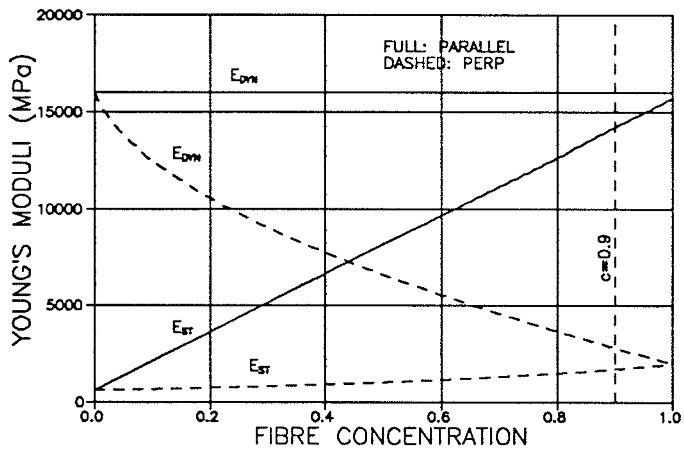

Figure 2. Young's moduli. $E^*_{DYN} = r^*(0)$, $E^*_{ST} = r(0.001\ day)$. Composite (c = 0.9) parallel: (E^*_{DYN}, E^*_{ST}) = (16000,14209) MPa - perp: (2807,1707) MPa.

4. Composite stiffness

Composite stiffnesses are given by Equation 5 where the "parallel" expression is well-known from composite theory and the "perpendicular" expression is derived by the author using a method suggested in (11) for composites reinforced by cylinders perpendicular to stress. A graphical representation of Equation 5 with moduli from Table 1 is presented in Figure 2. It is noticed that "static" stiffness moduli of realistic orders of magnitudes are obtained. The superscript * in Equation 5, Figure 2, and the following text refers to composite quantity.

5. Composite viscoelastic properties

The algorithms of determining the viscoelastic properties of a homogeneous material and the viscoelastic properties of a composite are very similar. The Young's modulus of the former material is replaced by the composite Young's modulus of the latter material. The procedure is summarized in Equation 6. The composite examples considered in subsequent sections are based on the phase properties given in Table 1.

$$E^{*} = E^{*}(E_M, E_F) = \begin{cases} (1-c)E_M + cE_F & \textit{Parallel} \\ E_M \dfrac{E_F - (E_F - E_M)(1-\sqrt{c})\sqrt{c}}{E_F - (E_F - E_M)\sqrt{c}} & \textit{Perpendicular} \end{cases} \tag{5}$$

$$\begin{aligned} & E^{*A} = E^{*A}(s) = E^{*}(E_M^A(s), E_F^A(s)) \\ & E_C^{*} = E_C^{*}(\omega) = E_R^{*} + iE_I^{*} = E^{*A}(i\omega) \Rightarrow \tan\delta^{*} = \frac{E_I^{*}}{E_R^{*}} \\ & c^{*}(t) = \mathcal{L}^{-1}\left(\frac{1}{sE^{*A}(s)}\right) \quad ; \quad r^{*}(t) = \mathcal{L}^{-1}\left(\frac{E^{*A}(s)}{s}\right) \end{aligned} \tag{6}$$

5.1 Complex stiffness and loss tangent

The numerical results obtained for complex moduli and loss tangents related to frequency ($f = \omega/(2\pi)$) are presented in Figures 3 and 4. Quantities at the very high and the very low frequencies are of rather academic interest. In the present context, however, they are needed in order to get a complete picture of the phenomenon considered.

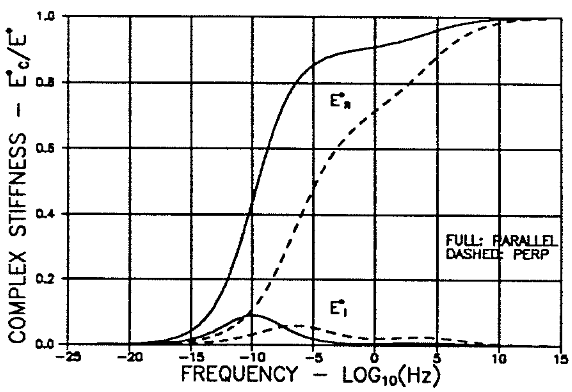

Figure 3. Complex Young's moduli

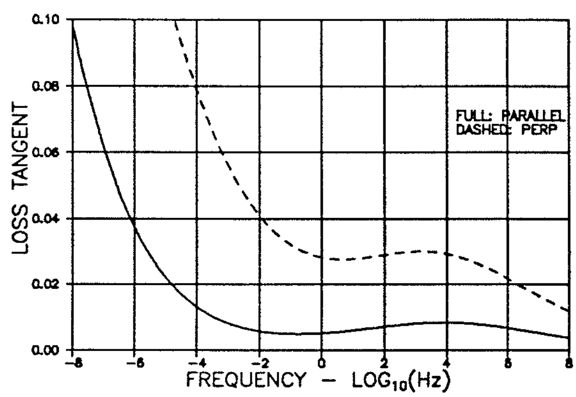

Figure 4. Loss tangents

5.2 Creep and relaxation

As previously mentioned, creep- and relaxation functions can in general only be determined by numerical means. For composites with PL-components with $b_k < 1/3$, however, good approximations can be obtained using the simple procedure described in Equation 7. The creep results of this approach are shown in Figures 5 and 6 normalized with respect to E^*_{DYN} and $E^*_{ST} = r(0.001 \text{ day})$ respectively. A high quality Power-Law approximation ($r^2 > 0.98$) of the data in Figure 6 is obtained by b^* and τ^* as presented in the legend of the figure.

$$c^* = c^*(t) \approx \frac{1}{E^*(1/c_M, 1/c_F)} \quad ; \quad r^* = r^*(t) \approx \frac{1}{c^*} \quad (PL\text{-}creep\ with\ b_k < 1/3) \qquad (7)$$

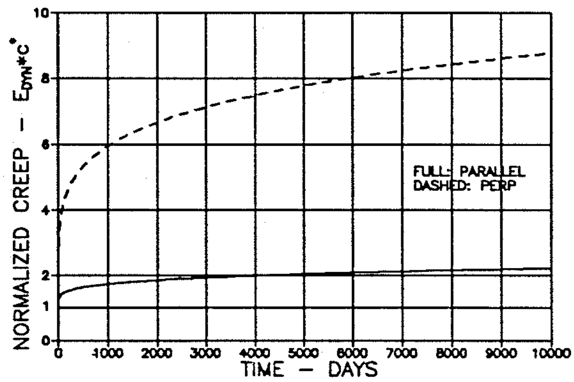

Figure 5. Creep functions based on $(E^*_{DYN}\parallel, E^*_{DYN}\perp) = (16000,2807)$ MPa.

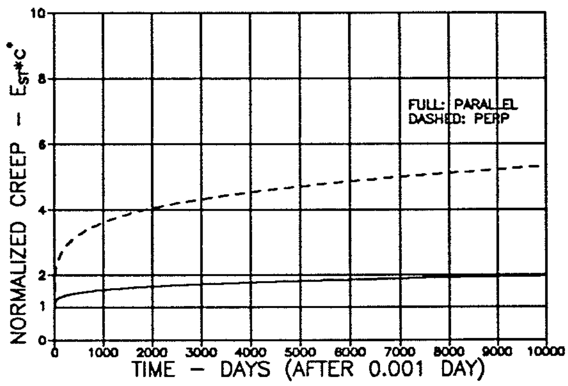

Figure 6. Creep functions based on $(E^*_{ST}\parallel, E^*_{ST}\perp) = (14209,1707)$ MPa. PL-approx: $(b^*,\tau^*)\parallel = (0.278, 17334$ days$)$, $(b^*,\tau^*)\perp = (0.245, 46$ days$)$.

5.3 Further viscoelastic information

Further rheological properties like relaxation and retardation spectra of the composite can be obtained as shown in the appendix at the end of the paper. The relaxation spectrum, $H(\tau)$, shown in Figure 7, for example, is obtained by the first approximation given for $H(\tau)$ in Equation A4.

6. Conclusion

The following assumptions have been made in the paper: Wood is a fiber composite with very long parallel fibres. The fiber volume concentration is 90 %. Fibres are plane-isotropic with stiffness- and creep properties as given in Table 1. The rate of creep is very slow (relaxation time is very high). The matrix (volume concentration 10 %) is isotropic as defined in Table 1. The creep rate is very fast (relaxation time is very low).

From Figures 1 - 4, Equation 1, and further observations made in (4) on energy loss by vibration of wood loaded parallel and perpendicular to grain is concluded that these assumptions are plausible. Creep and loss tangents perpendicular to grain are much greater than what is observed parallel to grain.

The main conclusion is obtained from Figure 6. For most practical purposes we can maintain the simple PL-creep description in wood design, meaning that load does not vary in time faster than time normally used in practice (≈ 1 min) to measure the static Young's modulus which is highly influenced by the fast reacting creep component.

The recognition of a dynamic Young's modulus (testing time 0) and a static Young's modulus (testing time ≈ 1 min) is important. Consequences of ignoring this fact are false and inconsistent b and τ quantities, for example, derived from experimental creep data. This feature has been discussed in more details by the author in (3).

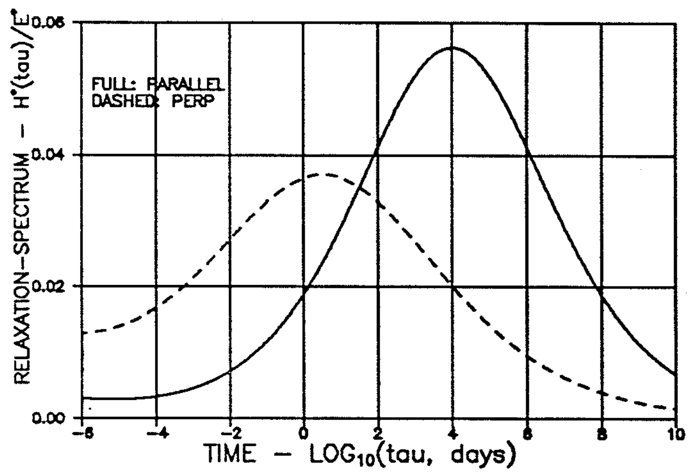

Figure 7. Relaxation spectra.

6.1 Future research

The acceptance of wood being a multi-phase material is very important in future creep research. Creep in wood is a composite reflection of creep in wood components. It is not the behavior of a homogeneous continuum.

When research, for example, is made on the influence of moisture on creep of wood approaches must be used which include studies on the moisture influence on the relaxation behavior of each "mechanism" responsible for the bulk behavior of wood.

Advantages of a composite approach in wood mechanical studies are many. In this paper, for example, creep studies will also reveal information on the influence of moisture content on the static Young's modulus. The composite results obtained may be generalized to include mechanical behavior of woods of any compositions.

The importance of considering the micro-structure in creep research is also emphasized from a lifetime point of view. Cracks propagate in local areas where they meet a minimum of resistance. Lifetime of wood structures, whether they are loaded mechanically (6) or by drying actions (12), depends on where cracks expand. Thus, creep research on local micro-levels (in crack front areas fx) is just as important as research on creep averages on macro levels.

APPENDIX: Auxiliary rheology relations

Complex stiffness, flexibility, and loss tangent

$$E_C = E_C(\omega) = E_R(\omega) + iE_I(\omega) \qquad \textit{Complex stiffness}$$
$$|E_C| = \sqrt{E_R^2 + E_I^2} \qquad \textit{Numerical stiffness}$$
$$J_C = J_C(\omega) = J_R(\omega) - iJ_I(\omega) \qquad \textit{Complex flexibility}$$
$$|J_C| = \sqrt{J_R^2 + J_I^2} \qquad \textit{Numerical flexibility} \tag{A1}$$

$$\tan\delta = \frac{E_I}{E_R} = \frac{J_I}{J_R} \qquad \textit{Loss tangent}$$
$$J_R = \frac{E_R}{|E_C|^2} \quad ; \quad J_I = \frac{E_I}{|E_C|^2} \quad ; \quad |J_C| = \frac{1}{|E_C|} \tag{A2}$$

Relaxation- and creep function

$$r(t) = \frac{2}{\pi}\int_0^\infty \frac{E_R(\omega)}{\omega}\sin(\omega t)\,d\omega \quad ; \quad c(t) = \frac{2}{\pi}\int_0^\infty \frac{J_R(\omega)}{\omega}\sin(\omega t)\,d\omega + \frac{t}{\eta} \tag{A3}$$

Viscosity $\eta = \infty$ *for Power-Law creeping components*

Relaxation- and retardation spectra

$$H(t) \approx -\frac{d\,r(t)}{d\log(t)} \approx -\frac{dE_R(1/t)}{d\log(t)} \approx \frac{2}{\pi}E_I\left(\frac{1}{t}\right)$$
$$L(t) \approx \frac{d\,c(t)}{d\log(t)} - \frac{t}{\eta} \approx \frac{dJ_R(1/t)}{d\log(t)} \approx \frac{2}{\pi}J_I\left(\frac{1}{t}\right) \tag{A4}$$

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