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LIFETIME AND FATIGUE OF WOOD AND OTHER BUILDING MATERIALS

SUBJECTED TO STATIC AND REPEATED LOADS

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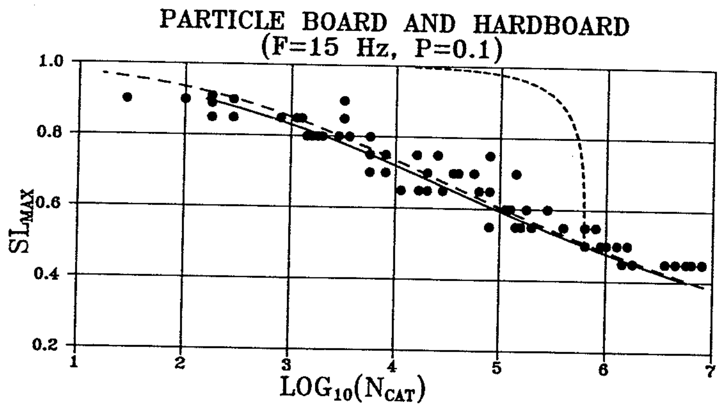

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Abstract: A lifetime theory has previously been developed by the present author which considers lifetime of wood and other viscoelastic building materials like concrete subjected to ramp- and deadload mainly (static fatigue). The theory is based on the concept of building materials behaving as a Damaged Viscoelastic Material and is therefore named the DVM-theory.

In the area of wood mechanical behavior which is especially considered in the paper the theory has frequently been shown to describe successfully the lifetime behavior of both clear wood and structural wood. Also other aspects of wood behavior like tertiary creep, strength reduction due to sustained load or drying and lifetime dependency on wood quality and humidity can be explained by the theory.

The DVM-theory is further developed in the work presented such that cyclic loading of viscoelastic materials can also be considered. Lifetime (real time or number of cycles) is predicted as a function of maximum load, load amplitude, fractional time under maximum load, and load frequency. The analysis includes prediction of residual strength during the process of load cycling - and it is shown how "Master graphs" can be constructed which are valid for any creep behavior (relaxation time), materials quality (grading, strength level), and mode of loading (e.g. tension, compression). It is hereby concluded that number of cycles to failure is a poor design criterion. A simple time criterion is much better.

The theory is successfully compared with data from experiments representing different wood products (and glass fiber reinforced epoxy). The hypothesis is made that the master graphs can be used on wood products in general. Future fatigue research projects are outlined.

Keywords: Fatigue, Lifetime prediction, Cyclic load, Static load, Wood, Viscoelastic materials.

I. INTRODUCTION

Fatigue in engineering material is usually defined as the progressive damage and failure that occurs when the material is subjected to repeated loads of a magnitude smaller than the static strength. This definition originates from early lifetime studies on elastic materials like many metals.

In the present paper which considers both elastic and viscoelastic materials the term, fatigue, has a broader meaning which at high frequency loading or in the absence of creep includes *Elastic Fatigue,* and at low frequency loading includes the so-called *Duration of Load* phenomenon which considers lifetime of viscoelastic materials subjected to constant loads. The term, *Static Fatigue,* is used synonymous with Duration of load effects.

Fatigue reduces the materials strength and lifetime to a degree which has to be considered in design of structures. The frequency of loading is hereby an important parameter when viscoelastic materials are considered. The number of load cycles to failure of wood, for example, may decrease 100 times lowering the frequency from 1 cycle per 10 seconds to 1 cycle per 2 hours. Thus, a number of cycles to failure is not a very good design criterion. A simple time criterion is much better. This fact which is considered in more details in the paper is often overlooked in fatigue research on building materials. One cannot in general accelerate fatigue tests on viscoelastic materials. Results needed for practical design can only be obtained involving theoretical research on the nature and mechanisms of the fatigue phenomenon. It is not realistic to think of experiments running under forecasted live conditions.

These introductory remarks indicate the topics of the paper: Relations are developed which predict fatigue lifetime (real time or number of cycles) and remaining strength of materials. Fatigue solutions for elastic materials subjected to repeated loads and static fatigue solutions for viscoelastic materials are hereby included as limit solutions.

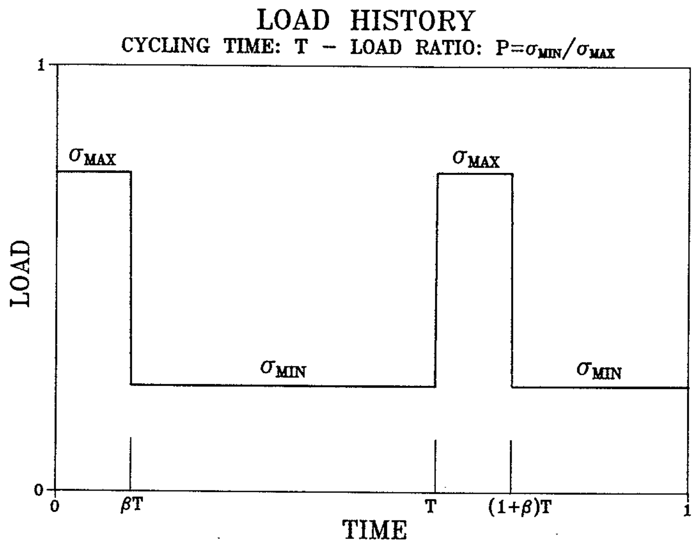

Figure 1. Square wave loading considered in fatigue analysis.

The load history considered in the paper is outlined in Figure 1. The terminology used in the paper to describe load is summarized below. Normalized load quantities are with respect to short time strength, σ_{CR}. Normalized time and load frequency are with respect to the creep relaxation time, τ, introduced in a subsequent section.

Load in general:	σ
Strength:	σ_{CR}
Load level:	$SL = \sigma/\sigma_{CR}$
Minimum load:	σ_{MIN}
Minimum load level	$SL_{MIN} = \sigma_{MIN}/\sigma_{CR}$
Maximum load:	σ_{MAX}
Maximum load level	$SL_{MAX} = \sigma_{MAX}/\sigma_{CR}$
Load ratio :	$p = \sigma_{MIN}/\sigma_{MAX} = SL_{MIN}/SL_{MAX}$
Load range:	$\Delta\sigma = \sigma_{MAX}-\sigma_{MIN} = (1-p)\sigma_{MAX}$
Load level range:	$\Delta SL= SL_{MAX}-SL_{MIN} = (1-p)SL_{MAX}$
Time	t
Cycling time:	T
Frequency	$f = 1/T$
Number of load cycles:	$N = t/T = f*t$
Fractional time under σ_{MAX}:	β
Relaxation time (creep)	τ
Non-dimensional time	$\langle t\rangle = t/\tau$
Non-dimensional frequency:	$\langle f\rangle = \tau*f = \tau/T$

II. BACKGROUND AND MATERIALS CONCEPT

Wood has a natural content of defects and defect nuclei (like knots and inherent cracks) - and wood exhibits time dependent behavior (creep). It is therefore logical to state the hypothesis that wood behaves like a *Damaged Viscoelastic Material* (1). On this basis a lifetime theory (referred to as the *DVM-theory*) has previously been developed by the present author (e.g. 2,3,4) which considers wood (and other building materials) subjected to ramp- and deadload mainly. It was hereby assumed that the bulk substance of the material considered behaves linear-viscoelastically.

The DVM-theory has been shown to describe successfully the lifetime behavior of both clear wood (5,6) and structural wood (7,8). Also other aspects of wood behavior like tertiary creep and stiffness-strength relations, strength reduction due to sustained load or drying, and lifetime dependency of wood quality and humidity can be explained by the theory (9,10,11,12). The important conclusion of the theory that lifetime of materials subjected to similar load levels (load/strength) increases with decreasing strength agree with early observations made by Madsen (13) from experimental lifetime studies on structural wood. The conclusion is strongly supported also by more recent experiments on clear wood (5,11) and plastic (14). It is shown in (11) that the DVM-theory also agree with experimental observations (15,16) that wet wood has a shorter lifetime than dry wood.

The results obtained by the DVM-theory are not bound to a defect system which literally consists of cracks. Dislocations, for example, not visible to the naked eye may also be the defect source. It is known that the effects of a climbing group of edge dislocations are described exactly by the same equations which govern the crack problem. At vital points of the analysis we therefore introduce the non-dimensional quantities, "damage", "damage ratio", and "damage rate" in stead of crack, crack length, and crack velocity respectively. In this way the analysis takes the form of a so-called theory of damage accumulation.

Incidentally, the introduction of non-dimensional quantities (or "levels") has another purpose which is pointed out in (2,4,11): The immediate results obtained in the analysis are based on the simplified materials concept of a plane-stressed isotropic material weakened by an opening mode crack. The non-dimensional results, however, are valid in general for orthotropic materials like wood where cracks are bound to follow the principal directions (17). Strength and relaxation time are the common denominators considering mode of loading (e.g. tension, shear) and other features like stress-strain states and environmental effects respectively defining the crack problem.

Basically this means that the theory presented applies when strength and creep have been determined according to the mode of loading considered. When lifetime of wood in compression is of interest we need information on compression strength and compression creep. When tensile lifetime is of interest we need information on tensile strength and tensile creep. Practically such information is a matter of course. Thus, the theory can be considered practically qualified in general to predict lifetime of wood (and other viscoelastic materials).

If, however, for some theoretical reason the influence of orthotropy on lifetime, for example, is of interest we only have to consider how the "common denominators" mentioned above are influenced by orthotropy. This matter has been considered by the author in (4,17).

The flexibility of the DVM-theory simultaneously to consider (identify) the influence on lifetime of *loading mode*, strength (materials quality, *grading*) and relaxation time (*temperature* and *humidity*) makes the theory very qualified in the field of static fatigue of wood. The theory is easy to apply (see Section VII) and the results agree with experimental evidence.

Other methods to predict static lifetime of viscoelastic materials are given in the literature. Recent reviews on lifetime prediction and wood are given in (18,19). Examples are methods based on empirical crack mechanics (20), viscoelastic energy accumulation (21), viscoelastic crack mechanics (22,23,24,25), damage accumulation (26,27), and chemical kinetics considerations (28). None of these methods, however, offer the flexibility of the DVM-method explained above - a property which is much appreciated when evaluating the mechanical behavior of natural materials like wood. A further advantage of the DVM-method to consider higher loads is mentioned in the subsequent section.

It is hereby justified that the DVM-concept of wood as defined by the author in (2,4) is a realistic basis of developing further the theory such that also fatigue lifetime under cyclic loading can be considered. The materials description given for wood applies in principle for many viscoelastic *building materials*. Therefore, the term "wood" is in many ways considered in this paper as a synonym for a number of building materials like concrete for example for which can be assumed that the bulk substance behaves *linear-viscoelastically*.

III. BASICS OF ANALYSIS

The *elastic-viscoelastic analogy* (29,30) is an important tool used in the present article to determine damage influence on the behavior of viscoelastic materials. The analogy can be formulated as follows: Similar solutions apply to a linear-elastic problem and to its exact but linear-viscoelastic duplicate. The only difference is that coefficients of elasticity in the elastic solution are represented by their viscoelastic counterparts (operators) in the viscoelastic solution.

This means, for example, that elastic displacements, u_{EL}, and viscoelastic displacements, u_{VISC}, are related by,

$$u_{VISC} = \int_{-\infty}^{t} C(t-\theta)\frac{du_{EL}}{d\theta}d\theta \qquad (1)$$

where t is time and C(t) is the normalized creep function defined by C(t) = E*c(t) where E and c(t) denote Young's modulus and the (conventional) creep function respectively.

Wood and a number of other important building materials (31,32) exhibit Power-Law creep which is generally described as follows in the literature,

$$c(t) = \frac{1}{E}\,(1 + a^{*}(t)^{b}) \quad ;\ (constants:\ a,\ b)$$

It has been shown by the present author (33) that this way of expressing creep is very unfortunate. No proper physical meaning can be given to the parameter a. A simple re-writing, however, was suggested in (33) which completely changes this feature. We rephrase: *Power law creep* means that viscoelasticity is defined by a creep function of the following type where τ and b denote *relaxation time* (or creep doubling time) and *creep power* respectively,

$$c(t) = \frac{1}{E}\,(1 + (\frac{t}{\tau})^{b}) \tag{2}$$

A complete rheological analysis of this expression has been made in (33) and the following suggestions were made from comparing it with experimental creep data from the literature: Relaxation time is the parameter most sensitive to changes in temperature and humidity. The creep power, b, is practically independent of climatic conditions. The consequences of these observations are obvious: Standard lifetime solutions can be developed based on non-dimensional time, $\langle t \rangle = t/\tau$.

The lifetime theory presented in this article is based primarily on Power law creep as described by Equation 2. In principle, however, the theory applies for any viscoelastic material. The equations presented in the paper can without greater effort be "translated" for this purpose. The mathematical complexity involved solving the expressions is, however, heavily increased. A more easy way to analyze numerically the fatigue behavior of materials not exhibiting Power-Law creep is to subdivide time in intervals where creep is adapted to follow a Power Law description. Such a method is demonstrated in (34) to predict fatigue behavior of concrete and other aging viscoelastic materials.

The damaged materials model applied is the *Dugdale crack model* considered in the following section. The expressions presented apply when load is smaller than practically 45 % of the un-cracked materials strength. In most cases this restriction on the fatigue theory subsequently developed is of no practical importance.

The load limitation of approximately 45 % which applies for any of the viscoelastic crack theories referred to in Section II is not required in the original version of the DVM-theory (2,4). It is therefore quite possible to develop further this theory to consider also repeated loads at higher load levels. In the present paper, however, we desist from doing this. The more heavy mathematics needed is generally out of proportion to what is practically gained considering the present quality of most commercial wood products.

The single crack model chosen is also a matter of avoiding disproportionate mathematical efforts. In principle it is possible to examine fatigue on the basis of a multi-crack model (11,17). However, the present knowledge of defect distribution in wood cannot justify such an approach. It is also questionable if more detailed studies would improve significantly the results of practical interest. Two features justify the use of a single-crack model: Failure in wood is very often observed to be released by single major defects. The lifetime studies previously referred to confirm that realistic lifetime predictions can be obtained on the basis of a single-crack materials concept.

Dugdale Crack

The Dugdale model of a cracked material (35) is shown in Figure 2. Load, σ, is applied at infinity perpendicular to the crack plane. The uniformly distributed cohesive stress, σ_l, at the crack front may be thought of as being the un-cracked materials strength. (Material pulled out into the crack front zone is considered to be stiff and perfectly plastic). The width of the crack front zone, R, and the crack front opening, δ, are expressed as follows at plane stress

$$\frac{R}{l} = \frac{\pi^2}{8}\left(\frac{\sigma}{\sigma_l}\right)^2 \quad ; \quad \frac{\delta}{l} = \frac{\pi\sigma^2}{E\sigma_l} \tag{3}$$

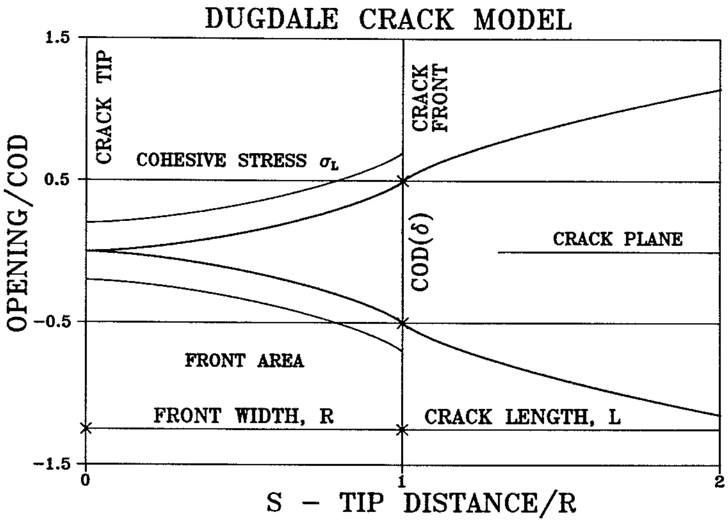

Figure 2. Dugdale crack model. Terminology used in the paper

Crack opening in general, v, and perpendicular to crack plane stress at the crack front are given by the following approximations presented in (17) introducing the non-dimensional coordinate, s = distance from crack tip divided by R,

$$v/\delta \approx \begin{cases} 0 & \text{when } s < 0 \\ s^2 & \text{when } 0 \le s \le 1 \\ 2\sqrt{s} - 1/s & \text{when } s > 1 \end{cases} \tag{4}$$

$$\sigma_Y/\sigma_l \approx \begin{cases} 1 - (2/\pi)\arctan\sqrt{-s} & \text{when } s < 0 \\ 1 & \text{when } 0 \le s \le 1 \\ 0 & \text{when } 1 < s \end{cases} \tag{5}$$

Two other parameters well-known from the crack mechanics literature (e.g. 36) are related to the crack front opening as shown in Equation 6 below. Γ is *strain energy release rate* and K is *stress intensity factor*.

$$\Gamma = \delta\sigma_l = \frac{\pi\sigma^2 l}{E} \; ; \; K = \sqrt{E\Gamma} = \sigma\sqrt{\pi l} \tag{6}$$

Strength, σ_{cr}, of a cracked material is predicted by Equation 3 introducing the following failure criterion: A crack becomes unstable when the crack front opening, δ, approaches a critical value, δ_{cr}. This criterion, sometimes named the critical COD (Crack Opening Displacement) criterion, can also be formulated with respect to critical strain energy release rate, $\Gamma_{CR} = \sigma_l\delta_{CR}$, or critical stress intensity factor, $K_{CR} = \sqrt{(E\Gamma_{CR})}$. Thus, strength of a material containing a crack of diameter $2l_o$ can be expressed as follows in three different ways,

$$\sigma_{CR} = \left(\frac{E\sigma_l\delta_{CR}}{\pi l_o}\right)^{1/2} = \left(\frac{E\Gamma_{CR}}{\pi l_o}\right)^{1/2} = \frac{K_{CR}}{(\pi l_o)^{1/2}} \tag{7}$$

all predicting a strength equal to the well-known Griffith load capacity (37).

Non-dimensional expressions: It is very convenient in crack analysis to normalize strength (σ_{CR}), load (σ), and damage size (l) with respect to theoretical strength (σ_l), strength (σ_{CR}), and initial crack size (l_o) respectively. In the order mentioned *strength level, load level,* and *damage ratio* respectively are defined by,

$$FL = \sigma_{CR}/\sigma_l \quad ; \quad SL = \sigma/\sigma_{CR} \quad ; \quad k = l/l_o \tag{8}$$

The normalized version of Equation 3 yields

$$R/l = (\pi^2/8)(FL*SL)^2 \quad ; \quad \delta/\delta_{CR} = k*SL^2 \tag{9}$$

The following expressions derived from the "δ terms" in Equations 3 and 7 are of interest in the analysis of crack propagation,

$$k_{CR} = 1/SL^2 \tag{10}$$

$$S_R = \sigma_{CR}(k)/\sigma_{CR} = 1/\sqrt{k} \tag{11}$$

The *critical damage ratio,* k_{CR}, is damage ratio at which the material considered will fail when exposed to a load level, $SL \leq 1$. Residual strength(level), S_R, is strength remaining during crack propagation. It is, strength, $\sigma_{CR}(k)$, at damage ratio k relative to strength, σ_{CR}, at initial damage ratio, $k_o = 1$.

Curiosum: The consequences of introducing another cohesive stress distribution in the Dugdale model than the one presently applied can be analyzed using the crack theory of Barenblatt (38). Such an analysis has been made in (18,39,4) assuming a "concentrated" and a linear cohesive stress variation respectively. It was concluded that the changes relative to the simple Dugdale solutions are too small to justify the more complicated model - especially when considering that we actually know nothing about the real stress distribution.

Crack closure: Until now it has implicitly been assumed that load (and consequently crack opening) is constant or increasing such that a uniformly distributed stress of cohesion agrees with the assumption of rigid-plastic materials behavior in the crack front zone. At decreasing load where the crack tries to return to its starting configuration this assumption cannot be maintained. Rice (40) modified the Dugdale solution also to consider a sudden reduction in load, $\Omega\sigma = \sigma_{MAX} - \sigma_{MIN}$, by superimposing (on the max load situation) a separate Dugdale load range solution where load is $-\Omega\sigma$ and cohesive stress is $-2\sigma_1$. A compressive theoretical strength, $\sigma_{C1} = \sigma_1$, is hereby assumed.

The Rice solution is illustrated in Figure 3. The reference quantities referred to are from Equation 3 with $\sigma = \sigma_{MAX}$,

$$\frac{R_{MAX}}{1} = \frac{\pi^2}{8}\left(\frac{\sigma_{MAX}}{\sigma_1}\right)^2 \quad ; \quad \frac{\delta_{MAX}}{1} = \frac{\pi\sigma_{MAX}^2}{E\sigma_1} \tag{12}$$

Crack opening is invariant at $s \le s_o$ where

$$1 - s_o = (1-p)^2/4 \tag{13}$$

Stress of cohesion is $-\sigma_1$ for $s \ge s_o$ and somewhere between σ_1 and $-\sigma_1$ at $s < s_o$. The displacement range, $\cap\delta = \delta_{MAX} - \delta_{MIN}$, is given by

$$\cap\delta/\delta_{MAX} = (1-p)^2/2 \tag{14}$$

The minimum crack front deformation, δ_{MIN}, is given by

$$\delta_{MIN}/\delta_{MAX} = 1 - (1-p)^2/2 \quad ; \quad (\delta_{MIN,REL}/\delta_{MAX} = p|p|) \tag{15}$$

The latter term in Equation 15 is the relaxed minimum δ if no crack closure effects were present. It is noticed that a constant reference width of the crack front zone (R_{MAX}) can only be maintained when $s_o \ge 0$, meaning $p > p_{MIN}$ where

$$p_{MIN} = -1 \quad ; \quad (\cap\delta/\delta_{MAX} = 2,\ s_o = 0) \tag{16}$$

$\cap\delta > \delta_{MAX}$ requires a sliding mode crack with symmetrical (with respect to $p = 0$) crack closure effects. Normally we expect a maximum $\cap\delta = \delta_{MAX}$ which according to Equation 14 is obtained at the following critical load ratio,

$$p_{CR} = 1 - \sqrt{2} = -0.4 \quad ; \quad (\cap\delta/\delta_{MAX} = 1,\ s_o = 1/2) \tag{17}$$

The following relations obtained by Equations 13 and 14 will be used in the subsequent analysis,

$$Z = (1-s_o)\frac{\cap\delta}{\delta_{MAX}} \quad ; \quad (\text{Rice: } Z = (1-p)^4/8) \tag{18}$$

$$1-s_o = \sqrt{Z/2} \quad ; \quad \cap_o = \cap\delta/\delta_{MAX} = \sqrt{2Z} \tag{19}$$

Rice's analysis disregards any contact of opposite crack surfaces at $s > 1$. Practically this is an over-simplification. The results, however, present valuable qualitative information which must be considered in fatigue analysis. We continue accepting the existence of a coordinate, s_o, below which crack opening is invariant. An area of alternating deflection is defined at $s > s_o$ where stress of cohesion alternates between σ_1 and $-\sigma_1$. The actual quantities, however, or combination of quantities, $1-s_o$ and $\cap\delta/\delta_{MAX}$, are left to be deduced from experiments.

The Rice analysis can easily be modified to consider a compressive theoretical strength, σ_{CL}, different from $-\sigma_L$. We desist from introducing such a modification as it will not change qualitatively the results already presented. The Rice crack closure analysis has been improved by Budiansky and Hutchinson (41) to consider some crack surface contact outside the crack front area. The results obtained in

(41) have been used by the present author to develop a fatigue theory for elastic and viscoelastic materials in (17). It was hereby concluded that such a theory can still not be developed fully satisfactory without substantial experimental information on $1-s_o$ and $\Omega\delta/\delta_{MAX}$.

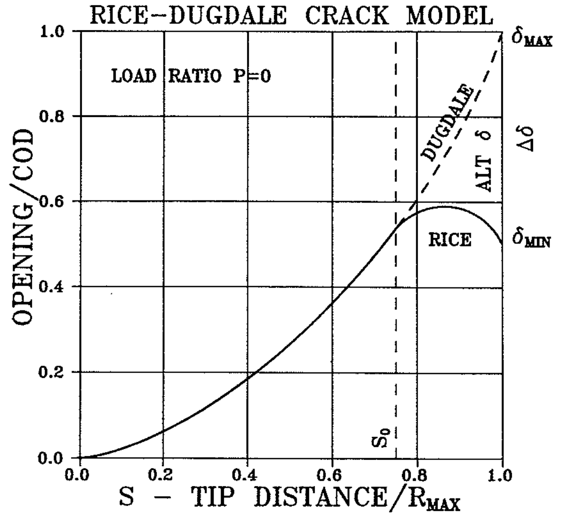

Figure 3. Crack opening predicted by the Rice-Dugdale crack closure model. Example: p = 0.

Failure criterion: The failure criterion previously applied to predict strength is generalized as follows also to apply when loads are alternating: Failure appears when total energy dissipation at the crack front approaches the critical energy release rate previously introduced. This means,

$$\Gamma = (\delta_{MAX}+\Sigma|\Omega\delta|)\sigma_1 = \delta_{MAX}\sigma_1(1+\Sigma(|\Omega\delta|/\delta_{MAX})) \rightarrow \Gamma_{CR} = \delta_{CR}\sigma_1 \qquad (20)$$

where Γ_{CR} and $\delta_{MAX}\sigma_1$ are related as follows by Equation 9

$$\frac{\Gamma_{CR}}{\sigma_1\delta_{MAX}} = \frac{1}{kSL_{MAX}^2} \qquad (21)$$

We recall that the Dugdale expressions presented apply when load is smaller than practically 45 % of the materials un-cracked strength. In the present terminology this means strength level, FL < 0.45.

IV. ELASTIC FATIGUE

The lifetime of an elastic material subjected to varying loads can be determined by the following procedure: We consider a location, X, defined by the immediate crack tip. The propagating crack will open up this location such that coherence fails after a period of time, Ω, which can be determined considering the energy dissipation involved in the opening process. The associate distance travelled by the crack is equal to the immediate crack front width, R_{MAX} (Eq. 12), meaning that crack (and damage) velocity is given by

$$\frac{dl}{dt} = \frac{R_{MAX}}{\Omega} \Rightarrow \frac{dk}{dN} = \frac{(\pi^2/8)(FL*SL_{MAX})^2k}{\Omega/T} \qquad (22)$$

from which lifetime can be obtained by integration between the initial damage ratio, $k = 1$, and the critical quantity, $k = k_{CR}$, given by Equation 10 (with $SL = SL_{MAX}$).

Assuming a locally constant rate of propagation position X will experience an opening history which can be subdivided in two parts: 1) a parabolically increasing opening history starting with $\delta = 0$ at time 0 and finishing with δ_{MAX} at time Ω (so far unknown) where the crack front is at position X - and 2) an alternating opening history starting with 0 amplitude at time $(1-s_o)\Omega$ and approaching an amplitude of $\Delta\delta$ at time Ω. The amplitudes are approximated to vary linearly between $(1-s_o)\Omega$ and Ω.

The energy dissipated at position X in this process is given by

$$\Gamma = \sigma_1 \delta_{MAX} [1 + \frac{\Delta\delta}{\delta_{MAX}} (1-s_o) \frac{\Omega}{T}] \tag{23}$$

where the terms in order correspond to the opening histories 1 and 2 previously considered. (Time under alternating opening is $(1-s_o)\Omega$. Each cycling time, T, contributes to dissipation with $2\Delta\delta\sigma_1$. Average amplitude is $\Delta\delta/2$).

As the crack front of a propagating crack is continuously failing we may determine Ω combining Equation 21 and Equation 23 with $\Gamma = \Gamma_{CR}$. We get

$$\frac{1 - kSL_{MAX}^2}{kSL_{MAX}^2} = \frac{\Omega}{T} \frac{\Delta\delta}{\delta_{MAX}} (1 - s_o) \tag{24}$$

by which crack velocity is determined as follows introducing Ω/T from Equation 22

$$\frac{dk}{dN} = \frac{\pi^2 FL^2}{8} \frac{(kSL_{MAX}^2)^2}{1 - kSL_{MAX}^2} \frac{\Delta\delta}{\delta_{MAX}} (1 - s_o) \tag{25}$$

or

$$\frac{dk}{dN} = \frac{\pi^2 FL^2}{8} \frac{(\Delta K/K_{CR})^4}{1 - (K_{MAX}/K_{CR})^2} \frac{\Delta\delta}{\delta_{MAX}} \frac{1 - s_o}{(1 - p)^4} \tag{26}$$

where stress intensity factors have been introduced according to the following list

Stress intensity factor (SIF): $K = \sigma\sqrt{(\pi l)}$
Critical SIF: $K_{CR} = \sigma_{CR}\sqrt{(\pi l_o)}$
Normalized SIF: $K/K_{CR} = SL\sqrt{k}$
Maximum SIF $K_{MAX} = \sigma_{MAX}\sqrt{(\pi l)}$
Minimum SIF $K_{MIN} = \sigma_{MIN}\sqrt{(\pi l)}$
SIF range: $\Delta K = K_{MAX}-K_{MIN} = \Delta\sigma\sqrt{(\pi l)} = (1-p)K_{MAX}$
Normalized SIF range: $\Delta K/K_{CR} = \Delta SL\sqrt{k} = (1-p)SL_{MAX}\sqrt{k}$

The empirically based, so-called *Paris-Erdogan-Elber Law* (42,43)

$$dl/dN \approx A^*(U^*\Delta K)^M \tag{27}$$

is very often met in the experimental literature on fatigue of materials (e.g. 44,45) - especially metals - where it frequently produces an excellent data fit. "A" is a materials "constant" which, however, to some degree is dependent on load level. The efficiency factor, "U", is also materials dependent. Elber (43) suggested $U \approx 0.5 + 0.4p$ on the basis of data from experiments ($p \geq -0.1$) on aluminum. "M" is a constant which for a number of metals has a magnitude close to 4. Many materials, however, exhibit M-values significantly different from 4. $M \approx 2.3$ was observed for a special steel in (46). The main rule seems to be $M \geq 4$ (47). For example, $M \approx 5$ and $M \approx 8.5$ were obtained for some polymers (48) and wood respectively (49). The hypothesis is suggested by the present author that M is texture dependent such that M increases with increasing roughness of the failure surface.

The quality of the Paris-Erdogan-Elber Law to fit data (where σ_{MAX} is not too close to σ_{CR}) is generally so convincing that any theory developed in the area of fatigue must be able of "predicting" it. Thus, by comparing Equations 26 and 27 we suggest

$$Z = \frac{\cap\delta}{\delta_{MAX}} (1 - s_o) = \frac{C}{8} (1 - p)^4 U^M \left(\frac{\cap K}{K_{CR}}\right)^{M-4} \tag{28a}$$

$$= \frac{C}{8} [U(1 - p)]^M (kSL_{MAX}{}^2)^{M/2-2} \tag{28b}$$

where the *damage rate constant, C,* and the *damage rate power, M,* are considered to be material dependent fatigue parameters which can be determined from experiments as shown in Section VIII.

Crack velocity can now be expressed by the following equation which complies well with the Paris-Erdogan-Elber expression,

$$\frac{dk}{dN} = \frac{\pi^2 C}{64} FL^2 \frac{(U^*\cap K/K_{CR})^M}{1 - (K_{MAX}/K_{CR})^2} \tag{29a}$$

$$= \frac{\pi^2 C}{64} FL^2 \frac{(U^*\cap SL)^M}{1 - kSL_{MAX}{}^2} k^{M/2} \tag{29b}$$

For the special case, $U \equiv 1$ and $(M,C) = (4,1)$ Equation 29a agrees with an expression obtained by Weertman (50) on the basis of dislocation theory. Rice (40) used his own crack closure theory to derive $dl/dN = \text{constant}^*(\cap K)^4$ which also compares positively with Equation 29a ($M = 4$).

Lifetime: Equations 29 can be solved analytically integrating between $k = 1$ and $k = k_{CR}$. The number of cycles to failure, N_{CAT}, is given by

$$G^*N_{CAT} = SL_{MAX}{}^{-2}\left[\frac{1-SL_{MAX}{}^{M-2}}{(M-2)SL_{MAX}{}^{M-2}} - \frac{1-SL_{MAX}{}^{M-4}}{(M-4)SL_{MAX}{}^{M-4}}\right] \tag{30}$$

where

$$G = C^*FL^2{}^*[U(1-p)]^M/13 \tag{31}$$

When M approaches 4 we get,

$$G^*N_{CAT} = SL_{MAX}{}^{-2}[SL_{MAX}{}^{-2} - 1 + 2^*\log_E(SL_{MAX})] \quad ;(M=4) \tag{32}$$

It is noticed that lifetime can be presented graphically by "master graphs" only considering SL_{MAX} and M.

Efficiency factor: The factor, $1 \geq U \geq 0$, is a measure of the efficiency of the crack closure mechanism. The δ-range, $\Omega\delta$, decreases with decreasing U which decreases crack velocity (increases N_{CAT}). We suggest the following efficiency factor,

$$U = 0.5*\begin{cases} MAX[1+p,\ 1] & \text{when } p \geq p_{CR} \\ MIN[1,\ (1-p_{CR})/(1-p)] & \text{when } p < p_{CR} \end{cases} \tag{33}$$

where $p_{CR} \approx -0.5$ is a critical load ratio below which lifetime is predicted to be independent of p (see Equations 30 and 31). The critical load ratio corresponds to p_{CR} in Equation 17 based on the Rice analysis. The efficiency factor suggested agrees well with the factor previously referred to given by Elber (43).

Residual strength: The materials strength decreases during the process of fatigue. The (relative) strength, S_R, still remaining after a certain number of load cycles, N, where damage ratio has become $k = k(N)$ is described very easily by Equation 11. This means, $S_R = (k(N))^{-1/2}$ which leads to the following relation integrating Equation 29 between $k = 1$ and $k = S_R^{-2}$. We get

$$G*N = SL_{MAX}^{-2}\left[\frac{1 - S_R^{M-2}}{(M-2)SL_{MAX}^{M-2}} - \frac{1 - S_R^{M-4}}{(M-4)SL_{MAX}^{M-4}}\right] \tag{34}$$

$$G*N = SL_{MAX}^{-2}[(1-S_R^{2})SL_{MAX}^{-2} + 2*\log_E(S_R)] \qquad ;(M=4) \tag{35}$$

V. FATIGUE CREEP

Lifetime of a viscoelastic material can be determined following a similar procedure as previously used in the analysis of the elastic fatigue phenomenon. Modifications of the method, however, have to be introduced which consider creep and how creep is influenced by the crack closure phenomenon. In the present section we will use the elastic-viscoelastic analogy to establish such modifications by looking at the following features: How does creep influence the opening of a position being penetrated by a moving crack at constant load - and how does crack closure influence creep in the crack front area of a resting crack. The general problem, i.e. the opening of a position being penetrated by a moving crack in a material subjected to repeated loads is then solved by combining the two separate solutions previously obtained.

It is noticed that creep in the present context (in a crack front area) refers to opening and/or sliding mode viscoelasticity, meaning that relaxation refers to perpendicular to grain tensile creep and parallel to grain shear creep more than it refers to parallel to grain creep like bending creep f.ex. This subject is discussed in further details in Section VIII.

Moving Crack Creep

As in Section IV we consider a position X defined by the immediate crack tip location in an elastic material subjected to a constant load. The crack is extended at constant speed such that X opens up from $u = 0$ at $t = 0$ to $u = \delta_{EL}$ at time $t = \Omega$. As the shape of the crack is parabolic (see Equation 4) the total opening history which X experiences is $u = \delta_{EL}(t/\Omega)^2$. The corresponding opening history, u_{VISC} of the crack in the duplicate viscoelastic system is determined by Equation 1. Assuming Power Law creep we get the following result relating δ_{EL} and $\delta_{VISC} = u_{VISC}(\Omega)$ - and defining the *moving crack creep function*, C_M

$$C_M(\Omega) = \frac{\delta_{VISC}}{\delta_{EL}} = 1 + (\frac{\Omega}{q\tau})^b = C(\Omega/q) \tag{36}$$

The moving crack creep function is observed to be a simple time shifted version of the basic creep function, C. The shift factor, q, is given by

$$q = [(1+b)(2+b)/2]^{1/b} \tag{37}$$

Resting Crack Closure Creep

The phenomenon of elastic crack closure with relaxing coherent stresses has been studied theoretically by the present author in (17b) and the Appendix of this paper. An important result is that approximately congruent relaxed crack closure profiles can be produced with relaxed coherent stresses not violating the materials theoretical strength (σ_1). (Congruent means that the total profile relaxes as the crack front opening, δ_{MIN}). This observation and the elastic-viscoelastic analogy are used in the following two-step procedure to establish and quantify an opening history of a crack in a viscoelastic material subjected to repeated loads as shown in Figure 1.

Step 1: We model the opening history by "generalizing" the history applying to the "elastic" Dugdale-Rice crack shown in Figure 3: ▪ Looking at the first load cycle, a creeping Dugdale opening $v = v_{MAX}(t)$ is developed at max load. At time βT load drops to minimum producing a "frozen" opening for $s < s_o$ ($v \equiv v_{MAX}(\beta T)$) and an opening at $s > s_o$ which differs increasingly from $v_{MAX}(\beta T)$ as $s \to 1$ such that a crack front opening of $\delta_{MIN}(\beta T) = \delta_{MAX}(\beta T) - \Omega\delta(\beta T)$ is obtained at $s = 1$. In the period of min load, βT - T, the crack profile is generally locked in position $v \equiv v(\beta T^+)$ (an exception is explained in the subsequent text). ▪ Starting the next cycle load jumps to maximum at $t = T$ and crack opening jumps to $v_{MAX}(T)$ from where it creeps to $v_{MAX}((1+\beta)T)$ just before load drops to minimum. In the period of min load, $(1+\beta)T$ - 2T the crack opening is locked in a similar way as in the first cycle, i.e. $v \equiv v((1+\beta)T^+)$. And so on ...

We define a creep-dependent δ-range as follows where $\Omega\delta$ is the elastic δ-range. The *δ-range creep factor*, W, is considered to be a materials property. W = 0 defines a δ-range independent of creep.

$$\Omega\delta(t) = \Omega\delta[1 + W^*(1 - \frac{\delta_{MAX}(0)}{\delta_{MAX}(t)})] \tag{38}$$

Two features are un-known in the opening history outlined: We do not know in details how creep developes and how big are the δ-jumps associated with loads going from minimum to maximum.

Step 2: We now proceed using the elastic-viscoelastic analogy expressed by Equation 1 considering a *fictitious* elastic duplicate from which is required that δ is invariably δ_{MAX} when load is maximum. It is noticed that the missing quantitative information previously mentioned on creep and δ-jumps at load increases are revealed by this supplementing boundery condition. An example of corresponding displacement histories produced by Equation 1 is shown in Figure 4 which is based on the Rice crack closure results given by Equations 14 and 15. Any displacement in the figure is normalized with respect to $\delta_{MAX} = \delta_{MAX}(0)$. The elastic duplicate deformation history varies at minimum load meaning that cohesive stresses reorganize ("relax") *simultaneously* in both duplicates.

The condition of the viscoelastic δ_{MIN} being constant cannot be maintained when the elastic δ_{MIN} during the relaxations process goes below the relaxed minimum front opening, $\delta_{MIN,REL}$, given by Equation 15. The condition previously introduced of a constant viscoelastic δ_{MIN} must then be replaced by a constant elastic $\delta_{MIN} \equiv \delta_{MIN,REL}$ which causes the viscoelastic δ_{MIN} to increase (this modification is the exception previously referred to concerning the crack profile being locked in the period of time where load is at its minimum).

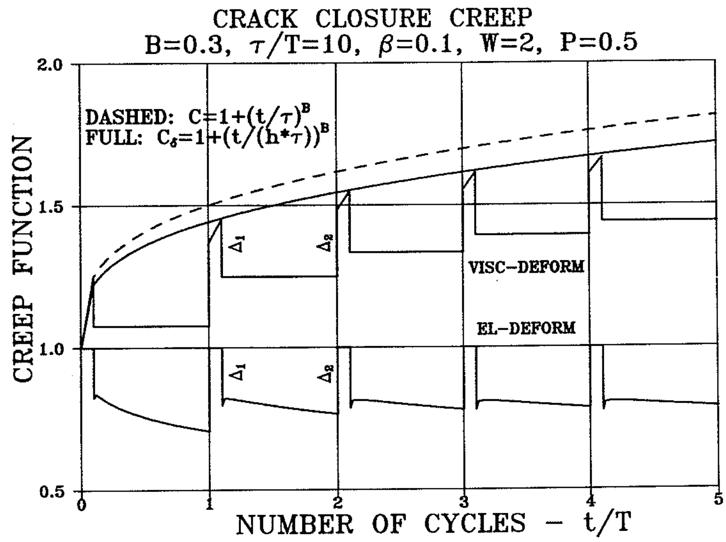

Figure 4. Corresponding crack opening histories in an elastic and a viscoelastic crack problem. The example shown is based on the Rice δ-range, $\cap\delta/\delta_{MAX} = (1-p)^2/2$ and $(W,p) = (2,0.5)$.

Crack closure creep function: It is convenient in the subsequent fatigue analysis to describe the time influence on crack closure by continuous functions. We therefore introduce the crack *closure creep function*, C_δ, and the *δ-range creep function*, $C_\cap$ respectively by the following expressions,

$$C_\delta(t) = \frac{\delta(t)}{\delta_{MAX}} \tag{39}$$

$$C_\cap(t) = \frac{\cap\delta(t)}{\cap\delta} = 1 + W\,\frac{C_\delta(t)-1}{C_\delta(t)} \tag{40}$$

where $\delta(t)$ is the graph which envelops a smoothed $\delta_{MAX}(t)$ and W is the δ-range creep factor previously introduced. The normalized δ-jumps, $\cap_1$, shown in Figure 4 at load reduction are given by Equations 40 with $\cap_o$ from Equations 19 and 28. We get

$$\eta_1 = \eta_1(t) = \eta_o C_\eta(t) \qquad ; \ \eta_o = \sqrt{2Z} \tag{41}$$

Approximation: The crack closure creep function can only be determined numerically. An approximate solution, however, can be established as follows shifting the original creep function, meaning $C_\delta(t) = C(t/h)$. The shift factor, h, can be determined considering the first cycle by elementary rheological rules. We get

$$C_\delta(T) \approx \frac{\delta_{MAX}(T)}{\delta_{MAX}(0)} \approx C(\beta T) - \eta_1 + \eta_2$$

$$\approx C(T) - \eta_1 C((1-\beta)T) - (\eta_2 - \eta_1) C((1-\beta)T/R) + \eta_2 \tag{42}$$

where η_1 and η_2 denote δ-jumps at load reduction at $t = \beta T$ and load increase at $t = T$ respectively. A relaxation factor of $R = 1$ can be used for the present purpose. We determine η_2 by this equation introducing Power Law creep. The result is

$$\eta_2 = MIN \left| \begin{array}{l} \dfrac{1 - \beta^b + \eta_1 \langle f\rangle^b}{\langle f\rangle^b + (1-\beta)^b} \\ 1 - p|p| \end{array} \right. \tag{43}$$

with non-dimensional frequency, $\langle f\rangle = f^* \tau$, and

$$\eta_1 \approx \eta_o \left(1 + W \frac{C(T)-1}{C(T)}\right) \tag{44}$$

The latter expression in Equation 43 considers the condition previously introduced that δ_{MIN} must not go below $\delta_{MIN,REL}$ given by Equation 15.

Finally, the shift factor, h, can be determined as follows,

$$C_\delta(T) \approx C(T/h) = 1 + \left(\frac{T}{h\tau}\right)^b = C((1-\beta)T) - \eta_1 + \eta_2 \quad => \tag{45}$$

$$h = [\beta^b + (\eta_2 - \eta_1)\langle f\rangle^b]^{-1/b} \tag{46}$$

Computer calculations show that a better h is obtained replacing the power, $-1/b$, with the following exponent,

$$e = -0.1(1+4\eta_1^{0.2})(1-\beta)/b \tag{47}$$

Moving Crack Closure Creep

The general problem of determining the opening of a position being penetrated by a moving crack in a viscoelastic material subjected to repeated loads is solved approximately combining the separate solutions previously considered. This means that the viscoelastic opening relative to its elastic counterpart is given by the following creep function,

$$C_{M\delta}(t) = C(t/(hq)) = 1 + \left(\frac{t}{hq\tau}\right)^b \tag{48}$$

The δ-range creep function defined in Equation 40 is modified accordingly, meaning that argument C_δ is replaced by $C_{M\delta}$.

VI. VISCOELASTIC FATIGUE

We can now determine lifetime of viscoelastic materials applying exactly the same procedure as used when elastic materials are considered. In energy calculations, however, we have to modified (amplify) deformations according to the effective creep functions established in Section V. Two states are examined. The initial, resting crack state where energy is dissipated at the constant damage ratio, $k = 1$. When energy dissipation has become critical at time t_S (N_S) the crack starts moving. The crack goes into the *propagating* state which ends at time t_{CAT} (N_{CAT}) where *catastrophic failure* occurs as the damage ratio approaches the critical quantity, $k = k_{CR}$.

Resting Crack

The energy dissipated at the "front fiber" of a resting crack *(damage ratio $k \equiv 1$)* is the sum of Γ_1 caused by the general increase of the crack front opening, and Γ_2 which is caused by the alternating crack front opening. The contributions are expressed as follows introducing resting crack closure creep from Section V.

$$\Gamma_1 = \sigma_1 \delta_{MAX} C_\delta(t) \tag{49}$$

$$\Gamma_2 = \{\frac{2\Omega_0 t}{T} C_\Omega(t/2) + (1-\beta)[C_\delta(t)-1]\} \sigma_1 \delta_{MAX} \tag{50}$$

where $\Gamma_2 = \Sigma(\Omega_1+\Omega_2)\sigma_L\delta_{MAX}$. The total dissipation is then

$$\Gamma = [1 + (2-\beta)[C_\delta(t)-1] + \frac{2t\sqrt{2Z}}{T}(1+W\frac{C_\delta(t/2)-1}{C_\delta(t/2)})]\ \sigma_1\delta_{MAX} \tag{51}$$

where $\Omega_0 = \sqrt{(2Z)}$ has been introduced from Equations 19 and 28b (with $k = 1$).

The time, t_S, it takes the crack to start propagating is given by Equations 21 (with $k = 1$) and Equation 51 letting $\Gamma \to \Gamma_{CR}$. The following expression is obtained introducing Power law creep and non-dimensional time, $\langle t_S\rangle = t_S/\tau$. We get

$$(2-\beta)(\frac{\langle t_S\rangle}{h})^b + 2\langle f\rangle\langle t_S\rangle\sqrt{2Z}\ (1+W\frac{(\langle t_S\rangle/h)^b}{1+(\langle t_S\rangle/h)^b}) = \frac{1-SL_{MAX}^2}{SL_{MAX}^2} \tag{52}$$

or in terms of load cycles, $N_S = t_S/T$

$$(2-\beta)(\frac{N_S}{h\langle f\rangle})^b + 2N_S\sqrt{2Z}\ (1+W\frac{[N_S/(h\langle f\rangle)]^b}{1+[N_S/(h\langle f\rangle)]^b}) = \frac{1-SL_{MAX}^2}{SL_{MAX}^2} \tag{53}$$

which generally have to be solved numerically. Two important cases, however, can be analyzed analytically:

Time to start of damage propagation when dead load is considered is obtained by Equation 52 introducing $SL_{MAX} \equiv SL$ ($T = \infty$, $\beta = 1$). We get

$$\langle t_S\rangle = (1/SL^2 - 1)^{1/b} \qquad (\langle f\rangle = 0,\ \text{dead load}) \tag{54}$$

Number of cycles to initiate failure propagation in an elastic material or in a viscoelastic material subjected to high load frequencies is obtained by Equation 53 introducing $\langle f\rangle = \infty$. We get

$$N_S = \frac{1/SL^2 - 1}{2\sqrt{2Z}} \qquad (\langle f \rangle = \infty,\ el.\ fatigue) \qquad (55)$$

Time to start of damage propagation is often ignored in studies on fatigue of materials because it is usually small compared with the lifetime subsequently experienced by the propagating crack (damage).

Curiosum: The dead load solution of time to start of damage propagation in Equation 54 can also be derived directly from Equations 6 and 7 replacing 1/E and Γ in the former expression with the creep function and the critical energy release rate respectively. We get

$$SL = [C(t_S)]^{-1/2} \quad \Rightarrow \quad t_S = C^{-1}(1/SL^2) \qquad (56)$$

applying for any viscoelastic material. The inverse creep function is denoted by C^{-1}. It is noticed that a crack will never start propagating when the load level considered is smaller than a *threshold value* SL_{TH} given by

$$SL_{TH} = [C(\infty)]^{-1/2} \qquad (57)$$

The materials primarily considered in this paper have a threshold load level $SL_{TH} = 0$ because the "end value" of power law creep is infinite. Thus, it is only a matter of time before these materials will fail catastrophically. Other materials with a finite end creep value will survive being loaded with $SL < SL_{TH}$. For example, if $C(\infty) = 3$ the material can be deadloaded safely with SL < 60 %.

Propagating Crack

The time, Ω, it takes a crack to propagate a distance, R_{MAX}, in a viscoelastic material is determined by energy considerations just as in Section IV looking at elastic fatigue. Displacements, however, appearing in the energy expression will amplify according to moving crack closure creep as explained in Section V. We get

$$\Gamma_{CR} = \sigma_1 \delta_{MAX}[C_{M\delta}(\Omega) + (1-s_o)\{\frac{\Pi_o \Omega C_\Pi(\Omega/2)}{T} + \frac{1-\beta}{2}(C_{M\delta}(\Omega)-1)\}] \qquad (58)$$

which can also be written as follows introducing s_o and Π_o according to Equations 19 and 28b and the information given in Equation 21 on Γ_{CR}

$$\frac{1-kSL_{MAX}^2}{kSL_{MAX}^2} = (C_{M\delta}(\Omega)-1)(1 + \frac{(1-\beta)\sqrt{Z}}{2\sqrt{2}}) + \frac{Z\Omega C_\Pi(\Omega/2)}{T} =$$

$$= (\frac{\Omega}{qh\tau})^b(1 + (1-\beta)\sqrt{Z/8}) + \frac{Z\Omega}{T}(1 + \frac{W(\Omega/(2qh\tau))^b}{1 + (\Omega/(2qh\tau))^b}) \qquad (59)$$

where the latter expression is obtained by introducing Power Law creep.

Ω can be eliminated in Equation 59 introducing damage velocity, dk/dN according to Equation 22 which can be written

$$\frac{\Omega}{T} = \Phi\ (FL^2 \frac{dN}{dk}) \qquad [\Phi = \frac{\pi^2}{8} kSL_{MAX}^2] \qquad (60)$$

Finally, Equation 59 can be written as follows relating *damage velocity to damage ratio*, k,

$$Y = A_1 X^b + A_2 X - A_3 = 0 \tag{61}$$

where

$$X = FL^2 * dN/dk \tag{62}$$

$$A_1 = [1 + (1-\beta)\sqrt{Z/8}]\ \theta^b \qquad ; \ [\theta = \frac{\Phi}{qh\langle f\rangle}] \tag{63}$$

$$A_2 = Z*\Phi(1 + \frac{W(\theta X/2)^b}{1 + (\theta X/2)^b} \qquad ;(X*dA_2/dX = \frac{WbZ\Phi(\theta X/2)^b}{(1 + (\theta X/2)^b)^2}) \tag{64}$$

$$A_3 = \frac{1 - kSL_{MAX}^2}{kSL_{MAX}^2} \tag{65}$$

with Z expressed by Equations 28b + 33. Fatigue parameters needed to establish Equation 61 are the *damage rate constant C, the damage rate power M, the critical load ratio p_{CR}, and the δ-range creep factor W.*

At a fixed damage ratio, k, Equation 61 can be solved easily applying the Newtons iteration principle,

$$X_{NEW} = X - \frac{Y}{dY/dX} = X - \frac{A_1 X^b + A_2 X - A_3}{bA_1 X^{b-1} + A_2 + X*dA_2/dX} \tag{66}$$

Lifetime: It is hereby indicated how lifetime, N_{CAT}, is determined numerically by Equation 61: Define a Δk (f.ex. $(k_{cr}-1)/500$). At a given $k = k_1$ we determine dN/dk as just described. At $k_2 = k_1 + \Delta k$ we have $N_2 = N_1 + (dN/dk)\Delta k$. A.s.o. between the initial damage ratio, k = 1, and $k = k_{cr}$ where damage rate becomes infinitely high.

An appropriate estimate of the initial value of X is obtained by Equations 54 (with $\Omega \approx t_S$) and Equation 60 (with k = 1). We get

$$X_o \approx \langle f\rangle \frac{(1/SL_{MAX}^2 - 1)^{1/b}}{SL_{MAX}^2} \tag{67}$$

Residual strength: The (relative) materials strength, S_R, still remaining after a certain number of load cycles, N, where damage ratio has become k = k(N) is predicted just as in Section IV considering elastic fatigue. This means

$$S_R = \sigma_{CR}(k)/\sigma_{CR} = 1/\sqrt{k(N)} \tag{68}$$

is evaluated along with k in the algorithm outlined above to predict lifetime. At $N \le N_S$ we have $S_R \equiv 1$ while $S_R = SL_{MAX}$ at $N = N_{CAT}$.

It is important to notice that lifetime and residual strength are determined non-dimensionally by Equations 61 and 68 respectively. This means that "master" lifetime and residual strength graphs can be constructed. For example, $FL^2 * N_{CAT}$ versus non-dimensional frequency, ⟨f⟩ and load level, SL_{MAX}, - or $FL^2 * \langle t_{CAT}\rangle$ versus ⟨f⟩ and SL_{MAX} where non-dimensional time to catastrophic failure is given by

$\langle t_{CAT} \rangle = t_{CAT}/\tau$. Another example is residual strength S_R versus $FL^2 {*} N$ (or $FL^2\langle t \rangle$), SL_{MAX}, and $\langle f \rangle$.

The elastic fatigue results presented in Section IV are obtained by Equation 61 with $\langle f \rangle \rightarrow \infty$, meaning τ and/or $f \rightarrow \infty$. The static viscoelastic fatigue results (Duration of Load) are obtained as shown in the following Section VII.

Reversed loading: The problem of reversed loading is solved by two lifetime calculations. One considering a tensile reference state and one considering a compressive state. The shorter lifetime predicted is the one of interest. Alternatively a reference state is chosen according to the observed mechanism responsible for failure during a short time test with $N_{CAT} \approx 10$.

Threshold: There are many speculations on the existence of a threshold on load alternation below which no fatigue failure will ever occur in wood and other viscoelastic materials. Information given by Kollmann and Côté (51) indicate a threshold load level ("endurance limit") of $SL_{TH} \approx 1/4$ for wood subjected to load variation with $p = -1$. (The corresponding threshold on load level range is $\Delta SL_{TH} \approx 50$ %). Kollmann and Côté state that the endurance limit for wood seems to be "as a rule higher than for most metals". Their conclusions are based on observations from experiments at relatively high load frequencies ($f > 15$ Hz). No experimental evidence is present which supports the threshold idea at arbitrary load frequency.

The existence of a threshold is widely accepted in the literature on metal fatigue. Irving and McCartney (44) suggested on an empirical basis that the threshold phenomenon can be considered practically by replacing the stress range intensity factor in Equation 27 according to $\Delta K^4 \Rightarrow \Delta K^2 {*} (\Delta K^2 - \Delta K_{TH}^2)$ where a threshold stress intensity factor range, ΔK_{TH}, is introduced as a materials constant (information given in (44) indicate $\Delta K_{TH}/K_{CR} \approx 10$ %). Examples given in (52) demonstrate, however, that ΔK_{TH} is not in general a materials constant for metals.

From these considerations we will continue developing a theory which predicts a fatigue threshold for viscoelastic materials when loaded at high frequencies. That is, we accept as a "must" that the observations made on metals and wood at high frequencies should be part of a fatigue theory for building materials in general. Referring to Equation 57 no threshold is expected at very low frequencies (simple creep failure) for the materials primarily considered in this paper. The meaning of the term "threshold" is generalized, now defining a transition limit below which lifetime becomes larger than "expected" (not necessarily infinitely high).

The theory is based on the hypothesis that the crack closure mechanism at very small $\Delta\delta$ is more efficient than expressed by Equations 19 and 28b. We suggest that these expressions only apply when $\Delta\delta/\delta_{CR}$ is greater than a certain *crack closure threshold value*, $D_{TH} = (\Delta\delta/\delta_{CR})_{TH}$. When smaller, $\Delta\delta$ must be reduced.

The δ-range relative to δ_{CR} is obtained by Equations 19 and 9. We get

$$\Delta\delta/\delta_{CR} = \sqrt{2Z}\, kSL_{MAX}^2 \geq D_{TH} = (\Delta\delta/\delta_{CR})_{TH} \tag{69}$$

where the $\cap\sigma$ criterion just formulated is introduced by the latter term. A Z-threshold is now defined by Equations 69 and 28b,

$$Z_{TH} = \frac{1}{2}\left(\frac{(\cap\sigma/\sigma_{CR})_{TH}}{kSL_{MAX}^2}\right)^2 \tag{70}$$

expressing the value of Z below which $\cap\sigma$ must be reduced. A simple way of doing so is to introduce an effective Z expressed in the following way from Z given by Equation 28b,

$$Z_{EFF} = Z*MIN(1, Z/Z_{TH}) \tag{71}$$

The traditional meaning of the term "threshold" is approached raising the term, Z/Z_{TH}, to a power, $n \rightarrow \infty$.

The following relations between crack closure threshold and thresholds at k = 1 for max load level and load level range respectively are obtained by Equations 70 and 28b. We get

$$SL_{MAX,TH} = \frac{[4*D_{TH}{}^2/C]^{1/M}}{U(1-p)} \tag{72}$$

$$\cap SL_{TH} = \frac{[4*D_{TH}{}^2/C]^{1/M}}{U} \tag{73}$$

The threshold values previously referred to applying for wood are obtained introducing D_{TH} = 0.0005 and (C,M,p_{CR}) = (3,9,-.5).

VII. STATIC VISCOELASTIC FATIGUE

Damage rate at static fatigue is obtained by Equation 61 with $\langle f\rangle \rightarrow 0$. We get

$$\frac{dk}{dt} = \frac{\pi^2 FL^2}{8q\tau}\,\frac{SL^2k}{[(SL^2k)^{-1}-1]^{1/b}} \tag{74}$$

from which the following relation is obtained between non-dimensional time, $\langle t\rangle = t/\tau$, and damage ratio, k,

$$\langle t\rangle FL^2 = \frac{8q}{\pi^2 SL^2}\int_\alpha^\beta \frac{x^{1/b}}{(1+x)}\,dx \qquad \beta=1/SL^2 - 1 \qquad \alpha=1/(kSL^2) - 1 \tag{75}$$

Lifetime: Catastrophic failure appears when the damage ratio approaches the critical value $k_{CR} = 1/SL^2$. Time to failure is then obtained by Equation 75 with lower limit α = 0. Relevant lifetime results for wood and other building materials are given below with the following abbreviations introduced

$$\mu = SL^{-2} - 1 \tag{76}$$

$$\langle t_{CAT}\rangle = t_{CAT}/\tau \tag{77}$$

and time to start of damage propagation ignored (Eq. 54),

$$\langle t_{CAT}\rangle FL^2 = \frac{2.4}{SL^2}[\mu - \log_E(\mu + 1)] \qquad [b=1]$$

$$\langle t_{CAT}\rangle FL^2 = \frac{2.7}{SL^2}\left[\frac{2\mu^{3/2}}{3} - 2(\sqrt{\mu} - \arctan(\sqrt{\mu}))\right] \qquad [b=2/3]$$

$$\langle t_{CAT}\rangle FL^2 = \frac{2.9}{SL^2}\left[\frac{\mu^2}{2} - \mu + \log_E(\mu + 1)\right] \qquad [b=1/2]$$

$$\langle t_{CAT}\rangle FL^2 = \frac{3.0}{SL^2}\left[\frac{2\mu^{5/2}}{5} - \frac{2\mu^{3/2}}{3} + 2(\sqrt{\mu} - \arctan(\sqrt{\mu}))\right] \qquad [b=2/5]$$

$$\langle t_{CAT}\rangle FL^2 = \frac{3.1}{SL^2}\left[\frac{\mu^3}{3} - \frac{\mu^2}{2} + \mu - \log_E(\mu + 1)\right] \qquad [b=1/3]$$

$$\langle t_{CAT}\rangle FL^2 = \frac{3.1}{SL^2}\left[\frac{2\mu^{7/2}}{7} - \frac{2\mu^{5/2}}{5} + \frac{2\mu^{3/2}}{3} - 2(\sqrt{\mu} - \arctan(\sqrt{\mu}))\right] \qquad [b=2/7]$$

$$\langle t_{CAT}\rangle FL^2 = \frac{3.2}{SL^2}\left[\frac{\mu^4}{4} - \frac{\mu^3}{3} + \frac{\mu^2}{2} - \mu + \log_E(\mu + 1)\right] \qquad [b=1/4]$$

$$\langle t_{CAT}\rangle FL^2 = \frac{3.3}{SL^2}\left[\frac{\mu^5}{5} - \frac{\mu^4}{4} + \frac{\mu^3}{3} - \frac{\mu^2}{2} + \mu - \log_E(\mu + 1)\right] \qquad [b=1/5]$$

Residual strength: The strength, S_R, remaining during the fatigue process is easily determined by the above mentioned lifetime integration introducing $k = S_R^{-2}$ (Eq. 11). This means that residual strength and time (with t_S ignored) are related through Equation 75 simply by introducing a lower limit of $\alpha = (S_R/SL)^2-1$. Residual strength starts at $S_R = 1$ and ends at $S_R = SL$ where $\alpha = 0$.

Curiosum: The lifetime Equation 75 is a special case of the more general theory given in (2,4) which applies for any creep function. For the sake of curiosity we give the following approximate general results,

$$\frac{dk}{dt} \approx 0.5\ FL^2 \frac{SL^2 k}{C^{-1}[(SL^2k)^{-1}]} \qquad (78)$$

$$t^*FL^2 \approx \frac{2}{SL^2}\int_{x_1}^{x_2} \frac{C^{-1}(x)}{x}\,dx \qquad x_2 = 1/SL^2, \quad x_1 = 1/(kSL^2) \qquad (79)$$

where the inverse normalized creep function is denoted by C^{-1}. Lifetime is determined introducing a lower integration limit of $x_1 = 1$.

VIII. APPLICATION OF THEORY AND EXPERIMENTS

The static fatigue results presented in the article have previously been shown to produce very realistic descriptions of experimental data. References on this point are given in Section II. In the following section it is demonstrated in Figures 6 to 13 that theoretically predicted lifetime of materials subjected to repeated loads also agrees with experimental evidence.

Experimental data are denoted in the figures by dots - theoretical data by lines. Heavy lines represent actual lifetimes while dashed lines denote limiting solutions at higher frequency loading (elastic fatigue) or at lower frequency loading (deadload lifetime) predicted by Equation 30 (no threshold) and Equation 75 respectively. Theoretical residual strength predictions at SL_{MAX} = 0.5 are shown by point-dashed lines. The not real wood materials considered (glass fiber re-inforced epoxy, particle board, hardboard, and Douglas-Fir fin-gerjoints) have been "treated" like wood when estimating the param-eters needed for the analysis. A more detailed estimate is not needed for the present purpose where these materials have been tested at relatively high frequencies. Normal laboratory climatic conditions have been assumed all over.

Parameters

Strength level: The strength level of wood can be estimated by a method developed in (5,17) combining elementary expressions consid-ering theoretical strength (e.g. 47) of materials and strength of Dug-dale crack models (2,4). The result presented in (5) and shown in Figure 5 is given by

$$FL = \frac{\sigma_{CR}}{\sigma_l} = \frac{2}{\pi} \arccos[\exp(-\frac{\pi^2}{8}\frac{a}{l_o})] \tag{80}$$

where l_o and a denote major crack radius and defect nucleus diame-ter respectively. The latter quantity refers to "virgin clear wood" where no real cracks have yet been developed. A number of inherent defect nuclei (weak areas), however, are present like pit concentra-tions, rays, ineffective overlapping zones or bad bonding between fibers. Reference strength, σ_l, is strength of virgin clear wood sub-stance between defect nuclei. Strength level can be considered inde-pendent of climatic conditions as long as a and l_o keep constant.

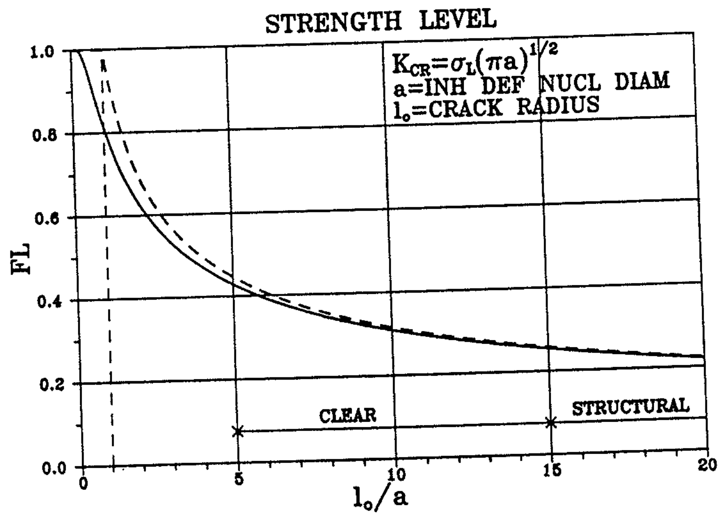

Figure 5. Strength ratio of wood. Inherent defect nu-cleus diameter, and crack radius are denoted by a (≈ 0.3 mm) and l_o respectively. Dashed line denotes "big knots" approximation, FL = $\sqrt{(a/l_o)}$.

The areas of clear wood and structural wood indicated in Figure 5 are based on the following estimates: Defect nucleus diameter, a ≈ 0.3 mm (estimated by the present author from micro structural pho-tographs), clear wood crack radius, l_o ≈ 1.5 mm (estimated from Schniewind and Lyon (53)), and structural crack radius, l_o ≈ 3 mm (minor knot radius). The term, "clear", should not be taken too rigorously. Badly treated clear wood may easily exhibit a "structural" wood strength level.

Creep parameters: It has previously been mentioned that creep in the present context refers to opening and/or sliding mode viscoelasticity in the vicinity of a crack, meaning that relaxation refers to perpendicular to grain tensile creep and parallel to grain shear creep more than it refers to parallel to grain creep like bending creep f.ex.

The following relaxation times are deduced from (54,55) applying for clear wood at normal laboratory conditions (Power law creep with b $\approx$ 0.23 - 0.28): $\tau \approx 10^5$ days, $\tau \approx 10^4$ days, $\tau \approx 10^3$ days, and $\tau \approx 10^2$ days for tension, bending, shear, and tension perp respectively.

This means that $\tau \approx 100 - 1000$ days should be appropriate in lifetime studies of wood. However, it is not. In the authors experience (11,17) a relaxation time of $\tau \approx 1 - 10$ days is more relevant. A natural explanation of this discrepancy is that lifetime prediction of wood cannot be based on bulk creep parameters. Creep of wood is probably a "small areas big events" phenomenon associated with boundary areas between fibers which also are the areas of minimum resistance to crack propagation.

MATERIAL	*MODE*	*b*	*LOG10(τ,days)*
Clear	*tension*	*1/4*	$2 \pm 1 + d$
	bending	-	$1 \pm 1 + d$
	compression	-	$0 \pm 1 + d$
	tension perp	*1/3*	$0 \pm 1 + d$
Structural	*tension* *bending* *compression*	*1/4*	$1 \pm 1 + d$

Table 1. Estimates of creep power, b, and relaxation time, τ. Moisture content and temperature at equilibrium: u $\approx$ 15 % and T $\approx$ 20 °C. Other conditions are considered by the parameter, d, defined in Equation 81.

Creep parameters suggested for lifetime studies of wood are given in Table 1 reproduced from (17). Temperature and moisture content at equilibrium are considered by the following parameter suggested in (33),

$$d \approx (15-u)/10 + (20-T)/15 \tag{81}$$

Dynamic climatic changes influence relaxation time dramatically - possibly by a factor less than 0.1 - 0.01. Creep power, b, can be considered independent of climate. "Clear wood" quantities in Table 1 include structural wood with a decisive clear wood failure mode.

δ-range creep factor: A δ-range creep factor of W $\approx$ 0 is generally assumed meaning that crack closure is not influenced by creep. The results in Figures 11 - 13, however, indicate that this simplification may not apply at loading perpendicular to grain.

Damage rate power and damage rate constant: The damage rate power, M, and damage rate constant, C, are estimated as follows applying a method developed in (17): Results of fatigue tests at higher frequencies (where creep effects are relatively small) are presented in a $\log_{10}(N_{CAT})$ - SL_{MAX} graph. Then, M is determined by

$$M \approx -10[1 + 2\log_{10}(B)] \quad ; \quad B = - \left. \frac{d[SL_{MAX}]}{d[\log_{10}(N_{CAT})]} \right|_{SL_{MAX} \approx 0.6} \tag{82}$$

and C from utilizing that this parameter is responsible for parallel shifting only of the lifetime graph (see Eq. 30). The results of Figure 9 on Pine heartwood in reversed bending have been used to estimate $M \approx 9$ and $C \approx 3$ (with $FL \approx 0.4$). It is noticed that the M estimate agrees well with M = 8.5 previously referred to (49).

Critical load ratio: A critical load ratio of $p_{CR} = -0.5$ is generally assumed as previously suggested in Section IV.

Crack closure threshold: This parameter is generally assumed to be $D_{TH} \approx 0.0005$ as determined in Section VI for wood and similar products.

Results

The experimental data shown in the following figures are from tests with periodically repeated load cycles. Frequencies $f \geq 1$ Hz refer to sinusoidal load variation, while data for $f < 1$ refer to square wave loading as shown in Figure 1. A fractional time of $\beta = 0.5$ under max load applies when not otherwise indicated. Theoretically, sinusoidal variations have been approximated by square waves with $\beta = 0.5$. This procedure is justified evaluating the influence of load variation on crack closure creep in Section V.

The data shown in Figures 11 - 13 are from experiments (56,57) to be reported in a subsequent article on artificially cracked clear Douglas-Fir specimens subjected to square wave tensile loading perpendicular to grain. The specimen size is 140 mm perpendicular to grain (load direction). The cross-section is 17*40 mm. A 10 mm parallel to grain crack is cut through the specimen in the center of the 40*140 mm face.

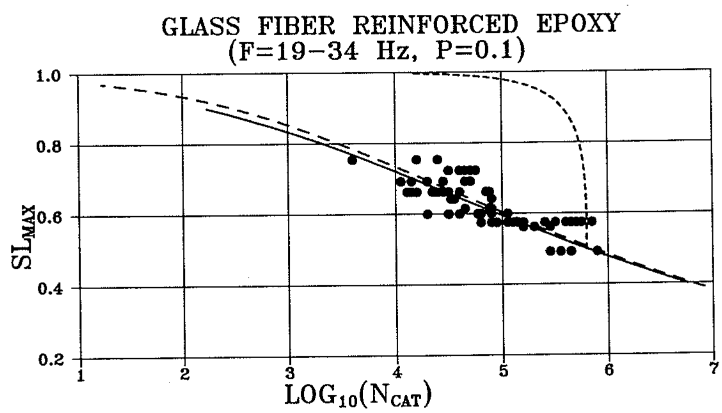

Figure 6. Fatigue lifetime of glass fiber reinforced epoxy. Tension in fiber direction. Experimental data by Hashin and Rotem (58). Theory: ▪ $(C, M, p_{CR}, W, D_{TH})$ = (3, 9, -0.5, 0, 0.0005) ▪ (FL, b, τ) = (0.4, 0.25, 1 day)

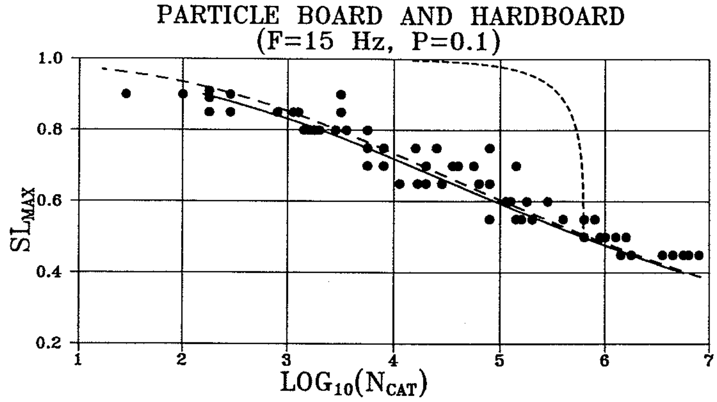

Figure 7. Fatigue lifetime of particle boards and hardboards in tension and interlaminar shear. Experimental data by McNatt (59,60). Theory: ▪ (C, M, p_{CR}, W, D_{TH}) = (3, 9, -0.5, 0, 0.0005) ▪ (FL, b, τ) = (0.4, 0.25, 1 day)

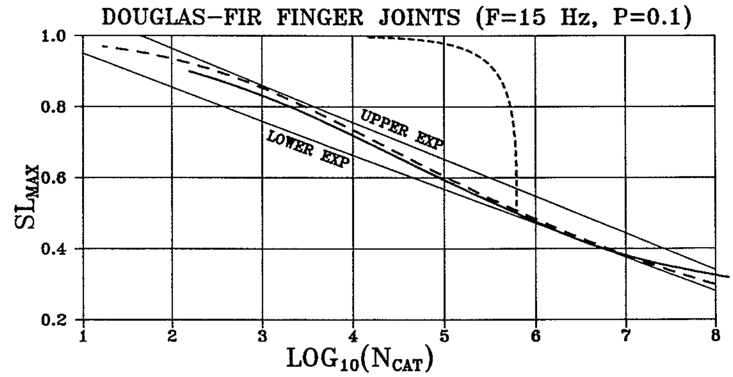

Figure 8. Fatigue lifetime of Douglas-Fir fingerjoints in tension parallel to grain. Experimental data by Bohannan and Kanvik (61) as bounded according to McNatt (62). Theory: ▪ (C, M, p_{CR}, W, D_{TH}) = (3, 9, -0.5, 0, 0.0005) ▪ (FL, b, τ) = (0.4, 0.25, 1 day)

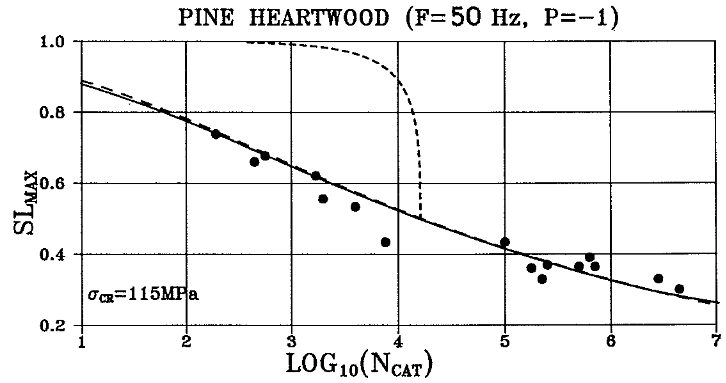

Figure 9. Fatigue lifetime of pine (heartwood) in reversed (rotational) bending. Experimental data by Kraemer (63) as represented in Kollmann and Côté (51). Theory: ▪ (C, M, p_{CR}, W, D_{TH}) = (3, 9, -0.5, 0, 0.0005) ▪ (FL, b, τ) = (0.4, 0.25, 1 day)

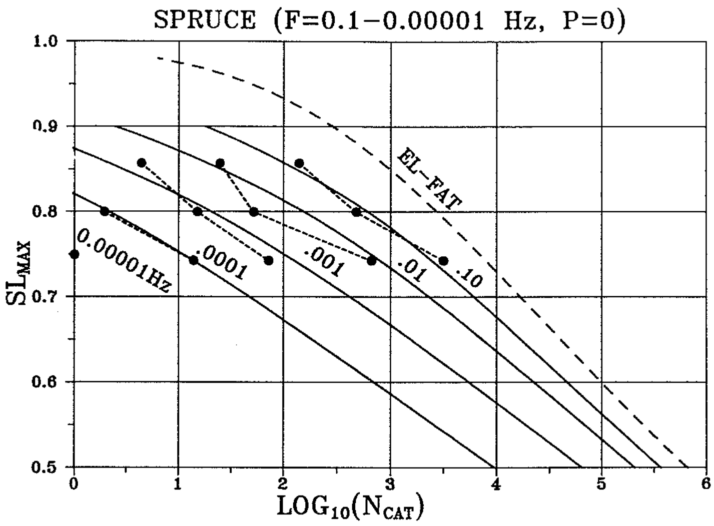

Figure 10. Fatigue lifetime of spruce compressed parallel to grain. Experimental data by Bach (64). Theory: ▪ (C, M, p_{CR}, W, D_{TH}) = (3, 9, −0.5, 0, 0.0005) ▪ (FL, b, τ) = (0.4, 0.25, 1 day)

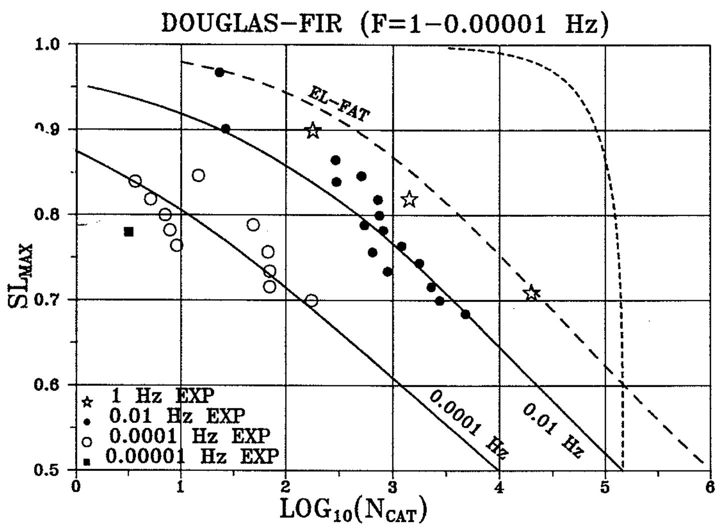

Figure 11. N-lifetime of artificially defected Douglas-Fir in tension perpendicular to grain. Experimental data by Nielsen and Madsen (56b, 0.01 Hz and 0.0001 Hz), Nielsen and Gray (57, 1 Hz), and McDowal (6, 10^{-5} Hz). Fractional time under max loading is 50 %. Theory: ▪ (C, M, p_{CR}, W, D_{TH}) = (5, 9, −0.5, 5, 0.0005) ▪ (FL, b, τ) = (0.25, 0.33, 1 day)

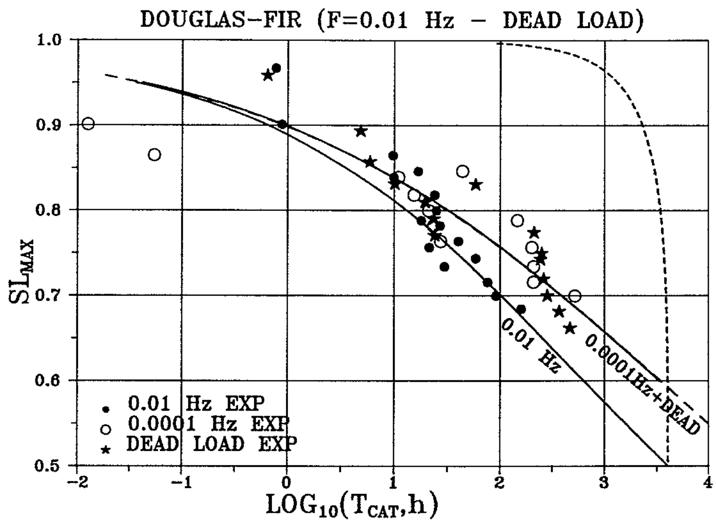

Figure 12. T-lifetime of artificially defected Douglas-Fir in tension perpendicular to grain. Same experiments and theoretical parameters as referred to in Figure 11.

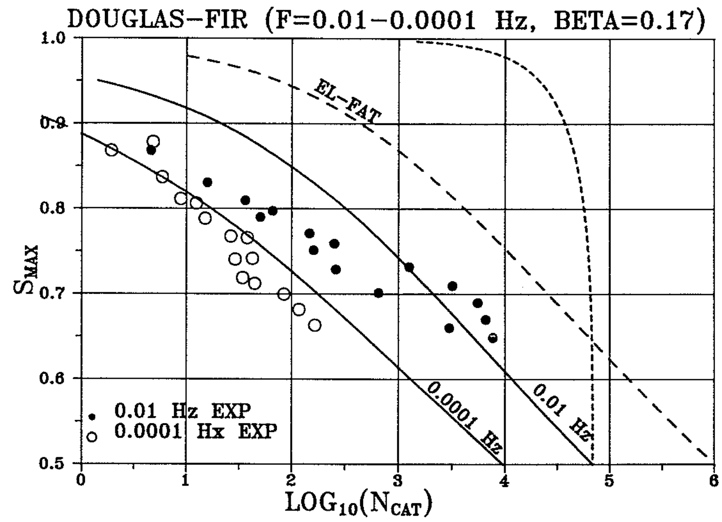

Figure 13. N-lifetime of artificially defected Douglas-Fir in tension perpendicular to grain. Experimental data by Nielsen and Madsen (56b). Fractional time under max loading is 17 %. Theory: ▪ (C, M, p_{CR}, W, D_{TH}) = (5, 9,-0.5, 20, 0.0005) ▪ (FL, b, τ) = (0.25, 0.33, 1 day).

IX. CONCLUSIONS AND FINAL REMARKS

The agreement between theoretical results and experimental data demonstrated in Section VIII, Figures 6 - 12, is very satisfactory. A variety of different wood and wood-related products (and glass fiber reinforced epoxy) at load ratios $-1 < p < 0.1$ reveal a remarkable "fellowship" in sharing fatigue parameters of the same orders of magnitudes.

It seems hereby made probable to suggest the hypothesis that the fatigue behavior of wood and wood related materials like particle-boards and fingerjoints loaded by tension, compression, bending, or shear parallel to fibers (flakes or fingers) can be described/predicted by one set of "master graphs" relating normalized quantities of lifetime, residual strength, load, load frequency, and relaxation time.

Such master graphs are shown in Figures 14 - 16 referring to on-and-off loading (p = 0). They are calculated as shown in the article with the following parameters: b = 0.25, β = 0.5, and (C, M, p_{CR}, W, D_{TH}) = (3, 9,-0.5, 0, 0.0005).

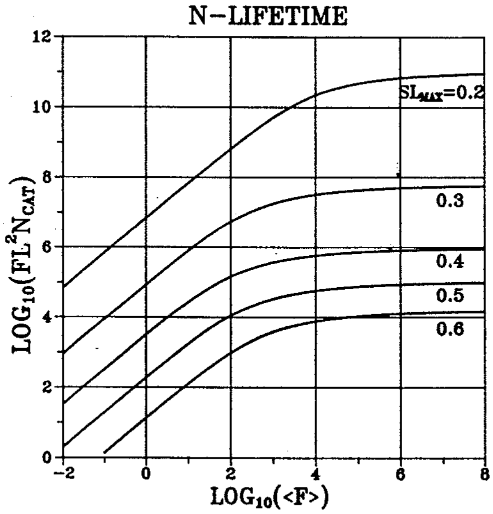

Figure 14. Suggested master-graph for number of cycles to catastrophic failure of wood and wood-related products subjected to on-and-off loading.

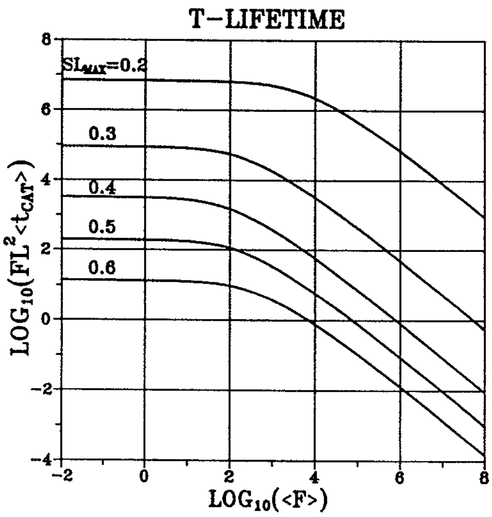

Figure 15. Suggested master-graph for real time to catastrophic failure of wood and wood-related products subjected to on-and-off loading.

It is noticed from the master graphs that lifetime is practically not influenced by creep at non-dimensional load frequencies, $\langle f \rangle > 10^6$. At the other hand lifetime is practically not influenced by load frequencies, $\langle f \rangle < 10$. Thus, lifetime is practically *elastic fatigue* (≈ *Equation 30*, $SL_{MAX} > SL_{MAX,TH}$) *at* $\langle f \rangle > 10^6$ and *deadload lifetime (Equation 75)* *at ⟨f⟩ < 10.* A *transition area,* 10 < ⟨f⟩ < 10^6, is defined where both creep and elastic fatigue mechanisms are active. From Figure 16 is observed that strength reduction starts becoming serious when the body considered has experienced 1/3 of its lifetime. The transition area just defined shifts to higher frequencies at increasing load ratio p, while the statement on strength keeps valid.

Example: Wood of quality FL = 0.4 and relaxation time τ = 1 day is used in a structural member which will experience a harmonic on-and-off load variation with one cycle of SL_{MAX} = 0.45 each 17 minutes (f = 0.001 Hz). ▪ *Conventional lifetime prediction based on accelerated test results:* $FL^2N_{CAT} = 10^{5.5}$ (Figure 14, right hand side) => $N_{CAT} = 10^{6.3}$ => t_{CAT} = 63 years. ▪ *DVM lifetime prediction:* $\langle f\rangle = f^*\tau = 10^{1.9}$ => $FL^2\langle t_{CAT}\rangle = FL^2(t_{CAT}/\tau) = 10^{2.5}$ (Figure 15) => t_{CAT} = 5 years ($N_{CAT} = 10^{5.2}$).

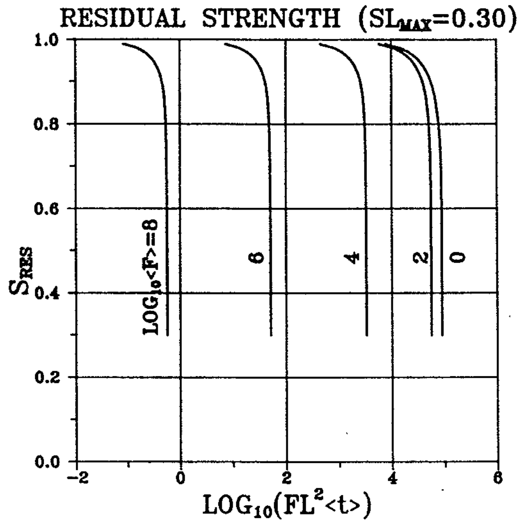

Figure 16. Suggested master-graph for strength decay in wood or wood-related product subjected to on-and-off loading.

Future research: Until now we have considered repeated loads primarily with a fractional time of β = 50 % under max load. There is no doubt that β different from 50 % at high frequencies will not change lifetime. At lower load frequencies, however, this statement is not that obvious. The experimental results shown in Figure 13 indicate that the δ-range creep factor, W, might change with β such that peak load variations (β = 0) might influence lifetime to be shorter than lifetime experienced when load is constantly at the peak load level. This indicates a very important research subject which presently is pursued experimentally by the present author and Borg Madsen at the University of British Columbia.

The master graphs shown in Figures 14 and 15 are valid at "room climate" primarily. The left hand side of the figures, however, which considers creep dominated static fatigue are also valid at other climatic conditions. Load frequency and lifetime are non-dimensional with respect to relaxation time which relates to moisture content and temperature through Equation 81. The right hand readings of the graphs are not immediately that general. We do not know, for example, how the "horizontal" and "transition" parts of the lifetime graphs in Figure 14 will shift at increasing moisture content in wood. More information is needed on how climatic conditions influence fatigue parameters in general, and at medium and higher frequencies in particular.

These considerations reveal other important research projects to be made in the future such that more detailed transition areas for example can be established between elastic fatigue and static fatigue lifetime. Experiments should include extreme temperatures and moisture contents. In general, for more safe estimates of fatigue parameters we need further experimental research on fatigue versus materials quality (e.g. grading, slope of grain), fatigue versus load ratio (including reversed loading), fatigue versus mode of loading, and on the existence of thresholds. And finally, basic research must be made on viscoelasticity in crack front areas. It should hereby not be forgotten that climatic changes occur at exactly these areas as a result of heat being generated in the process of crack closure. Especially at higher load frequencies this phenomenon might influence significantly the effective relaxation time - and lifetime consequently.

Appendix

Crack closure relaxation

The following relations have been developed theoretically by the present author in (17b) to examine the influence of relaxing coherent stresses on crack closure displacements. The distribution of coherent stresses, σ_{COH}, is $\sigma_{COH} \equiv \sigma_T$ at $0 \le s \le s_o$ and $\sigma_{COH} \equiv \sigma_F$ at $s_o < s \le 1$. The numerical quantities $|\sigma_F|$ and $|\sigma_T|$ are $\le \sigma_1$ which is the theoretical strength of the material considered. Terminology corresponds to the one applied in the main text of the paper. Crack opening displacement δ_{MAX} (and crack front width R_{MAX}) are the Dugdale quantities from Equation 12.

Crack closure $(\sigma_F/\sigma_1 = -1)$:

$$\frac{v_{CL}}{\delta_{MAX}} \approx \begin{cases} s^2 & ;(0 \le s \le s_o) \\ (1+p)(2\sqrt{s} - \frac{1+p}{2s}) - s^2 & ;(s_o \le s \le 1) \end{cases} \tag{A1}$$

$$\frac{\Omega\delta}{\delta_{MAX}} = \frac{1}{2}(1-p)^2 \tag{A2}$$

$$s_o \approx \left(\frac{1+p}{2}\right)^{2/3} \quad ; \quad \frac{\sigma_T}{\sigma_1} = 2\left(\frac{1+p}{2}\right)^{2/3} - 1 \tag{A3}$$

Deviations from the Rice crack closure solutions presented in Section III of the main text are due to the different assumptions of coherent stress distribution. This feature is of no significance in the present context. Reasons have been given in Sections III and IV that the results of crack closure theories should not be used too rigorously in fatigue analysis of materials

Relaxing state ***($p \geq \sigma_F/\sigma_1 > -1$):***

$$\frac{v_{REL}}{\delta_{MAX}} \approx \begin{cases} \left[\dfrac{\sigma_F}{\sigma_1} + 2\,\dfrac{p - (\sigma_F/\sigma_1)}{1 + p}\right] * s^2 & ;(0 \leq s \leq s_o) \\ \dfrac{\sigma_F}{\sigma_1}\, s^2 + \left(p - \dfrac{\sigma_F}{\sigma_1}\right)\left(2\sqrt{s} - \dfrac{1+p}{2s}\right) & ;(s_o < s \leq 1) \end{cases} \tag{A4}$$

$$\frac{v_{REL}}{\delta_{MAX}} \approx K\,\frac{v_{CL}}{\delta_{MAX}} \quad ; \; K = \frac{p(3-p) - (1-p)(\sigma_F/\sigma_1)}{1 + 2p - p^2} \tag{A5}$$

$$\frac{\sigma_T}{\sigma_1} = \frac{p-(1-\sqrt{s_o})(\sigma_F/\sigma_1)}{\sqrt{s_o}} \tag{A6}$$

It is noticed that relaxed crack closure profiles are predicted by Equation A5 to be approximately congruent. This means that crack opening at any location (s) varies approximately by the same "relaxation factor", K = $K(\sigma_F/\sigma_1)$, from the initial closure profile (v_{CL}) defined at K(-1) = 1. An example of crack closure with relaxing coherent stresses is given in Figure A1.

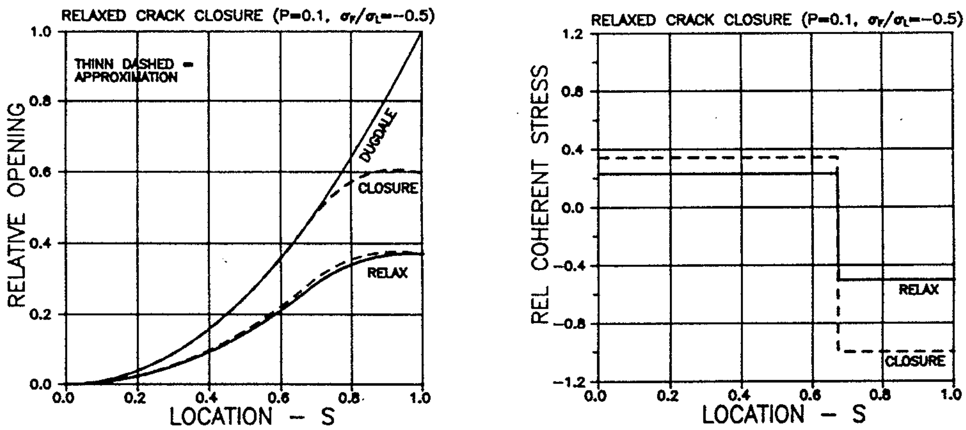

Figure A1. Crack closure at load ratio p = 0.1. Relaxation of front coherent stress from σ_F/σ_1 = −1 to −0.5.

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