

FUGTTRANSPORT BESKREVET SOM ENDIMENSIONAL
LINEÆR DIFFUSION

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FORORD

Ved Laboratoriet for Bygningsmaterialer indgår emnet fugtbinding og fugtvandring i bygningsmaterialer i det langsigtede forskningsprogram.

Den foreliggende rapport omhandler fugttransport i porøse bygningsmateriale under isoterme forhold beskrevet ved lineær diffusionsteori. Rapporten giver et basisgrundlag for de fugtvandringsberegninger, der opstilles senere i projektet.

Arbejdet er udført som led i FTU-projektet "Fugt i byggematerialer" (J.nr. 5.17.3.6.13), som udføres ved Laboratoriet for Bygningsmaterialer i perioden 1987-89.

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Projektleder

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1. SAMMENFATNING

Denne rapport omhandler fugttransport i porøse bygningsmaterialer under isoterme forhold beskrevet ved lineær diffusionsteori. Det skal fra starten slås fast, at det oftest er en stor forenkling af virkeligheden at beskrive fugttransport i porøse bygningsmaterialer som lineær diffusion. Lineær diffusionsteori kan dog være et godt hjælpeværktøj til at bestemme størrelsesordenen af fugttransporten.

Rapporten beskriver tre klassiske analytiske løsninger til den lineære endimensionale diffusionsligning, nemlig plade-, cylinder- og kugletilfældet med konstant vandindhold i overfladen og konstante vandindhold til tidspunkterne $t = 0$ og $t \rightarrow \infty$.

Da de analytiske løsninger numerisk set kan være vanskelige at evaluere, er der fremstillet EDB-programmerne CYLDIF og CARDIF, der løser den endimensionale lineære diffusionsligning numerisk ved hjælp af endelige differensers metode i henholdsvis cylinderkoordinater, kartesiske og kuglekoordinater. Numerisk løsning har endvidere den fordel, at diffusionsligningen kan løses for næsten vilkårlige rand- og begyndelsesbetingelser.

Løsningerne til diffusionsligningen i de tre tilfælde beskrives i form af kurver og tabeller. Kurverne er fugtprofiler optegnet for en række Fourier-tal.

Endelig er der udarbejdet analytiske beskrivelser af middelfugtindholdet i de tre behandlede geometrier som funktion af Fourier-tallet.

2. DEN ENDIMENSIONALE LINEÆRE DIFFUSIONSLIGNING

2.1 Opstilling af den endimensionale lineære diffusionsligning

Diffusion af f.eks. vanddamp i porøse bygningsmaterialer eller af atomer i metaller kan beskrives ved hjælp af Ficks 1. lov som

$$J = -D \frac{\partial c}{\partial x} \quad (2.1)$$

hvor

- J = flux af det, som diffunderer
- D = diffusivitet
- c = koncentrationen af det, som diffunderer
- x = stedkoordinaten

Opstilles kontinuitetsligningen for det i figur 1 viste kontrolvolumen, fås jvf. Crank /1/

$$\begin{aligned} \text{fraført} - \text{tilført} + \text{oplagret} &= 0 & \Leftrightarrow \\ (\int J + \frac{\partial J}{\partial x} dx) dy dz - J dy dz + \frac{\partial c}{\partial t} dx dy dz &= 0 & \Leftrightarrow \\ \frac{\partial J}{\partial x} dx dy dz + \frac{\partial c}{\partial t} dx dy dz &= 0 & \Leftrightarrow \\ \frac{\partial J}{\partial x} + \frac{\partial c}{\partial t} &= 0 & (2.2) \end{aligned}$$

Indsættes 2.1 i 2.2, fås den endimensionale ulineære partielle differential-ligning

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial x} \left(D \frac{\partial c}{\partial x} \right) \quad (2.3)$$

Regnes D for uafhængig af c, fås den tilsvarende lineære ligning

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2} \quad (2.4)$$

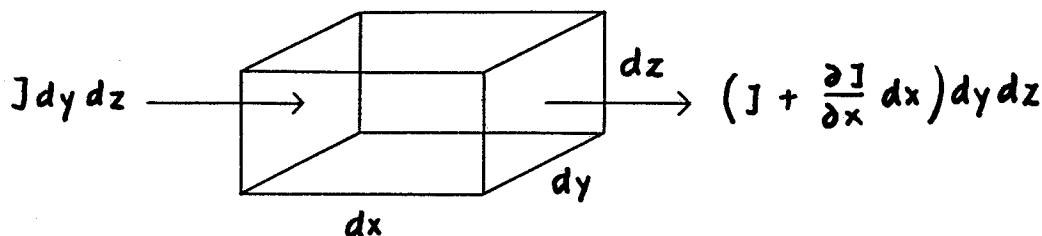
Opstilles 2.4 i cylinderkoordinater for endimensional symmetrisk diffusion i en cylinder, fås

$$\frac{\partial c}{\partial t} = D \left(\frac{1}{r} \frac{\partial c}{\partial r} + \frac{\partial^2 c}{\partial r^2} \right) \quad (2.5)$$

hvor r er den radiære stedkoordinat.

Opstilles 2.4 for endimensional symmetrisk diffusion i en kugle, fås ligningen

$$\frac{\partial c}{\partial t} = D \left(\frac{2}{r} \frac{\partial c}{\partial r} + \frac{\partial^2 c}{\partial r^2} \right) \quad (2.6)$$



Figur 1. Kontrolvolumen. Planparallelt tilfælde.

2.2 Analytisk løsning af den endimensionale lineære diffusionsligning

Ligningerne 2.4, 2.5 og 2.6 kan kun løses analytisk i få tilfælde og da kun for specielle rand- og begyndelsesbetingelser. I det følgende gennemgås løsninger til 2.4, 2.5 og 2.6, hvor der som drivende potentiiale er anvendt vandindholdet u_{∞} i et porøst bygningsmateriale. Løsningerne er givet for randbetingelsen konstant vandindhold u_{∞} i overfladen og begyndelsesbetingelsen et jævnt fordelte vandindhold u_0 gennem hele materialet. Løsninger findes f.eks. i Crank [1] og er opnået ved separation af de variable (f.eks. gennemgået i [2]).

Der forudsættes i det følgende altså begyndelsesbetingelsen (BB)

$$u(r, t) = u_0 \quad , \quad t < 0$$

og randbetingelsen (RB)

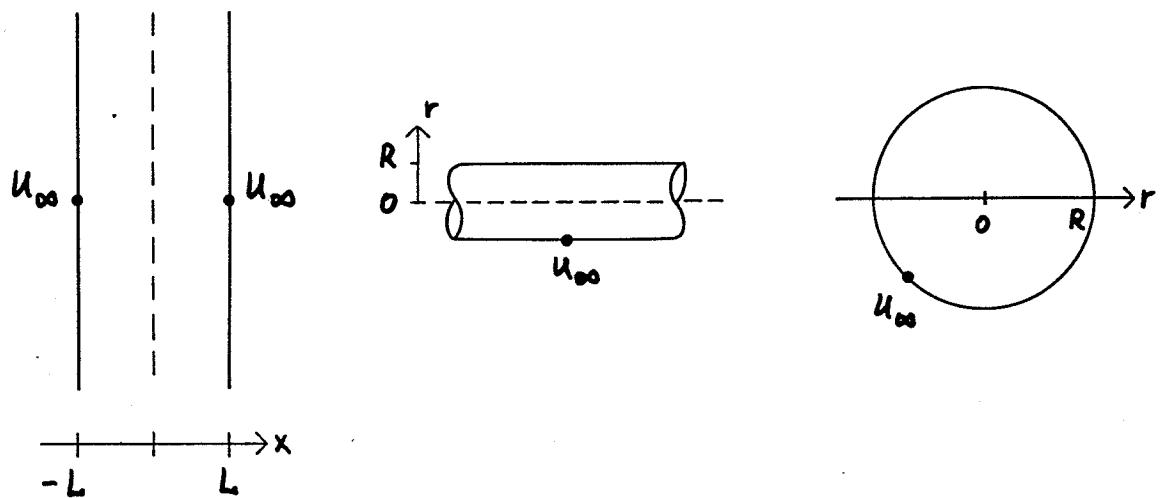
$$u(R, t) = u_{\infty} \quad , \quad t \geq 0$$

idet

r = stedkoordinat [m]

t = tiden [s]

R = karakteristisk dimension for det betragtede legeme [m]



Figur 2. De anvendte betegnelser i plade-, cylinder- og kugletilfældet.

Pladetilfældet

$$\frac{u(r,t) - u_{\infty}}{u_0 - u_{\infty}} = - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)} e^{-[(2n-1)\pi/(2L)]^2 Dt} \cdot \cos\left(\frac{2n-1}{2L}\pi x\right) \quad (2.7)$$

Cylindertilfældet

$$\frac{u(r,t) - u_\infty}{u_0 - u_\infty} = 2 \sum_{n=1}^{\infty} \frac{1}{\beta_n} e^{-(\beta_n/R)^2 Dt} \frac{J_0(\beta_n(r/R))}{J_1(\beta_n)} \quad (2.8)$$

J_0 = Besselfunktionen af 0. orden 1. art

J_1 = Besselfunktionen af 1. orden 1. art

β_n = den n'te rod af J_0 , altså β_n er givet ved ligningen $J_0(\beta_n) = 0$

Kugletilfældet

$$\frac{u(r,t) - u_\infty}{u_0 - u_\infty} = \frac{2R}{\pi r} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} e^{-(n\pi/R)^2 Dt} \sin\left(\frac{n\pi r}{R}\right) \quad (2.9)$$

2.3 Numerisk løsning af den endimensionale lineære diffusionsligning

Da de analytiske løsninger numerisk set kan være vanskelige at evaluere, er der fremstillet EDB-programmerne CYLDIF og CARDIF, der løser den endimensionale lineære diffusionsligning numerisk ved hjælp af endelige differensers metode i henholdsvis cylinderkoordinater, kartesiske og kuglekoordinater. Numerisk løsningen har endvidere den fordel, at diffusionsligningen kan løses for næsten vilkårlige rand- og begyndelsesbetingelser.

2.3.1 Cylindertilfældet

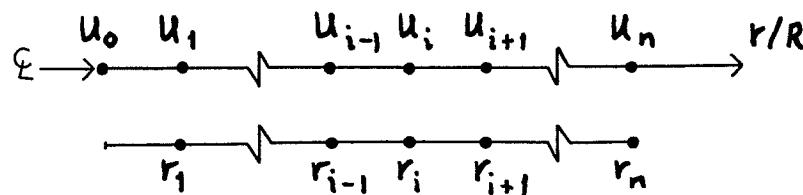
Til den numeriske løsning af ligning 2.5 er anvendt en eksplisit endelige differensers metode som beskrevet i f.eks. /3/. Metoden er, at vandindholdet i cylinderen beregnes i et endeligt antal punkter n i cylinderens radius, som det er vist i figur 3. Der er anvendt et fast stedsskridt i beregningerne nemlig

$$\Delta r = 1/n$$

Afstanden fra centrum af cylinderen til de enkelte punkter på cylinderens radius er

$$r_i = i(1/n)$$

hvor r_i er afstanden fra centrum til det i 'te punkt. Det bemærkes, at $r_0 = 0$ og $r_n = 1$, altså at r_i betegner en dimensionsløs radius.



Figur 3. Netinddeling i cylinderen.

De partielle afledede i ligning 2.5 erstattes med endelige differenstilnærmelser, og man får

$$\frac{u_i^{t_1 + \Delta t} - u_i^{t_1}}{\Delta t} = D \left(\frac{1}{r_i} \frac{u_{i+1}^{t_1} - u_{i-1}^{t_1}}{2 \Delta r} + \frac{u_{i+1}^{t_1} + u_{i-1}^{t_1} - 2u_i^{t_1}}{(\Delta r)^2} \right) \quad (2.10)$$

hvor

$u_i^{t_1}$ = vandindhold i cylinderen i cirklen med radius r_i til tidspunktet $t = t_1$,

Δt = tidsskridt

Vandindholdet i det i 'te netpunkt på tidsniveauet $t = t_1 + \Delta t$, $u_i^{t_1 + \Delta t}$, beregnes altså ud fra vandindholdet i selve punktet og de to omkringliggende punkter på tidsniveauet $t = t_1$.

Værdien af u_0, u_1, \dots, u_n til tidspunktet $t = 0$ angiver begyndelsesbetingelsen ved den numeriske løsning af den partielle differentialligning (begyndelsesvandindholdet i konstruktionen). Værdien af u_n (konstant eller varierende som funktion af tiden) for $t \geq \Delta t$ angiver randbetingelsen til den numeriske løsning af den partielle differentialligning 2.5.

Der er et problem ved bestemmelse af u_0 , d.v.s. af vandindholdet i cylinderens centrum, idet r_i i differensligningen 2.10 da bliver $r_0 = 0$, og $1/r_0$ bliver udefineret. Det er dog muligt at bestemme u_0 eksplisit, hvilket bliver vist i bilag 1 ved et ræsonnement, som bygger på analogien diffusion af temperatur i en cylinder. Her skal blot anføres resultatet

$$u_0^{t_1 + \Delta t} = u_0^{t_1} - \frac{4D\Delta t}{(\Delta r)^2} \left(u_0^{t_1} + \left(\frac{2}{\ln 2} - 4 \right) u_1^{t_1} + \left(3 - \frac{2}{\ln 2} \right) u_2^{t_1} \right) \quad (2.11)$$

For den eksplisitte formulering af endelige differensers metode gælder det, at der er grænser for, hvor store tidsskridt Δt man kan anvende i beregningerne, eller med andre ord hvor "hurtigt" man kan regne frem i tiden. Det vil ikke blive vist her, hvordan det maksimale Δt bestemmes (en metode til bestemmelse af det maksimale Δt , hvor den numeriske metode er stabil, kan findes i [5], p. 17), men blive anført, at for differensligningen 2.10 er stabilitetskravet

$$1 - \frac{2D\Delta t}{(\Delta r)^2} > 0 \quad (2.12)$$

og for 2.11 er stabilitetskravet

$$1 - \frac{4D\Delta t}{(\Delta r)^2} > 0 \quad (2.13)$$

2.13 er det skrappeste af de to stabilitetskrav, og det bliver derfor det gældende.

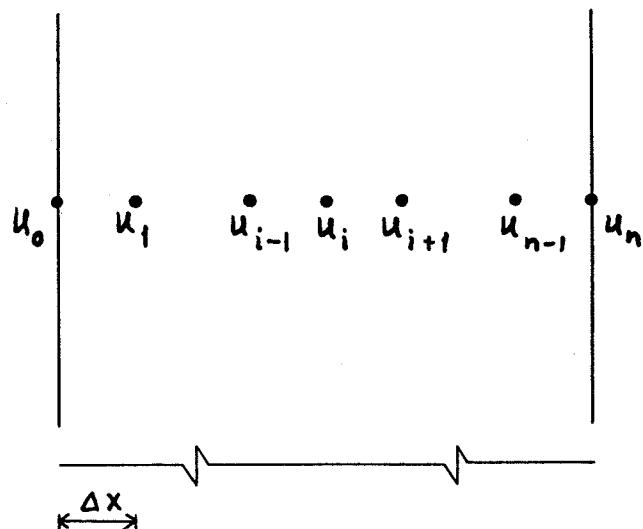
Den numeriske metode er implementeret i programmet CYLDIF. Programmet er listet i bilag 2. Den numeriske metode er kontrolleret ved sammenligning med den analytiske løsning (2.8), kontrollen er anbragt som bilag 3. Kontrollen viser, at implementeringen af den numeriske metode er i orden.

2.3.2 Pladetilfældet

Til den numeriske løsning af ligning 2.4 er anvendt en implicit endelige differensers metode som beskrevet i f.eks. /1/, /3/, /4/ og /5/. Metoden er, at vandindholdet i pladen beregnes i et endeligt antal punkter n i pladens tværsnit, som det er vist i figur 4. Der er anvendt et fast steds-skridt i beregningerne, nemlig $x = 1/n$.

Der er anvendt Crank-Nicolsons implicitte metode, der anvendt på ligning 2.4 giver differensligningen

$$\frac{u_i^{t_1 + \Delta t} - u_i^{t_1}}{\Delta t} = \frac{D}{2(\Delta x)^2} [u_{i-1}^{t_1 + \Delta t} - 2u_i^{t_1 + \Delta t} + u_{i+1}^{t_1 + \Delta t} + u_{i-1}^{t_1} - 2u_i^{t_1} + u_{i+1}^{t_1}] \quad (2.14)$$



Figur 4. Netinddeling af plade.

Indsættes $R = \Delta t D / (\Delta x)^2$ i 2.14 og leddene omarrangeres, fås

$$-R u_{i-1}^{t_1 + \Delta t} + (2R + 2) u_i^{t_1 + \Delta t} - R u_{i+1}^{t_1 + \Delta t} = \\ R u_{i-1}^{t_1} + (2 - 2R) u_i^{t_1} + R u_i^{t_1} \quad (2.15)$$

Af 2.15 ses det, at vandindholdet i det i 'te netpunkt på tidsniveauet $t = t_1 + \Delta t$, $u_i^{t_1 + \Delta t}$, beregnes altså ud fra vandindholdet i selve punktet og de to omkringliggende punkter på tidsniveauet $t = t_1$, samt de to omkringliggende punkters vandindhold på tidsniveauet $t = t_1 + \Delta t$.

I det u_0 og u_n i figur 4 angiver randbetingelserne, kan 2.15 opstilles for punkterne u_1, u_2, \dots, u_{n-1} . I matrixformulering fås da ligningssystemet

$$\begin{matrix} (2R+2) & (-R) \\ (-R) & (2R+2) & (-R) \\ & \ddots & \ddots & \ddots & \ddots & \ddots \\ & & & (-R) & (2R+2) & (-R) \\ & & & & (-R) & (2R+2) \end{matrix} \begin{bmatrix} u_1^{t_1 + \Delta t} \\ u_2^{t_1 + \Delta t} \\ \vdots \\ u_{n-2}^{t_1 + \Delta t} \\ u_{n-1}^{t_1 + \Delta t} \end{bmatrix} = \\ \begin{bmatrix} R u_0^{t_1 + \Delta t} + R u_0^{t_1} + (2 - 2R) u_1^{t_1} + R u_2^{t_1} \\ R u_1^{t_1} + (2 - 2R) u_2^{t_1} + R u_3^{t_1} \\ \vdots \\ R u_{n-3}^{t_1} + (2 - 2R) u_{n-2}^{t_1} + R u_{n-1}^{t_1} \\ R u_{n-2}^{t_1} + (2 - 2R) u_{n-1}^{t_1} + R u_n^{t_1} + R u_n^{t_1 + \Delta t} \end{bmatrix} \quad (2.16)$$

Vandindholdet i punkterne $u_1 - u_{n-1}$ på tidsniveauet $t_1 + \Delta t$ bestemmes altså ved løsning af ligningssystemet 2.16. Da koefficientmatricen på venstresiden af 2.16 er en tridiagonalmatrix, kan ligning 2.16 løses ved hjælp af en speciel simpel algoritme kaldet tridiagonalgoritme eller Thomas' algoritme (beskrevet i [4] og [6]). Da koefficientmatricen på venstresiden af

2.16 endvidere er konstant, bliver algoritmen til ligningsløsningen yderligere simpel.

For den implicitte metode kan det vises, at den er ubetinget stabil for enhver størrelse af tidsskridtet Δt (se /4/ og /5/).

Den numeriske metode er implementeret i programmet CARDIF. Programmet er listet i bilag 4. Den numeriske metode er kontrolleret ved sammenligning med tabel 2.4 givet i /5/. Kontrollen er listet i bilag 5. Da tabellen i bilag 5 viser overensstemmelse med tabel 2.4 i /5/, er den numeriske metode implementeret tilfredsstillende.

2.3.3 Kugletilfældet

Det kan vises, at ligningen jvf. 2.6

$$\frac{\partial u}{\partial t} = D \left(\frac{2}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2} \right) \quad (2.6')$$

ved transformationen $w = ur$ kan føres over i

$$\frac{\partial w}{\partial t} = D \frac{\partial^2 w}{\partial r^2} \quad (2.4')$$

hvilket er ligningen for pladetilfældet. Transformationen er vist i bilag 6.

Altså fås den numeriske løsning til 2.6' ud fra den numeriske løsning til 2.4', idet

$$u = \frac{r}{w} \quad \text{for } w \neq 0$$

Den numeriske løsning til 2.6' fås altså ved hjælp af programmet CARDIF.

2.4 Middelvandindhold

Ønsker man at beregne middelvandindholdet i en konstruktion, er det bemerket at have analytiske udtryk for dette. For de i dette notat 3 behandlede tilfælde plade-, cylinder- og kugletilfælde er der i dette afsnit angivet de analytiske udtryk for middelvandindholdet.

2.4.1 Middelvandindhold i pladetilfældet

Middelvandindholdet $u_m(t)$ er i pladetilfældet givet ved udtrykket

$$\frac{u_m(t) - u_\infty}{u_0 - u_\infty} = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} e^{-[(2n-1)\pi/(2L)]^2 Dt} \quad (2.17)$$

2.17 er udledt i bilag 7.

2.4.2 Middelvandindhold i cylindertilfældet

Middelvandindholdet $u_m(t)$ er i cylindertilfældet givet ved udtrykket

$$\frac{u_m(t) - u_\infty}{u_0 - u_\infty} = 4 \sum_{n=1}^{\infty} \frac{e^{-(\beta_n/R)^2 Dt}}{\beta_n^2} \quad (2.18)$$

2.18 er udledt i bilag 8.

2.4.3 Middelvandindhold i kugletilfældet

Middelvandindholdet $u_m(t)$ er i kugletilfældet givet ved udtrykket

$$\frac{u_m(t) - u_\infty}{u_0 - u_\infty} = \frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-(n\pi/R)^2 Dt} \quad (2.19)$$

2.19 er udledt i bilag 9.

2.4.4 Program MEANMOIS til beregning af middelvandindhold

Til beregning af middelvandindholdet i de tre tilfælde givet ved ligningerne 2.17, 2.18 og 2.19 er udviklet programmet MEANMOIS. Programmet er listet i bilag 10. MEANMOIS beregner middelvandindholdet for et givet antal Fourier-tal, hvor Fourier-tallet F_o er givet ved

$$F_o = Dt/\delta^2$$

hvor δ er en karakteristisk længde, nemlig den halve bredde L i pladetilfældet, og radius R i cylinder- og kugletilfældet.

MEANMOIS bruger rødderne β_n af Bessel funktionen J_0 af 0. orden 1. art til beregning af middelvandindholdet i cylindertilfældet. For små F_0 -tal skal der medtages flere led i rækkeudviklingen end der normalt er angivet rødder β_n i litteraturen. Det har derfor været nødvendigt at beregne flere rødder til β_n . Beregningsmetode og resultat er beskrevet i bilag 11.

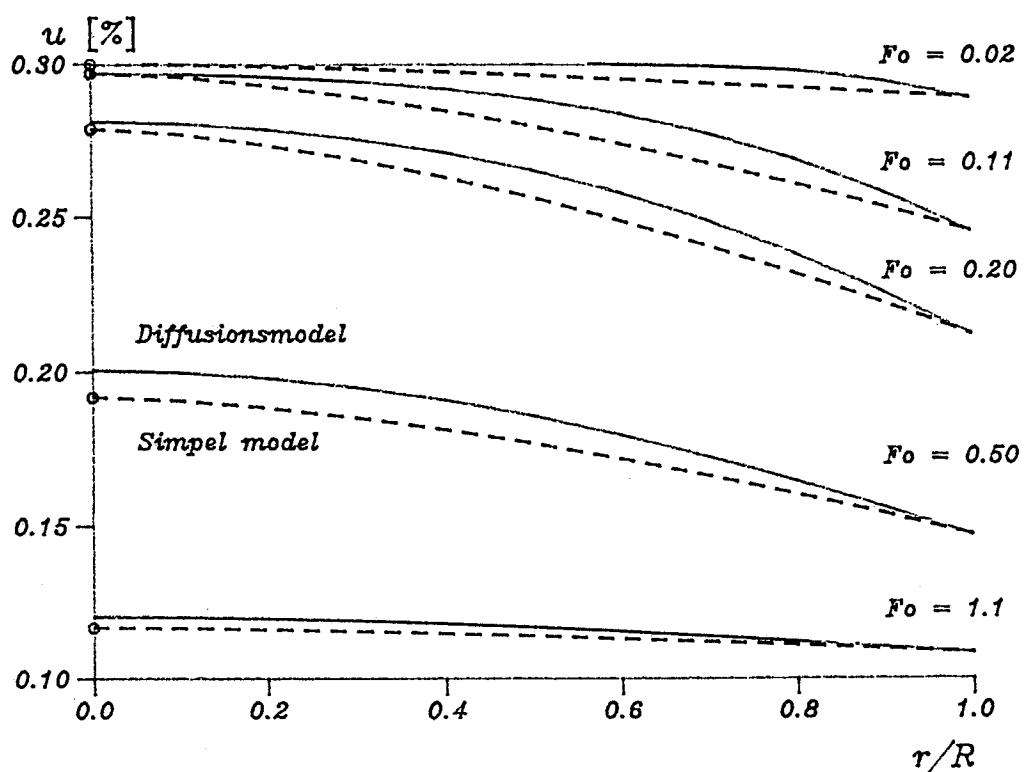
En udskrift fra en kørsel med MEANMOIS er anbragt som bilag 12.

3. KØRSLER MED DE UDVIKLEDE EDB-PROGRAMMER

I dette afsnit beskrives uddata fra EDB-programmerne CYLDIF, CARDIF og MEANMOIS kort.

3.1 Uddata fra CYLDIF

Programmet CYLDIF er benyttet ved udarbejdelsen af rapporten /7/. Fra denne er figur 5 hentet, der er fremstillet ved hjælp af uddata fra CYLDIF. De fuldt optrukne linier er fugtprofiler beregnet ved hjælp af CYLDIF. Der er her specificeret et eksponentielt aftagende fugtindhold i cylinderens overflade som randbetingelse. Cylinderen kan f.eks. være en træmast under udtørring.



Figur 5. Sammenligning af fugtprofiler beregnet ved forskellige metoder, fra [7].

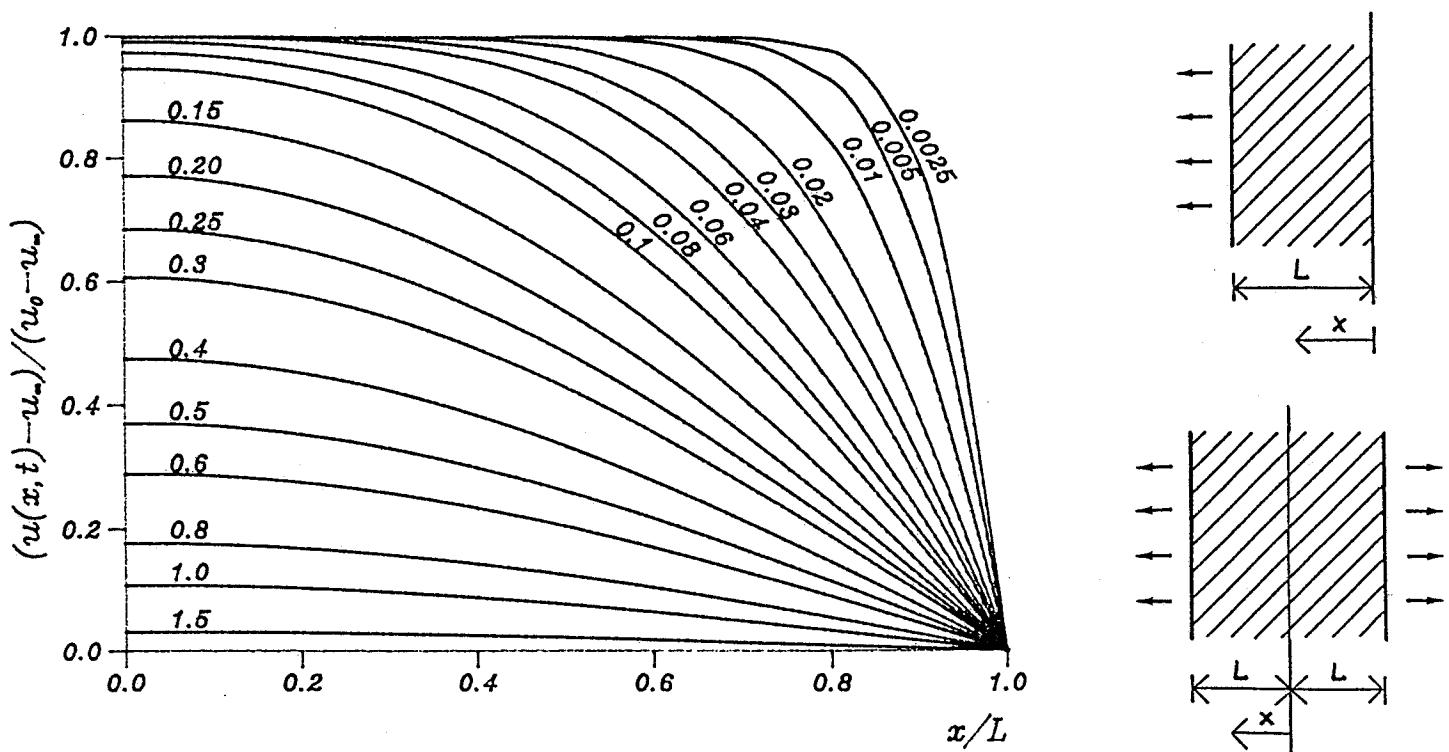
3.2 Uddata fra CARDIF

Uddata fra CARDIF foreligger dels i tabelform og dels i form af kurver. I figur 6 er vist et diagram, hvor man ved hjælp af to dimensionsløse størrelser kan bestemme fugtprofilet for en given plade med en given tykkelse og diffusionskoefficient som funktion af tiden.

Tallene foreligger også i tabelform. I bilag 13 er

$$\frac{u(r,t) - u_\infty}{u_0 - u_\infty} \quad (3.1)$$

tabelleret for $0 \leq Fo \leq 0.5$ med spring i Fo på 0.00125.



Figur 6. Fugtfordeling til givne tidspunkter i en uendeligt lang plade med startvandindhold u_0 og overfladevandindhold u_∞ . Tallene på kurverne er $Fo = Dt/L^2$.

I bilag 14 er 3.1 tabellag for $0 \leq Fo \leq 1.5$ med spring i Fo på 0.01.

3.3 Uddata fra MEANMOIS

Et eksempel på uddata fra MEANMOIS er anbragt i bilag 12. På grundlag af disse uddata er der fremstillet kurver over middelvandindholdet i plade-, cylinder- og kugletilfældet som vist i figur 7. Man går ind i diagrammet med den dimensionsløse størrelse

$$Fo = Dt/\delta^2$$

hvor δ er den karakteristiske dimension i henholdsvis plade-, cylinder- og kugletilfældet. Man kan da på de respektive kurver aflæse størrelsen U_m , der er givet som

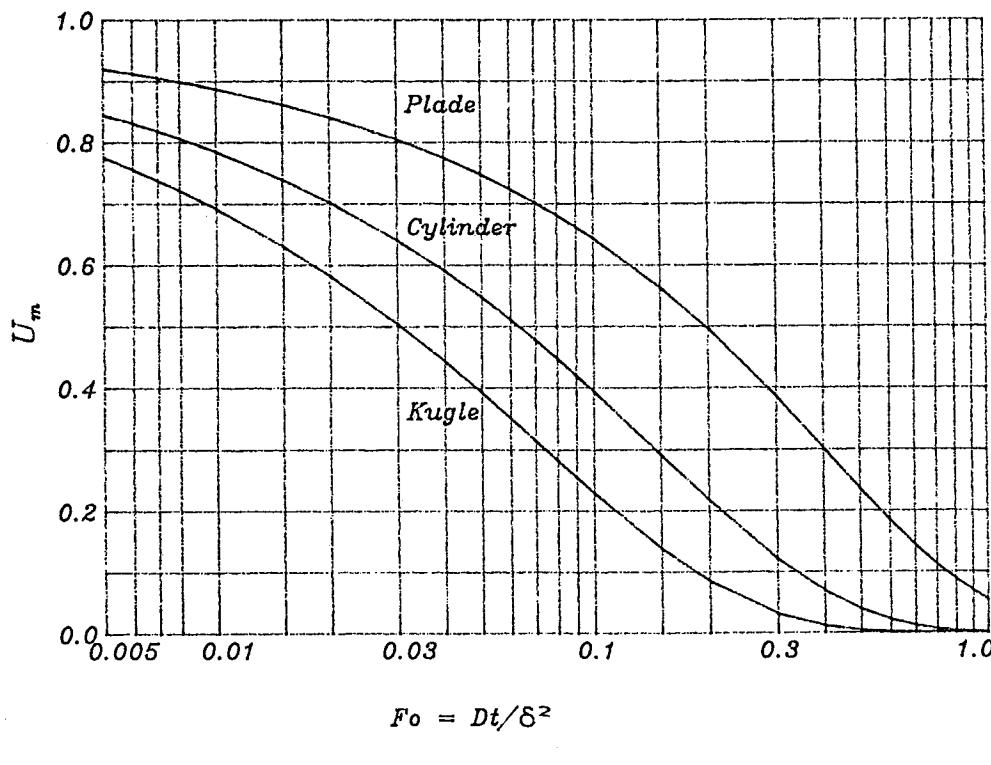
$$U_m = \frac{u_m(t) - u_\infty}{u_\infty - u_0}$$

hvor

$u_m(t)$ = konstruktionens middelvandindhold

u_∞ = slutvandindhold

u_0 = begyndelsesvandindhold



Figur 7. Kurver over middelvandindhold.

4. LITTERATURFORTEGNELSE

- /1/ Crank, J.: "The Mathematics of Diffusion". Oxford University Press, Oxford, 1975.
- /2/ Hansen, E.: "Sædvanlige differentialligninger fra fysikken". Polyteknisk Forlag, Lyngby, 1976.
- /3/ Hansen, P.N.: "Termiske beregningsmetoder". Laboratoriet for Varmeisolering, Danmarks tekniske Højskole, Notat til kurset Varmeisolering II, Lyngby, 1978.
- /4/ Ames, W.F.: "Numerical Methods for Partial Differential Equations". John Wiley, London, 1977.

- /5/ Nielsen, Hans Bruun: "Numerisk løsning af partielle differentialligninger". Numerisk Institut, Danmarks tekniske Højskole, Hæfte 39, Lyngby, 1977.
- /6/ Nielsen, Hans Bruun: "Løsning af lineære ligningssystemer". Numerisk Institut, Danmarks tekniske Højskole, Hæfte 23, Lyngby, 1972.
- /7/ Hansen, M.H., og Hansen, K.K.: "Sammenligning af simpel model for fugttransport og fugttransport beskrevet ved lineær diffusions-teori". Laboratoriet for Bygningsmaterialer, Danmarks tekniske Højskole, Teknisk Rapport 178/88, Lyngby, 1988.

BILAG 1. DIFFERENSLIGNING FOR u_o

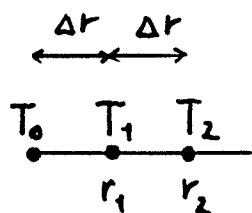
Dette bilag beskriver, hvordan der kan opstilles en differensligning med endelige differensers metode til bestemmelse af vandindholdet i centrum af en cylinder, hvori der foregår endimensional fugttransport beskrevet ved diffusionsligningen

$$\frac{\partial u}{\partial t} = D \left(\frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2} \right) \quad (2.5)$$

Der er valgt en eksplisit endelige differensers metode. Bestemmelsen sker ud fra et ræsonnement, som bygger på analogien diffusion af temperatur i en cylinder.

Opstilles differensligningen 2.10 for temperaturen i punktet T_1 , fås (jvf. figur B1)

$$\rho c_p \frac{T_1^{t_1+\Delta t} - T_1^{t_1}}{\Delta t} = \lambda \left(\frac{1}{r_1} \frac{T_2^{t_1} - T_0^{t_1}}{2\Delta r} + \frac{T_2^{t_1} + T_0^{t_1} - 2T_1^{t_1}}{(\Delta r)^2} \right) \quad (B1)$$



Figur B1. Punktbetegnelse.

Energifluxen for punkt 1 til punkt 2 er

$$\dot{q}_{1 \rightarrow 2} = \frac{2\pi\lambda(T_1 - T_2)}{\ln(r_2/r_1)} \quad (B2)$$

Arealflade 1 (vedrørende fladebetegnelse, se figur B2).

$$\begin{aligned} A_1 &= \pi \left(\left(\frac{r_1 + r_2}{2} \right)^2 - \left(\frac{r_1}{2} \right)^2 \right) \\ &= \frac{\pi r_2}{2} \left(\frac{r_2}{2} + r_1 \right) \end{aligned} \quad (B3)$$

Arealflade 0

$$A_0 = \pi \frac{r_1^2}{4} \quad (B4)$$

Energitilvækst flade 1 for $t_1 \rightarrow t_1 + \Delta t$

$$\Delta \dot{q}_1 = A_1 \frac{\rho c_p}{\Delta t} (T_1^{t_1 + \Delta t} - T_1^{t_1}) \quad (B5)$$

Energitilvækst flade 0 for $t_1 \rightarrow t_1 + \Delta t$

$$\Delta \dot{q}_0 = A_0 \frac{\rho c_p}{\Delta t} (T_0^{t_1 + \Delta t} - T_0^{t_1}) \quad (B6)$$

En energibalance for flade 1 og 0 giver

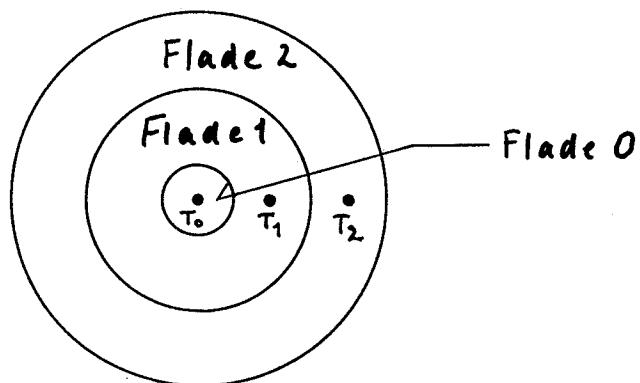
$$\Delta \dot{q}_0 + \Delta \dot{q}_1 + \dot{q}_{1 \rightarrow 2} = 0 \quad (B7)$$

Indsættes B6, B5 og B2 i B7, fås

$$A_0 \frac{\rho C_p}{\Delta t} (T_o^{t_1 + \Delta t} - T_o^{t_1}) + A_1 \frac{\rho C_p}{\Delta t} (T_1^{t_1 + \Delta t} - T_1^{t_1}) + \frac{2\pi\lambda(T_1^{t_1} - T_2^{t_1})}{\ln(r_2/r_1)} = 0 \quad (B8)$$

Indsættes B1, B3 og B4 i B8, fås

$$\pi \frac{r_1^2}{4} \frac{\rho C_p}{\Delta t} (T_o^{t_1 + \Delta t} - T_o^{t_1}) + \frac{\pi r_2}{2} \left(\frac{r_2}{2} + r_1 \right) \cdot \lambda \left(\frac{1}{r_1} \frac{T_2^{t_1} - T_o^{t_1}}{2\Delta r} + \frac{T_2^{t_1} + T_o^{t_1} - 2T_1^{t_1}}{(\Delta r)^2} \right) + \frac{2\pi\lambda(T_1^{t_1} - T_2^{t_1})}{\ln(r_2/r_1)} = 0 \quad (B9)$$



Figur B2. Fladebetegnelse.

For $r_2/r_1 = 2$, altså konstant r konstant, bliver B9

$$\frac{r_1^2}{4} \frac{\rho c_p}{\Delta t} \left(T_o^{t_1+\Delta t} - T_o^{t_1} \right) + 2r_1^2 \lambda \left(\frac{1}{r_1} \frac{T_2^{t_1} - T_o^{t_1}}{2\Delta r} + \frac{T_2^{t_1} + T_o^{t_1} - 2T_1^{t_1}}{(\Delta r)^2} \right) + \frac{2\lambda(T_1^{t_1} - T_2^{t_1})}{\ln 2} = 0 \quad (B10)$$

Idet $r = r_1$, bliver B10

$$\begin{aligned} & \frac{r_1^2}{4} \frac{\rho c_p}{\Delta t} \left(T_o^{t_1+\Delta t} - T_o^{t_1} \right) + \lambda \left(T_2^{t_1} - T_o^{t_1} + 2(T_2^{t_1} + T_o^{t_1} - 2T_1^{t_1}) \right) + \frac{2\lambda(T_1^{t_1} - T_2^{t_1})}{\ln 2} = 0 \Leftrightarrow \\ & \frac{r_1^2}{4} \frac{\rho c_p}{\Delta t} \left(T_o^{t_1+\Delta t} - T_o^{t_1} \right) + \lambda \left(T_o^{t_1} + \left(\frac{2}{\ln 2} - 4 \right) T_1^{t_1} + \left(3 - \frac{2}{\ln 2} \right) T_2^{t_1} \right) = 0 \Leftrightarrow \\ & T_o^{t_1+\Delta t} = T_o^{t_1} - \frac{4\rho c_p \Delta t}{r_1^2} \left(T_o^{t_1} + \left(\frac{2}{\ln 2} - 4 \right) T_1^{t_1} + \left(3 - \frac{2}{\ln 2} \right) T_2^{t_1} \right) \quad (B11) \end{aligned}$$

Da $r_1 = r$, og når der anvendes vandindholdet u som potentiale, fås

$$u_o^{t_1+\Delta t} = u_o^{t_1} - \frac{4D\Delta t}{(\Delta r)^2} \left(u_o^{t_1} + \left(\frac{2}{\ln 2} - 4 \right) u_1^{t_1} + \left(3 - \frac{2}{\ln 2} \right) u_2^{t_1} \right) \quad (B12)$$

Stabilitetskravet for B12 er

$$1 - \frac{4D\Delta t}{(\Delta r)^2} > 0$$

C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C NUMERISK L/SNING AF DIFFUSIONSLIGNING
C CYLINDERKOORDINATER
C 1-DIMENSIONALT
C EKPLICIT METODE
C FAST LAGTYKKELSE
C FAST TIDSSKRIDT
C
C MORTEN HJORSLEV HANSEN
C LABORATORIET FOR BYGNINGSMATERIALER 1987
C
C VERSION 2 6/10-1987
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
REAL*4 U(0:50),UNY(0:50)
OPEN(6,FILE='A:CYLDIF.UD',STATUS='NEW')
OPEN(7,FILE='A:CYLDIF.UD1',STATUS='NEW')
OPEN(8,FILE='A:CYLDIF.UD2',STATUS='NEW')
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C STYRING AF NETINDDELING
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
N=10
N1=N-1
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C DIFFUSIONSKOEFFICIENT
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
ALFA=1.0E-7
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C NETINDDELING
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
DELTAR=1.0/FLOAT(N)
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C TIDSSKRIDT
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
DELTI=20000.0
TID=0.0
TIDMAX=550.*DELTI
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C BEGYNDELSESBETINGELSER
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
USTART=0.3
USLUT=0.1
DO 10 I=0,N

```

      U(I)=USTART
10     UNY(I)=USTART
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C      RANDBETINGELSER
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
U(N)=USTART
UNY(N)=U(N)
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C      UDSKRIVNING AF BEGYNDELESFUGTINDHOLD
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      WRITE(6,25) TID,(U(I),I=0,4)
25 FORMAT(' ',F9.0,5F11.6)
      WRITE(7,30) TID,(U(I),I=5,10)
30 FORMAT (' ',F9.0,6F11.6)
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C      BEREGNINGSL/KKE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C      OPSKRIVNING AF TID
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      5 TID=TID+DELTI
      U(N)=URB(TID)
      DO 20 I=1,N1
          UNY(I)=(U(I+1)+U(I-1)-2*U(I))/(DELTAR**2)
          UNY(I)=UNY(I)+(U(I+1)-U(I-1))/(2*FLOAT(I)*DELTAR**2)
20      UNY(I)=U(I)+UNY(I)*ALFA*DELTI
      UNY(0)=U(0)-(4*ALFA*DELTI/DELTAR**2)*(U(0)+(2.0 ALOG(2.0)
$           -4.0)*U(1)+(3.0-2.0 ALOG(2.0))*U(2))
      FO=ALFA*TID
      ARG=(U(0)-U(5))/(U(0)-U(N))
      IF (ARG.LE.0.0) THEN
          XN=0.0
      ELSE
          XN=ALOG(ARG)/ ALOG(0.5)
      ENDIF
      IF ((USTART-U(0)).LE.0.0) THEN
          ALFAC=0.0
      ELSE
          ALFAC=-TID/ ALOG(1.0-(USTART-U(0))/(USTART-USLUT))
      ENDIF
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C      OMBYTNING AF FUGTINDHOLD
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      DO 40 I=0,N1
40      U(I)=UNY(I)
C

```


BILAG 3

KONTROL AF CYLDIF

Tids-punkt [s]	Radius [-]	"Analytisk"** løsning u [-]	Numerisk løsning u [-]	Numerisk løsning (DP)* u [-]
$r = 1/10 \quad t = 900 \text{ s}$				
54000	0	0.295344954	0.29538113	0.29538259
89100	0	0.290003032	0.29004008	0.29004228
89100	0.5	0.286741080	0.28670764	0.28670928
89100	0.9	0.281318333	0.28131056	0.28131102
$r = 1/10 \quad t = 900$				
10800	0	0.299998841	0.29999942	0.29999975
21600	0	0.29977652	0.29979372	0.29979360
21600	0.5	0.296859640	0.29675990	0.29676044
21600	0.9	0.284163338	0.28410345	0.28410371
$r = 1/10 \quad t = 450$				
10800	0	0.299998841	0.29999715	0.29999763
21600	0	0.299776562	0.29975724	0.29975834
21600	0.5	0.296859640	0.29676867	0.29676978
21600	0.9	0.284163338	0.28414148	0.28414199
$r = 1/20 \quad t = 225$				
10800	0	0.299998841	0.29999757	0.29999890
21600	0	0.299776562	0.29977918	0.29978162
21600	0.5	0.296859640	0.29683250	0.29683478
21600	0.9	0.284163338	0.28414708	0.28414807

* DP = double precision

**Jvf. ligning 2.8

PROGRAM CARDIF
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C NUMERISK LØSNING AF DIFFUSIONSLIGNING
C KARTESISKE KOORDINATER
C 1-DIMENSIONALT
C IMPLICET METODE
C FAST LAGTYKKELSE
C FAST TIDSSKRIDT
C
C MORTEN HJORSLEV HANSEN
C LABORATORIET FOR BYGNINGSMATERIALER 1987
C
C VERSION 1.0 23/9-87
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
REAL*8 U(50),UNY(50),A(50),B(50),C(50),D(50),E(50),G(50)
REAL*8 ALFA,DELTAX,DELTI,TID,TIDMAX,R,VRB,HRB
OPEN(6,FILE='A:CARDIF.UD',STATUS='OLD')
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C STYRING AF NETINDDELING
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
N=10
N1=N-1
N2=N-2
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C DIFFUSIONSKOEFFICIENT
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
ALFA=1.0D0
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C NETINDDELING
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
DELTAX=1.0D0/DFLOAT(N)
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C TIDSSKRIDT
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
DELTI=DELTAX**2
TID=0.0D0
TIDMAX=10.0D0*DELTI
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C OPSTILLING AF KOEFFICIENTMATRIX
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
R=ALFA*DELTI/DELTAX**2
B(1)=2.0D0+2.0D0*R
C(1)=-R

```

DO 1000 I=2,N2
  A(I)=-R
  B(I)=B(1)
1000   C(I)=-R
      A(N1)=-R
      B(N1)=B(1)
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C      GAUSSELIMINATION AF KOEFFICIENTMATRIX
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
CALL FAKT(A,B,C,E,G,N1)
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C      BEGYNDELSSES BETINGELSER
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C      DO 10 I=1,N1
C 10    U(I)=1.0D0
      U(1)=0.2D0
      U(2)=0.4D0
      U(3)=0.6D0
      U(4)=0.8D0
      U(5)=1.0D0
      U(6)=0.8D0
      U(7)=0.6D0
      U(8)=0.4D0
      U(9)=0.2D0
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C      UDSKRIVNING AF OVERSKRIFT
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      WRITE(6,28)
28 FORMAT('X = 0.0',3X,'0.1',4X,'0.2',4X,'0.3',4X,'0.4',4X,'0.5',4X,
     &'0.6',4X,'0.7',4X,'0.8',4X,'0.9',4X,'1.0')
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C      UDSKRIVNING AF FUGTINDHOLD
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      5 WRITE(6,25) TID
25 FORMAT(' ', 'TIDSPUNKT = ',F9.3,'S')
C      WRITE(6,28)
C 28 FORMAT(' ',19X,'X/L = ',5X,'0.0',6X,'0.1',6X,'0.2',6X,'0.3',6X,
C     &'0.4',6X,'0.5',6X,'0.6',6X,'0.7',6X,'0.8',6X,'0.9',6X,'1.0'//
C     &9X,'T',12X,'FO')
C
      5 FO=4.0D0*TID
      WRITE(6,30) VRB(TID),(U(I),I=1,N1),HRB(TID)
30 FORMAT(11F7.4//)
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C      BEREGNINGSLØKKE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      D(1)=R*VRB(TID)+(2.0D0-2.0D0*R)*U(1)+R*U(2)

```

```

      DO 35 I=2,NZ
35      D(I)=R*U(I-1)+(2.0D0-2.0D0*R)*U(I)+R*U(I+1)
      D(N1)=R*U(N2)+(2.0D0-2.0D0*R)*U(N1)+R*HRB(TID)
      CALL LOES(A,D,E,G,UNY,N1)

```

32

BACKSUBSTITUTION

```

X(N)=F(N)
DO 20 I=N1,1,-1
20      X(I)=F(I)-E(I)*X(I+1)
      RETURN
      END

```

C FUNCTION VRB
C GIVER RANDBETINGELSEN I VENSTRE SIDE AF PLADEN SOM EN FUNKTION AF
C TIDEN

DOUBLE PRECISION FUNCTION VRB(TID)

```

      DOGEE TID
      REAL*8 TID
      VRB=0.0DO
      RETURN
      END
```

C FUNCTION HRB
C GIVER RANDBETINGELSEN I HØJRE SIDE AF PLADEN SOM EN FUNKTION AF
C TIDEN

DOUBLE PRECISION FUNCTION HBB(TID)

```

      DOGEE TID
      REAL*8 TID
      HRB=0.0DO
      RETURN
      END

```

Bilag 5 Kontrol af CARDIF

X = 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
TIDSPUNKT = .000S										
.0000	.2000	.4000	.6000	.8000	1.0000	.8000	.6000	.4000	.2000	.0000
TIDSPUNKT = .010S										
.0000	.1989	.3956	.5834	.7381	.7691	.7381	.5834	.3956	.1989	.0000
TIDSPUNKT = .020S										
.0000	.1936	.3789	.5397	.6461	.6921	.6461	.5397	.3789	.1936	.0000
TIDSPUNKT = .030S										
.0000	.1826	.3515	.4902	.5843	.6152	.5843	.4902	.3515	.1826	.0000
TIDSPUNKT = .040S										
.0000	.1683	.3218	.4461	.5267	.5555	.5267	.4461	.3218	.1683	.0000
TIDSPUNKT = .050S										
.0000	.1538	.2932	.4047	.4770	.5019	.4770	.4047	.2932	.1538	.0000
TIDSPUNKT = .060S										
.0000	.1399	.2664	.3672	.4321	.4546	.4321	.3672	.2664	.1399	.0000
TIDSPUNKT = .070S										
.0000	.1270	.2418	.3330	.3916	.4119	.3916	.3330	.2418	.1270	.0000
TIDSPUNKT = .080S										
.0000	.1153	.2193	.3019	.3550	.3733	.3550	.3019	.2193	.1153	.0000
TIDSPUNKT = .090S										
.0000	.1045	.1989	.2738	.3219	.3385	.3219	.2738	.1989	.1045	.0000

BILAG 6

TRANSFORMATION AF KUGLETILFÆLDET TIL PLADETILFÆLDET

Givet den lineære partielle differentialligning for endimensional diffusion i en kugle

$$\frac{\partial u}{\partial t} = D \left(\frac{\partial^2 u}{\partial x^2} + \frac{2}{x} \frac{\partial u}{\partial x} \right) \quad (*)$$

Ved anvendelse af transformationen $w = ux$ fås for $x = 0$

$$u = \frac{w}{x}$$

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t} \left(\frac{w}{x} \right) = \frac{1}{x} \frac{\partial w}{\partial t}$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x} \frac{w}{x} = \frac{1}{x} \frac{\partial w}{\partial x} - \frac{w}{x^2}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x} \frac{\partial u}{\partial x} = \frac{\partial}{\partial x} \left(\frac{1}{x} \frac{\partial w}{\partial x} - \frac{w}{x^2} \right)$$

$$= -\frac{1}{x^2} \frac{\partial w}{\partial x} + \frac{1}{x} \frac{\partial^2 w}{\partial x^2} - \frac{\partial w}{\partial x} \frac{1}{x^2} + \frac{2}{x^3} w$$

Ved at indsætte i (*) fås

$$\frac{\partial w}{\partial t} = D \frac{\partial^2 w}{\partial x^2}$$

hvilket er den lineære partielle differentialligning for endimensional diffusion i en plade.

BILAG 7

MIDELVANDINDHOLD I PLADETILFÆLDET

Fugtfordelingen i uendelig plade med tykkelsen 2δ

$$\frac{u - u_\infty}{u_0 - u_\infty} = - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{(2n-1)} e^{-[(2n-1)\pi/2\delta]^2 \alpha t} \cdot \cos\left(\frac{2n-1}{2\delta} \pi x\right) (-1)^n$$

$$u(x,t) = u_\infty + (u_0 - u_\infty) f(x,t)$$

$$f(x,t) = - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{(2n-1)} e^{-[(2n-1)\pi/2\delta]^2 \alpha t} \cdot \cos\left(\frac{2n-1}{2\delta} \pi x\right) (-1)^n$$

Middelfugtindholdet er givet ved

$$\begin{aligned} \bar{u} &= \frac{1}{\delta - 0} \int_0^\delta u(x,t) dx \\ &= \frac{1}{\delta} \int_0^\delta u(x,t) dx \\ &= \frac{1}{\delta} \int_0^\delta [u_\infty + (u_0 - u_\infty) f(x,t)] dx \\ &= \frac{1}{\delta} \left[u_\infty x \right]_0^\delta + \frac{u_0 - u_\infty}{\delta} \int_0^\delta f(x,t) dx \\ &= u_\infty + \frac{u_0 - u_\infty}{\delta} \int_0^\delta f(x,t) dx \end{aligned}$$

$$\begin{aligned}
 & \int_0^\delta -\frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1} \cos\left(\frac{2n-1}{2\delta}\pi x\right) e^{-[(2n-1)\pi/2\delta]^2 \alpha t} dx = \\
 & -\frac{4}{\pi} \sum_{n=1}^{\infty} \int_0^\delta \frac{(-1)^n}{2n-1} \cos\left(\frac{2n-1}{2\delta}\pi x\right) e^{-[(2n-1)\pi/2\delta]^2 \alpha t} dx = \\
 & -\frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1} e^{-[(2n-1)\pi/2\delta]^2 \alpha t} \left[\frac{2\delta}{(2n-1)\pi} \sin\left(\frac{2n-1}{2\delta}\pi x\right) \right]_0^\delta = \\
 & -\frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1} e^{-[(2n-1)\pi/2\delta]^2 \alpha t} \frac{2\delta}{(2n-1)\pi} \sin(n-\frac{1}{2})\pi = \\
 & -\frac{8\delta}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)^2} e^{-[(2n-1)\pi/2\delta]^2 \alpha t} \sin(n-\frac{1}{2})\pi = \\
 & \frac{8\delta}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} e^{-[(2n-1)\pi/2\delta]^2 \alpha t}
 \end{aligned}$$

n	$(n-\frac{1}{2})\pi$	$\sin(n-\frac{1}{2})\pi$	$(-1)^n$	$(-1)^n \sin(n-\frac{1}{2})\pi$
1	$\frac{\pi}{2}$	1	-1	-1
2	$\frac{3\pi}{2}$	-1	1	-1
3	$\frac{5\pi}{2}$	1	-1	-1
4	$\frac{7\pi}{2}$	-1	1	-1

$$\begin{aligned}
 \bar{u} &= u_\infty + \frac{u_0 - u_\infty}{\delta} \frac{8\delta}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} e^{-[(2n-1)\pi/2\delta]^2 \alpha t} \\
 &= u_\infty + 8 \frac{u_0 - u_\infty}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} e^{-[(2n-1)\pi/2\delta]^2 \alpha t}
 \end{aligned}$$

BILAG 8

MIDDELVANDINDHOLD I CYLINDERTILFÆLDET

Fugtfordelingen i en uendeligt lang cylinder med radius R er givet som

$$\frac{u - u_\infty}{u_0 - u_\infty} = 2 \sum_{n=1}^{\infty} \frac{1}{\beta_n} e^{-(\beta_n/R)^2 \alpha t} \frac{J_0[\beta_n(r/R)]}{J_1(\beta_n)}$$

Middelfugtindholdet i cylinderen er givet som

$$\begin{aligned}\bar{u} &= \frac{\int_0^{2\pi} \int_0^R u(r,t) r dr d\theta}{\int_0^{2\pi} \int_0^R r dr d\theta} \\ &= \frac{2\pi \int_0^R u(r,t) r dr}{\pi R^2} \\ &= \frac{2}{R^2} \int_0^R u(r,t) r dr\end{aligned}$$

$$\begin{aligned}
 \int_0^R u(R,t) r dr &= \int_0^R [u_\infty + (u_0 - u_\infty) 2 \sum_{n=1}^{\infty} \frac{1}{\beta_n} e^{-(\beta_n/R)^2 \alpha t} \frac{J_0[\beta_n(r/R)]}{J_1(\beta_n)}] r dr = \\
 &= \left[\frac{1}{2} u_\infty r^2 \right]_0^R + 2(u_0 - u_\infty) \int_0^R \sum_{n=1}^{\infty} \frac{1}{\beta_n} e^{-(\beta_n/R)^2 \alpha t} \frac{J_0[\beta_n(r/R)]}{J_1(\beta_n)} r dr = \\
 &= \left[\frac{1}{2} u_\infty r^2 \right]_0^R + 2(u_0 - u_\infty) \sum_{n=1}^{\infty} \int_0^R \frac{1}{\beta_n} e^{-(\beta_n/R)^2 \alpha t} \frac{J_0[\beta_n(r/R)]}{J_1(\beta_n)} r dr = \\
 &= \frac{1}{2} u_\infty R^2 + 2(u_0 - u_\infty) \sum_{n=1}^{\infty} \frac{1}{\beta_n} \frac{e^{-(\beta_n/R)^2 \alpha t}}{J_1(\beta_n)} \int_0^R J_0[\beta_n(r/R)] r dr = \\
 &= \frac{1}{2} u_\infty R^2 + 2(u_0 - u_\infty) \sum_{n=1}^{\infty} \frac{1}{\beta_n} \frac{e^{-(\beta_n/R)^2 \alpha t}}{J_1(\beta_n)} \left(\frac{R}{\beta_n} \right)^2 \int_0^R \frac{r \beta_n}{R} J_0(\beta_n(r/R)) d\left(\frac{r \beta_n}{R}\right) = \\
 &= \frac{1}{2} u_\infty R^2 + 2(u_0 - u_\infty) \sum_{n=1}^{\infty} \frac{1}{\beta_n} \frac{e^{-(\beta_n/R)^2 \alpha t}}{J_1(\beta_n)} \left(\frac{R}{\beta_n} \right)^2 \left[\frac{r \beta_n}{R} J_1[\beta_n(r/R)] \right]_0^R = \\
 &= \frac{1}{2} u_\infty R^2 + 2(u_0 - u_\infty) \sum_{n=1}^{\infty} \frac{1}{\beta_n} \frac{e^{-(\beta_n/R)^2 \alpha t}}{J_1(\beta_n)} R^2 J_1(\beta) = \\
 &= \frac{1}{2} u_\infty R^2 + 2(u_0 - u_\infty) R^2 \sum_{n=1}^{\infty} \frac{e^{-(\beta_n/R)^2 \alpha t}}{\beta_n^2}
 \end{aligned}$$

dvs.

$$\bar{u} = u_\infty + 4(u_0 - u_\infty) \sum_{n=1}^{\infty} \frac{e^{-(\beta_n/R)^2 \alpha t}}{\beta_n^2}$$

Integration af J_0 jvf. 12/

BILAG 9

MIDDELVANDINDHOLD I EN KUGLE

Fugtfordelingen i en kugle er givet ved

$$\frac{u - u_\infty}{u_0 - u_\infty} = \frac{2}{\pi} R \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} e^{-(n\pi/R)^2 \alpha t} \sin\left(\frac{n\pi}{R} r\right) \Leftrightarrow$$

$$u(r, t) = u_\infty + (u_0 - u_\infty) f(r, t)$$

hvor

$$f(r, t) = \frac{2}{\pi} R \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} e^{-(n\pi/R)^2 \alpha t} \sin\left(\frac{n\pi}{R} r\right)$$

idet

$$x^2 + y^2 + z^2 = r^2 \Leftrightarrow$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

Middelfugtindholdet i kuglen er givet ved udtrykket

$$\begin{aligned} \bar{u}(t) &= \frac{\iiint_A u(r, t) dx dy dz}{vol(A)} = \\ &= \frac{\int_0^R \int_0^{2\pi} \left[\int_0^\pi [u_\infty + (u_0 - u_\infty) \frac{2}{\pi} R \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} e^{-(n\pi/R)^2 \alpha t} \sin\left(\frac{n\pi}{R} r\right)] r^2 \sin\theta dr d\theta d\phi \right] dr d\phi}{\frac{4}{3} \pi R^3} \\ &= \frac{\frac{4}{3} \pi R^3 u_\infty + 8(u_0 - u_\infty) \frac{R^3}{\pi} \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-(n\pi/R)^2 \alpha t}}{\frac{4}{3} \pi R^3} \\ &= u_\infty + \frac{6}{\pi^2} (u_0 - u_\infty) \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-(n\pi/R)^2 \alpha t} \end{aligned}$$

$$\begin{aligned}
& \int_0^R \int_0^{2\pi} \left[\int_0^\pi [u_\infty + (u_0 - u_\infty) \frac{2}{\pi} \frac{R}{r} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} e^{-(n\pi/R)^2 \alpha t} \sin\left(\frac{n\pi}{R} r\right)] r^2 \sin\theta d\theta \right] dr d\phi \\
&= \int_0^R \int_0^{2\pi} \left[[u_\infty + (u_0 - u_\infty) \frac{2}{\pi} \frac{R}{r} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} e^{-(n\pi/R)^2 \alpha t} \sin\left(\frac{n\pi}{R} r\right)] r^2 (-\cos\theta) \right] dr d\phi \\
&= \int_0^R \left[\int_0^{2\pi} \left[2r^2 u_\infty + (u_0 - u_\infty) \frac{4}{\pi} R r \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} e^{-(n\pi/R)^2 \alpha t} \sin\left(\frac{n\pi}{R} r\right) \right] d\phi \right] dr \\
&= 2\pi \int_0^R \left[2r^2 u_\infty + (u_0 - u_\infty) \frac{4}{\pi} R r \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} e^{-(n\pi/R)^2 \alpha t} \sin\left(\frac{n\pi}{R} r\right) \right] dr \\
&= 2\pi \left[2 \frac{r^3}{3} u_\infty \right]_0^R + 2\pi (u_0 - u_\infty) \frac{4}{\pi} R \int_0^R r \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} e^{-(n\pi/R)^2 \alpha t} \sin\left(\frac{n\pi}{R} r\right) dr \\
&= \frac{4}{3}\pi^2 R^3 u_\infty + 8(u_0 - u_\infty) R \int_0^R r \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} e^{-(n\pi/R)^2 \alpha t} \sin\left(\frac{n\pi}{R} r\right) dr \\
&= \frac{4}{3}\pi^2 R^3 u_\infty + 8(u_0 - u_\infty) R \left[-\frac{rR}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} e^{-(n\pi/R)^2 \alpha t} \cos\left(\frac{n\pi}{R} r\right) + \right. \\
&\quad \left. \frac{R^2}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3} e^{-(n\pi/R)^2 \alpha t} \sin\left(\frac{n\pi}{R} r\right) \right]_0^R \\
&= \frac{4}{3}\pi^2 R^3 u_\infty + 8(u_0 - u_\infty) R \left[-\frac{R^2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} e^{-(n\pi/R)^2 \alpha t} \cos(n\pi) + \right. \\
&\quad \left. \frac{R^2}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3} e^{-(n\pi/R)^2 \alpha t} \sin(n\pi) \right] \\
&= \frac{4}{3}\pi^2 R^3 u_\infty + 8(u_0 - u_\infty) R \left[-\frac{R^2}{\pi} \sum_{n=1}^{\infty} \frac{-1}{n^2} e^{-(n\pi/R)^2 \alpha t} \right] \\
&= \frac{4}{3}\pi^2 R^3 u_\infty + 8(u_0 - u_\infty) \frac{R^3}{\pi} \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-(n\pi/R)^2 \alpha t}
\end{aligned}$$

Mellemlægning:

$$\int r \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} e^{-(n\pi/R)^2 \alpha t} \sin\left(\frac{n\pi}{R}r\right) dr =$$

$$\begin{aligned} & r \int \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} e^{-(n\pi/R)^2 \alpha t} \sin\left(\frac{n\pi}{R}r\right) dr - \int -\frac{R}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} e^{-(n\pi/R)^2 \alpha t} \cos\left(\frac{n\pi}{R}r\right) dr \\ & = -\frac{rR}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} e^{-(n\pi/R)^2 \alpha t} \cos\left(\frac{n\pi}{R}r\right) + \frac{R}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} e^{-(n\pi/R)^2 \alpha t} \int \cos\left(\frac{n\pi}{R}r\right) dr \\ & = -\frac{rR}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} e^{-(n\pi/R)^2 \alpha t} \cos\left(\frac{n\pi}{R}r\right) + \frac{R}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} e^{-(n\pi/R)^2 \alpha t} \frac{R}{n\pi} \sin\left(\frac{n\pi}{R}r\right) \\ & = -\frac{rR}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} e^{-(n\pi/R)^2 \alpha t} \cos\left(\frac{n\pi}{R}r\right) + \frac{R^2}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3} e^{-(n\pi/R)^2 \alpha t} \sin\left(\frac{n\pi}{R}r\right) \end{aligned}$$

$$\int \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} e^{-(n\pi/R)^2 \alpha t} \sin\left(\frac{n\pi}{R}r\right) dr =$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} e^{-(n\pi/R)^2 \alpha t} \int \sin\left(\frac{n\pi}{R}r\right) dr =$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} e^{-(n\pi/R)^2 \alpha t} \left(-\frac{R}{n\pi} \cos\left(\frac{n\pi}{R}r\right) \right) =$$

$$-\frac{R}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} e^{-(n\pi/R)^2 \alpha t} \cos\left(\frac{n\pi}{R}r\right)$$

n	$n\pi$	$n+1$	$(-1)^{n+1}$	$\cos(n\pi)$	$\sin(n\pi)$	$(-1)^{n+1} \cos(n\pi)$
1	π	2	1	-1	0	(-1)
2	2π	3	-1	1	0	(-1)
3	3π	4	1	-1	0	(-1)
4	4π	5	-1	1	0	(-1)
5	5π	6	1	-1	0	(-1)

C
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C PROGRAM TIL BEREGNING AF ANALYSTISKE UDTRYK TIL BESKRIVELSE AF
C MIDDELFGUTINDHOLDET I EN PLADE, EN CYLINDER OG EN KUGLE.
C

C MORTEN HJORSLEV HANSEN
C LABORATORIET FOR BYGNINGSMATERIALER SEPTEMBER 1987
C VERSION 1.0 19/10-87
C

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INTEGER I, N, IER, NFO, NPLADE, NCYL, NKUGLE
REAL*8 FO(35), BETA(20), RODARG, PI, ARG
REAL*8 UPLADE, UKUGLE, UCYL
OPEN(6,FILE='A:MEANMOIS.UD', STATUS='OLD')

C
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C INITIALISERING
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PI=3.141592653589793

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C ANTAL FOURIERTAL
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NFO=35

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C R/DDER AF BESELFFUNKTIONEN AF 0' TE ORDEN
C

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BETA(1)=2.4048D0
BETA(2)=5.5201D0
BETA(3)=8.6537D0
BETA(4)=11.7915D0
BETA(5)=14.9309D0
BETA(6)=18.0711D0
BETA(7)=21.2116D0
BETA(8)=24.3525D0
BETA(9)=27.4935D0
BETA(10)=30.6346D0
BETA(11)=33.7758D0
BETA(12)=36.9171D0
BETA(13)=40.0584D0
BETA(14)=43.1998D0
BETA(15)=46.3412D0
BETA(16)=49.4826D0
BETA(17)=52.6241D0
BETA(18)=55.7655D0

C
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C FOURIERTAL
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FO(1)=0.0015D0
FO(2)=0.002D0

FO(3)=0.003D0
FO(4)=0.004D0
FO(5)=0.005D0
FO(6)=0.006D0
FO(7)=0.007D0
FO(8)=0.008D0
FO(9)=0.009D0
FO(10)=0.01D0
FO(11)=0.015D0
FO(12)=0.02D0
FO(13)=0.03D0
FO(14)=0.04D0
FO(15)=0.05D0
FO(16)=0.06D0
FO(17)=0.07D0
FO(18)=0.08D0
FO(19)=0.09D0
FO(20)=0.10D0
FO(21)=0.15D0
FO(22)=0.2D0
FO(23)=0.3D0
FO(24)=0.4D0
FO(25)=0.5D0
FO(26)=0.6D0
FO(27)=0.7D0
FO(28)=0.8D0
FO(29)=0.9D0
FO(30)=1.0D0
FO(31)=1.1D0
FO(32)=1.2D0
FO(33)=1.3D0
FO(34)=1.4D0
FO(35)=1.5D0

```
UCYL=UCYL*4.0D0
UKUGLE=0.0D0
C      WRITE(6,400)
C 400 FORMAT(' DETTE ER F/R L/KKE 3 ')
      DO 40 J=1,NKUGLE
          ARG=DFLOAT(J)
40      UKUGLE=UKUGLE+1.0D0/(ARG**2*DEXP((ARG*PI)**2*FO(I)))
      UKUGLE=UKUGLE*6.0D0/(PI**2)
      WRITE(6,50) FO(I),UPLADE,UCYL,UKUGLE,NPLADE,NCYL,NKUGLE
50      FORMAT(4F9.6,3I7)
10      CONTINUE
      STOP
      END
```

BILAG 11

NULPUNKTER FOR BESELFFUNKTIONEN AF 0. ORDEN 1. ART

Nulpunkterne β_n er beregnet efter formlen

$$\beta_n = \frac{1}{4} \pi a \left\{ 1 + \frac{2}{\pi^2 a^2} - \frac{62}{3\pi^4 a^4} + \frac{15116}{15\pi^6 a^6} - \frac{12544474}{105\pi^8 a^8} + \frac{8368654292}{315\pi^{10} a^{10}} \right\}$$

$$a = 4n - 1$$

hvor β_n er det n'te nulpunkt for J_0 , jvf. CRC Standard Mathematical Tables 26th Edition, p. 402.

n	β_n [CRC]	β_n [formel]
1	2.4048	2.4133
2	5.5201	5.5201
3	8.6537	8.6537
4	11.7915	11.7915
5	14.9309	14.9309
6	18.0711	18.0711
7	21.2116	21.2116
8		24.3525
9		27.4935
10		30.6346
11		33.7758
12		36.9171
13		40.0584
14		43.1998
15		46.3412
16		49.4826
17		52.6241
18		55.7655

Bilag 12

Udskrift fra en kørsel med MEANMOIS

FO	UM PLADE	UM CYLINDER	UM KUGLE	N PLADE	N CYLINDER	N KUGLE
.001500	.956298	.914109	.873394	97	18	96
.002000	.949537	.901104	.854612	84	18	83
.003000	.938196	.879438	.823588	68	18	68
.004000	.928635	.861334	.797905	59	18	59
.005000	.920211	.845507	.775635	53	18	52
.006000	.912596	.831299	.755788	48	18	48
.007000	.905593	.818317	.737779	45	18	44
.008000	.899075	.806307	.721224	42	18	41
.009000	.892953	.795091	.705858	39	18	39
.010000	.887162	.784540	.691486	37	18	37
.015000	.861802	.738996	.630407	31	18	30
.020000	.840423	.701450	.581269	26	18	26
.030000	.804559	.640245	.503677	22	18	21
.040000	.774324	.590416	.442972	19	18	18
.050000	.747687	.547893	.393060	17	17	16
.060000	.723605	.510585	.350814	15	15	15
.070000	.701459	.477254	.314377	14	14	14
.080000	.680846	.447091	.282538	13	13	13
.090000	.661487	.419538	.254458	12	13	12
.100000	.643177	.394189	.229521	12	12	11
.150000	.563050	.291875	.138734	10	10	9
.200000	.495912	.217862	.084504	8	8	8
.300000	.386764	.122036	.031475	7	7	6
.400000	.302118	.068436	.011731	6	6	5
.500000	.236050	.038382	.004372	5	5	5
.600000	.184435	.021526	.001630	5	5	4
.700000	.144107	.012073	.000607	4	4	4
.800000	.112597	.006771	.000226	4	4	4
.900000	.087977	.003798	.000084	4	4	3
1.000000	.068740	.002130	.000031	4	3	3
1.100000	.053710	.001195	.000012	4	3	3
1.200000	.041966	.000670	.000004	3	3	3
1.300000	.032790	.000376	.000002	3	3	3
1.400000	.025620	.000211	.000000	3	3	3
1.500000	.020018	.000118	.000000	3	3	3

Pladetilfældet. Tabel over $(u(r,t) - u_{\infty}) / (u_0 - u_{\infty})$ for $0 \leq F_0 \leq 0.5$ med spring på 0.00125.

$$X/L = 0.0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.6 \quad 0.7 \quad 0.8 \quad 0.9 \quad 1.0$$

.0128125	.99510	.99297	.93642	.87943	.78733	.65142	.46881	.24610
.0512500	.99455	.99227	.96870	.93332	.87506	.78190	.64566	.46388
.0525000	.98423	.99153	.98303	.96473	.93021	.87071	.77656	.64003
.0537500	.0134375	.99396	.98333	.98075	.96268	.92708	.86639	.77131
.0550000	.0137500	.99044	.98993	.98050	.96060	.92394	.86211	.76614
.0562500	.0140625	.99267	.98907	.97917	.95849	.92079	.85786	.76106
.0575000	.0143750	.99196	.98876	.98525	.97348	.94975	.90814	.84121
.0625000	.0156250	.0587500	.99122	.98818	.97780	.95635	.91763	.85364
.0637500	.0159375	.0587500	.98420	.9877	.97197	.94751	.90498	.83714
.0650000	.0162500	.0600000	.99044	.98724	.97042	.94524	.90182	.83311
.0612500	.0153125	.98962	.98627	.97496	.95198	.91131	.84532	.74632
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.0837500	.3350000	.55662	.52947	.49615	.45063	.39398	.08725
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.0865625	.3462500	.54146	.53482	.51504	.48261	.43830	.08457
.0868750	.3475000	.53980	.53318	.51346	.48112	.43695	.08431
.0871875	.3487500	.53815	.53154	.51189	.47965	.43560	.08405
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.0881250	.3525000	.5375000	.53322	.52667	.50719	.47524	.08327
.0884375	.3537500	.53158	.52506	.50563	.47378	.43027	.08301
.0887500	.3550000	.52995	.52345	.50408	.47233	.42894	.08276
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.0918750	.3675000	.51393	.50761	.48883	.45802	.41593	.08023
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.0971875	3887500	48178	43467	34503	22157	28684	15083	.07636
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.1015625	4112500	4112500	4112500	4112500	4112500	41125	37806	.27387
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.1021875	4137500	4137500	4137500	4137500	4137500	41379	37573	.27387
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.1050000	4200000	45168	44613	45165	44941	40747	37000	.32341
.1053125	4212500	45030	44476	45026	43358	40622	36886	.32242
.1056250	4225000	44891	44339	44888	43224	40497	36773	.32143
.1059375	4237500	45308	44750	45304	44625	40373	36660	.32044
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.1093750	4375000	43265	42732	43265	41148	39029	35439	.30976
.1106250	4425000	42735	42210	42735	41530	38909	35330	.30881
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.11109375	4437500	42604	42080	42604	40520	37963	34470	.30129

• 1112500	• 4450000	• 42473	• 41951	• 37846	• 24969	• 24965
• 1115625	• 4462500	• 42343	• 41822	• 41693	• 40148	• 37614
• 1118750	• 4475000	• 42213	• 41221	• 41183	• 41057	• 39535
• 1121875	• 4487500	• 42083	• 41565	• 40024	• 37498	• 37039
• 1125000	• 4500000	• 41954	• 41438	• 39901	• 37383	• 33944
• 1128125	• 4512500	• 41825	• 41310	• 39779	• 37268	• 33840
• 1131250	• 4525000	• 41697	• 41697	• 39657	• 37153	• 33736
• 1134375	• 4537500	• 41568	• 41057	• 39535	• 37039	• 33632
• 1137500	• 4550000	• 41441	• 40931	• 39413	• 36925	• 33529
• 1140625	• 4562500	• 41313	• 40805	• 39292	• 36812	• 33425
• 1143750	• 4575000	• 41187	• 40680	• 39171	• 36699	• 33323
• 1146875	• 4587500	• 41060	• 40555	• 39051	• 36586	• 33220
• 1150000	• 4600000	• 40934	• 40430	• 38931	• 36474	• 33118
• 1153125	• 4612500	• 40808	• 40306	• 38811	• 36362	• 33016
• 1156250	• 4625000	• 40683	• 40182	• 38692	• 36250	• 32915
• 1159375	• 4637500	• 40558	• 40059	• 38573	• 36138	• 32814
• 1162500	• 4650000	• 40433	• 39935	• 38455	• 36027	• 32713
• 1165625	• 4662500	• 40309	• 39813	• 38337	• 35917	• 32612
• 1168750	• 4675000	• 40185	• 39690	• 38219	• 35806	• 32512
• 1171875	• 4687500	• 40062	• 39569	• 38101	• 35696	• 32412
• 1175000	• 4700000	• 39939	• 39447	• 37984	• 35587	• 32313
• 1178125	• 4712500	• 39816	• 39326	• 37868	• 35477	• 32213
• 1181250	• 4725000	• 39694	• 39205	• 37751	• 35368	• 32114
• 1184375	• 4737500	• 39572	• 39084	• 37635	• 35260	• 32016
• 1187500	• 4750000	• 39450	• 38964	• 37520	• 35151	• 31917
• 1190625	• 4762500	• 39329	• 38845	• 37404	• 35043	• 31819
• 1193750	• 4775000	• 39208	• 38725	• 37289	• 34935	• 31721
• 1196875	• 4787500	• 39087	• 38606	• 37175	• 34828	• 31624
• 1200000	• 4800000	• 38967	• 38488	• 37061	• 34721	• 31527
• 1203125	• 4812500	• 38848	• 38369	• 36947	• 34614	• 31430
• 1206250	• 4825000	• 38728	• 38252	• 36833	• 34508	• 31333
• 1209375	• 4837500	• 38609	• 38134	• 36720	• 34402	• 31237
• 12121875	• 4887500	• 38137	• 37668	• 36271	• 33981	• 30855
• 1212500	• 4850000	• 38491	• 38017	• 36607	• 34296	• 31141
• 1215625	• 4862500	• 38372	• 37900	• 36495	• 34191	• 31045
• 1218750	• 4875000	• 38255	• 37784	• 36383	• 34086	• 30950
• 1221875	• 4887500	• 38137	• 37668	• 36271	• 33981	• 30855
• 1225000	• 4900000	• 38020	• 37552	• 36159	• 33877	• 30760
• 1228125	• 4912500	• 37903	• 37436	• 36048	• 33772	• 30665
• 1231250	• 4925000	• 37787	• 37321	• 35937	• 33669	• 30571
• 1234375	• 4937500	• 37670	• 37207	• 35827	• 33565	• 30477
• 1237500	• 4950000	• 37555	• 37092	• 35717	• 33462	• 30383
• 1240625	• 4962500	• 37439	• 37903	• 36978	• 33359	• 30290
• 1243750	• 4975000	• 37324	• 36865	• 35498	• 33257	• 30197
• 1246875	• 4987500	• 37210	• 36752	• 35389	• 33155	• 30104
• 1250000	• 5000000	• 37095	• 36639	• 35280	• 33053	• 30012

Pladetilfældet. Tabel over $(u(r,t) - u_{\infty}) / (u_{\infty} - u_{\infty})$ for
 $0 \leq F_o \leq 1.5$ med spring i F_o på 0.01.

X/L =	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
T	FO										
0.0000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
.0025000	.999999	.999998	.999995	.99980	.99926	.99924	.98969	.96152	.85641	.46410	.00000
.0050000	.99991	.99984	.99951	.99840	.99487	.98405	.95238	.86672	.66838	.38120	.00000
.0075000	.99951	.99919	.99782	.99385	.98320	.95651	.89559	.77510	.58403	.31310	.00000
.0100000	.99826	.99734	.99377	.98469	.96397	.92082	.84051	.70962	.51835	.27560	.00000
.0125000	.99544	.99354	.98668	.97117	.94026	.88436	.79268	.65594	.47220	.24764	.00000
.0150000	.99042	.98730	.97666	.95464	.91495	.84965	.75070	.61286	.43586	.22702	.00000
.0175000	.98293	.97855	.96421	.93633	.88949	.81733	.71419	.57690	.40687	.21068	.00000
.0200000	.97303	.96750	.94985	.91703	.86457	.78758	.68208	.54650	.38288	.19744	.00000
.0225000	.96099	.95449	.93407	.89725	.84052	.76020	.65365	.52032	.36266	.18639	.00000
.0250000	.94718	.93987	.91723	.87731	.81743	.73496	.62823	.49746	.34529	.17699	.00000
.0275000	.93193	.92399	.89964	.85740	.79531	.71157	.60531	.47724	.33014	.16886	.00000
.0300000	.91559	.90718	.88156	.83765	.77410	.68979	.58444	.45916	.31674	.16172	.00000
.0325000	.89843	.88969	.86317	.81814	.75375	.66939	.56529	.44281	.30477	.15538	.00000
.0350000	.88071	.87173	.84462	.79892	.73416	.65019	.54757	.42779	.29394	.14968	.00000
.0375000	.86262	.85350	.82605	.78003	.71529	.63202	.53107	.41416	.28406	.14450	.00000
.0400000	.84431	.83513	.80754	.76149	.69707	.61476	.51559	.40142	.27496	.13975	.00000
.0425000	.8170000	.82593	.78916	.74331	.67946	.59828	.50099	.38951	.26651	.13537	.00000
.0450000	.1800000	.80758	.79842	.77099	.72550	.66240	.58250	.48715	.37832	.25862	.00000
.0475000	.1900000	.78933	.78024	.75306	.70808	.64586	.56734	.47397	.36774	.25120	.00000
.0500000	.2000000	.77125	.76226	.73540	.69103	.629980	.55275	.46138	.35768	.24418	.00000
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.0550000	.2200000	.73579	.72705	.70100	.65807	.59904	.52506	.43768	.33891	.23115	.00000
.0575000	.2300000	.71847	.70989	.68429	.64216	.58429	.51187	.42648	.33009	.22505	.00000
.0600000	.2400000	.70146	.69304	.66792	.62661	.56993	.49909	.41566	.32160	.21920	.00000
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.0700000	.2800000	.63673	.622898	.60590	.56800	.51614	.45152	.37566	.29038	.19778	.00000
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.1300000	.5200000	.35311	.34876	.33583	.31462	.28567	.24969	.20756	.06096
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.1375000	.5500000	.32796	.32393	.30433	.28511	.25888	.22627	.18809	.05390
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.1425000	.5700000	.31220	.31220	.29944	.28970	.27141	.24644	.21539	.05259
.1450000	.5800000	.30461	.30086	.28086	.28266	.26481	.24044	.21015	.05131
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