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Stresses in Wood Caused by Drying

by

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PREFACE AND ABSTRACT

It is well-known that drying, i.e. moisture loss, builds up tensile shrinkage stresses in wood which may be high enough to violate the natural strength of the materials structure. This means that cracks develop which reduce considerably the final strength of dry wood.

It is also known that fast drying is more severe in this respect than slow drying. This observation is the basis of any empirical rule which have been suggested in drying technology to optimize economy (time) and quality of wood.

Obviously these two observations on the drying behavior of wood are consistent with the concept of wood being a viscoelastic material weakened by inherent defect nuclei (e.g. bad fiber bondings, pit concentrations and rays) which will develop into cracks when exposed to some critical stress.

Assuming linear viscoelasticity the present report contributes to the theoretical research on drying technology by presenting relationships between moisture loss and internal stresses in wood.

In a subsequent publication these results will be correlated to crack mechanics such that drying can be related directly to the final strength of wood.

The work reported has been carried out late 1985 at the Building Materials Laboratory, Technical University of Denmark, as part of a research project on "The mechanical Durability of Wood" fonded by the Danish Technical Research Council (StvF-16-3785.B-172).

Stresses in Wood Caused by Drying

1. INTRODUCTION

It is well-known that drying, i.e. moisture loss, builds up tensile shrinkage stresses in wood which may be high enough to violate the natural strength of the materials structure. This means that cracks develop which reduce considerably the final strength of dry wood.

It is also known that fast drying is more severe in this respect than slow drying. This observation is the basis of any empirical rule which have been suggested in drying technology to optimize economy (time) and quality of wood.

Obviously these two observations on the drying behavior of wood are consistent with the concept of wood being a viscoelastic material weakened by inherent defect nuclei (e.g. bad fiber bondings, pit concentrations and rays) which will develop into cracks when exposed to some critical stress.

The present report contributes to the theoretical research on drying technology by an analysis explaining the relationship between moisture loss and the internal stress state of wood. Such an analysis is the first step towards a theoretical prediction of strength as related to drying procedures. A next step which will be considered in a subsequent publication is to integrate crack mechanics.

The stress analysis presented is made on a drying pole. In principle, however, the same method can be applied to any wood member. Two principal assumptions are made on structure and stress-strain response:

1) The traditional concept of wood structure is maintained: Wood is a cylindrically homogeneous and cylindrically orthotropic material. The axis of symmetry is the trunk pith (in this report the pole axis). The three principal axes referred to with respect to homogeneity and orthotropy are defined by the R(adial), T(angential) and L(ongitudinal) directions of wood.

2) The mechanical behavior of wood can be described linear-viscoelastically according to the Power Law description (e.g. 1).

2. MOISTURE CONTENT

The moisture content of the pole is assumed to be axial-symmetrically distributed as defined by the parabolic profile shown in Figure 2.1.

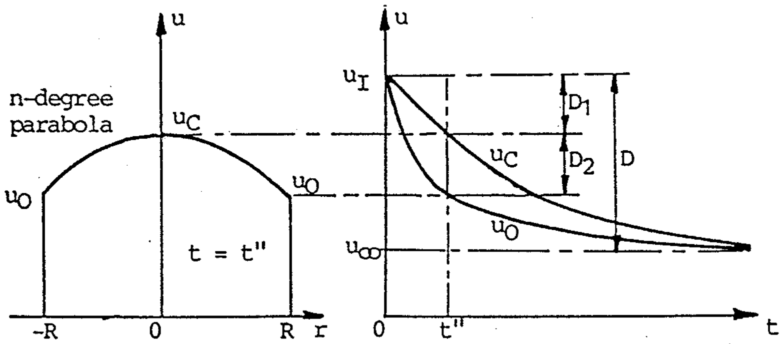

Figure 2.1. Distribution and time dependency of pole moisture content, u. Location coordinate is denoted by r. The pole radius is R.

The time-dependency of the moisture content is defined by the moisture history at the center, $u_C = u_C(t)$ and at the outside of the pole, $u_O = u_O(t)$.

For simplicity we consider the pole to have a location independent moisture content both at t = 0 and at $t \rightarrow \infty$. The initial quantity, u_I, is less than or equal to the moisture content, u_F = 25 - 30 %, at fiber saturation. The final value, u_∞ (also $\leq u_F$), is the target moisture content. Both moisture histories are assumed to decrease exponentially such that

$$u_C = u_I - D*(1 - e^{-t/\alpha_C})$$
$$u_O = u_I - D*(1 - e^{-t/\alpha_O}) \tag{2.1}$$

where the parameters, α_C and α_O, define the drying rate at the center and at the outside respectively of the pole. The total moisture loss, D, as $t \to \infty$ is given by,

$$D = u_I - u_\infty \tag{2.2}$$

The moisture differences, D_1 and D_2, illustrated in Figure 2.1 are of special interest for the stress analysis made in the following sections. We get from Equation 2.1

$$D_1 = u_I - u_C = D*(1 - e^{-t/\alpha_C})$$
$$D_2 = u_C - u_O = D*(e^{-t/\alpha_C} - e^{-t/\alpha_O}) \tag{2.3}$$

The moisture loss, M, at an arbitrary location, r, in the pole is given by

$$M = M(r) = D_1 + D_2(\frac{r}{R})^n \tag{2.4}$$

where n, according to Figure 2.1, refers to the n-degree parabolic moisture distribution in the pole.

3. SHRINKAGE

It is assumed that shrinkage strain, ϵ_{SH}, of wood and moisture content, u, are related linearely according to

$$d\epsilon_{T,SH}/du = s_T;\ d\epsilon_{R,SH}/du = s_R;\ d\epsilon_{L,SH}/du = s_L \tag{3.1}$$

where the subscripts, T, R and L, refer to directions previously defined. Notice that strain increases with increasing moisture content. In the following analysis we will relate shrinkage strain to moisture loss, M, meaning that $d\epsilon_{i,SH}/dM = -s_i$, (i = R,T,L).

The shrinkage coefficients have the following orders of magnitude when moisture content is given in %,

$$s_T \approx 4*10^{-3}/\% \ ;\ s_R \approx 2*10^{-3}/\% \ ;\ s_L \approx 10^{-4}/\% \tag{3.2}$$

The shrinkage ratio, S, used in the stress analysis is defined by

$$S = s_R/s_T \approx 0.5 \qquad (3.3)$$

4. ELASTIC STRESS ANALYSIS

In the linear range a stress analysis with respect to shrinkage is similar to a thermal stress analysis. This is because both moisture and temperature have similar strain responses. In the case of temperature we only have to replace (u,s) with (T,α) where T is temperature and α is the coefficient of thermal expansion. At the same time, of course, subscript, sh, changes to T.

The analysis presented below follows the stress analysis of a homogeneous and linear elastic pole exposed to an axial symmetric temperature field presented by Timoshenko and Goodier (2). Essential modifications, however, have been introduced which consider anisotropy with respect to both elasticity and materials response to shrinkage. The analysis is at first based on the assumption of plane stress, meaning that $\sigma_L \equiv 0$. Subsequently, however, modifications are given which generalize the solutions obtained to be valid also when plane strain (fixed end and free end) situations are considered.

The practical results of the analysis applying to real wood poles are <u>summarized in Section 4.3</u>.

Conditions of equilibrium require in general

$$\frac{d\sigma_R}{dr} + \frac{\sigma_R - \sigma_T}{r} = 0 \qquad (4.1)$$

where σ_i is stress in the i-direction.

Conditions of compatibility require in general

$$\begin{aligned} \epsilon_R &= \frac{du_R}{dr} \\ \epsilon_T &= \frac{u_R}{r} \end{aligned} \qquad (4.2)$$

where u_R is the radial deformation and ϵ_i is strain in the i-direction.

4.1 PLANE STRESS (CIRCULAR DISK)

The plane stress constitutive equation of a cylindrically orthotropic material requires

$$\epsilon_R + Ms_R = \frac{\sigma_R}{E_R} - \mu_{TR}\frac{\sigma_T}{E_T} \tag{4.3}$$

$$\epsilon_T + Ms_T = \frac{\sigma_T}{E_T} - \mu_{RT}\frac{\sigma_R}{E_R} \tag{4.4}$$

where M is moisture loss according to Equation 2.4. E_i is Young's modulus in the i-direction, while μ_{ij} is Poisson's ratio referring to strain, $-\mu_{ij}/E_i$, in the j-direction for extension, $1/E_i$, in the i-direction. The following relation applies between Young's moduli and Poisson's ratios for orthotropic materials

$$\frac{\mu_{ij}}{E_i} = \frac{\mu_{ji}}{E_j} \tag{4.5}$$

When Equations 4.3 and 4.4 are solved with respect to stresses we get

$$\sigma_R = \frac{E_R}{1-\mu^2}[\epsilon_R + Ms_R + \mu_R(\epsilon_T+Ms_T)] \tag{4.6}$$

$$\sigma_T = \frac{E_T}{1-\mu^2}[\epsilon_T + Ms_T + \mu_T(\epsilon_R+Ms_R)] \tag{4.7}$$

where the following abbreviations have been introduced,

$$\mu_T = \mu_{RT}$$

$$\mu_R = \mu_{TR} \tag{4.8}$$

$$\mu = \sqrt{\mu_{RT}\mu_{TR}} = \sqrt{\mu_T\mu_R}$$

Another abbreviation used in the following text is the so-called stiffness ratio, N, defined by

$$N = \frac{E_T}{E_R} = \frac{\mu_R}{\mu_T} \ (\approx 0.5) \Rightarrow E_T = NE_R \text{ and } \mu_R = N\mu_T \tag{4.9}$$

where the alternative definition is obtained by Equations 4.5 and 4.8. The order of magnitude given for N is adequate for many types of wood.

Equations 2.4, 4.1, 4.6 and 4.7 combine as follows

$$\frac{d^2u_R}{dr^2} + \frac{1}{r}\frac{du_R}{dr} - N\frac{u_R}{r^2} = F(r) \tag{4.10}$$

where F(r) is given by

$$F(r) = \frac{s_T}{r}[(NA_T - A_R)*D_1 + (NA_T - (1+n)A_R)(\frac{r}{R})^n*D_2] \quad (4.11)$$

N is the stiffness ratio from Equation 4.9 and

$$A_R = S+\mu_R \quad ; \quad A_T = 1+S\mu_T \quad (4.12)$$

where the shrinkage ratio, S, is defined by Equation 3.3.

A solution of u_R to the homogeneous differential equation, Equation 4.10 with $F(r) \equiv 0$, is given by

$$u_{R,1} = r^{\sqrt{N}} \quad (4.13)$$

Another solution to Equation 4.12 is obtained by $u_{R,1}$ using a method given in (3, p.398). We get

$$u_{R,2} = u_{R,1} \int \frac{1}{r u_{R,1}^2} dr = -\frac{1}{2\sqrt{N}} r^{-\sqrt{N}} \quad (4.14)$$

The complete solution, $u_{R,H}$, to the homogeneous Equation 4.10 ($F(r) \equiv 0$) is now the linear combination of $u_{R,1}$ and $u_{R,2}$,

$$u_{R,H} = C_1 r^{\sqrt{N}} + C_2 \frac{1}{2\sqrt{N}} r^{-\sqrt{N}} \quad (4.15)$$

where C_1 and C_2 are constants.

A particular solution, $u_{R,I}$, to the non-homogeneous Equation 4.10 is, according to (3, p.398), given by

$$u_{R,I} = \int_{r_o}^{r} \theta F(\theta)[u_{R,2}(r)u_{R,1}(\theta) - u_{R,1}(r)u_{R,2}(\theta)]d\theta \quad (4.16)$$

We get (with $r_o = 0$),

$$\frac{u_{R,I}}{rs_T} = \frac{NA_T - A_R}{1-N}*D_1 + \frac{NA_T - (1+n)A_R}{(1+n)^2 - N}(\frac{r}{R})^n*D_2 \quad (4.17)$$

The complete solution to the non-homogeneous differential equation 4.10 is now $u_R = u_{R,H} + u_{R,I}$ giving

$$\frac{u_R}{rs_T} = \frac{NA_T - A_R}{1-N}*D_1 + \frac{NA_T - (1+n)A_R}{(1+n)^2 - N}(\frac{r}{R})^n*D_2 + Cr^{\sqrt{N}-1} \quad (4.18)$$

where $C_2 = 0$ has been introduced in order to get $u_R(0) = 0$.

The constant, $C = C_1$, appearing in Equation 4.18 has to be determined from the boundary condition, $\sigma_R(R) = 0$.

The radial stress, σ_R, is determined by Equation 4.6 with strains, ϵ_R and ϵ_T, derived from Equation 4.2 together with Equation 4.18.

We get

$$\frac{\epsilon_R}{s_T} = \frac{NA_T - A_R}{1-N} * D_1 + (1+n)\frac{NA_T-(1+n)A_R}{(1+n)^2-N}\left(\frac{r}{R}\right)^n * D_2 + C\sqrt{N} r^{\sqrt{N}-1} \tag{4.19}$$

and

$$\frac{\epsilon_T}{s_T} = \frac{NA_T - A_R}{1-N} * D_1 + \frac{NA_T-(1+n)A_R}{(1+n)^2-N}\left(\frac{r}{R}\right)^n * D_2 + C\sqrt{N} r^{\sqrt{N}-1} \tag{4.20}$$

which produce C as outlined above: $\sigma_R(R) = 0 \rightarrow$

$$C = (1-\mu) s_T \sqrt{N} * R^{1-\sqrt{N}}\left[\frac{1-S}{1-N} * D_1 + \frac{1+n-S}{(1+n)^2-N} * D_2\right] \tag{4.21}$$

where the shrinkage ratio, S, has been introduced from Equation 3.3. The Poisson's ratio, μ, is defined by Equation 4.8.

When C from Equation 4.21 is introduced into Equation 4.18 we finally get the solution for the <u>radial deformation</u> of a pole exposed to an axial symmetric moisture distribution. The corresponding <u>radial and tangential stresses</u> are given by Equations 4.6, 4.7, 4.19, 4.20 and 4.21. We get

<u>Final plane stress results:</u>

$$\frac{u_R}{r s_T} = \frac{1}{1-N}\left[N - \mu_R - (1-\mu_R)S - (1-S)(\sqrt{N}-\mu_R)\left(\frac{r}{R}\right)^{\sqrt{N}-1}\right] * D_1$$
$$+ \frac{1}{(1+n)^2-N}\left[\left((N-(1+n)\mu_R) - (1+n-\mu_R)S\right)\left(\frac{r}{R}\right)^n\right.$$
$$\left. - (1+n-S)(\sqrt{N}-\mu_R)\left(\frac{r}{R}\right)^{\sqrt{N}-1}\right] * D_2 \tag{4.22}$$

$$\frac{\sigma_R}{s_T E_T} = \frac{1-S}{1-N}\left[1 - \left(\frac{r}{R}\right)^{\sqrt{N}-1}\right] * D_1$$
$$+ \frac{1+n-S}{(1+n)^2-N}\left[\left(\frac{r}{R}\right)^n - \left(\frac{r}{R}\right)^{\sqrt{N}-1}\right] * D_2 \tag{4.23}$$

$$\frac{\sigma_T}{s_T E_T} = \frac{1-S}{1-N}\left[1-\sqrt{N}\left(\frac{r}{R}\right)^{\sqrt{N}-1}\right]*D_1$$

$$+ \frac{1+n-S}{(1+n)^2-N}\left[(1+n)\left(\frac{r}{R}\right)^{n} - \sqrt{N}\left(\frac{r}{R}\right)^{\sqrt{N}-1}\right]*D_2 \quad (4.24)$$

It is noticed that Poisson ratios appear only in the deformation expression.

Another note should be made: As the stiffness ratio, N, is less than or equal to 1 Equations 4.23 and 4.24 predict infinite stress values at the center (r = 0) of the pole. This feature, however, is of minor practical significance. We only have to think of a very small area at the center where wood is in a state of flow. This corresponds to introducing a small, finite value of r_0 in Equation 4.16.

Pole surface:

The situation at the pole surface (r = R) is of special interest. We get $\sigma_R = 0$ and

$$\frac{u_R(R)}{Rs_T} = -(S+\sqrt{N})\left[\frac{D_1}{1+\sqrt{N}} + \frac{D_2}{1+n+\sqrt{N}}\right] \quad (4.25)$$

$$\frac{\sigma_T(R)}{E_T s_T} = \frac{1-S}{1+\sqrt{N}}*D_1 + \frac{1+n-S}{1+n+\sqrt{N}}*D_2 \quad (4.26)$$

At uniform moisture distribution ($D_2 = 0$) the average shrinkage coefficient, $s_{AV} = u_R(R)/(-RD_1)$, of the pole cross-section is immediately derived from Equation 4.25. We get

$$s_{AV} = \frac{s_R + s_T\sqrt{N}}{1 + \sqrt{N}} \quad (4.27)$$

4.12 PLANE ISOTROPY:

Often wood can be approximated elastically as being a plane isotropic material. This means that the TR-plane is considered isotropic with the elastic coefficients E_{90} and μ_{90}. Practically we may use the following averages,

$$E_{90} = \sqrt{E_R E_T}$$
$$\mu_{90} = \sqrt{\mu_{RT}\mu_{TR}} \quad (4.28)$$

Notice that $\mu_{90} = \mu$ as defined in Equation 4.8.

The plane isotropic suolutions for deformation and stresses are obtained by Equations 4.22 -4.24 introducing N = 1 and coefficients of elasticity as given by Equation 4.28.

The transformation is made utilizing that

$$\left(\frac{r}{R}\right)^{\sqrt{N}-1} \rightarrow 1 + (\sqrt{N}-1)\log_e\left(\frac{r}{R}\right) \quad \text{when } N \rightarrow 1 \qquad (4.29)$$

We get

$$\frac{u_R}{r s_T} = -\frac{1-S}{2}\left[1 - (1-\mu_{90})\log_e\left(\frac{r}{R}\right)\right]*D_1$$

$$-\frac{1}{n(2+n)}\left[(1-\mu_{90})(1+n-S)\right.$$

$$\left.-\{(1-(1+n)\mu_{90})-(1+n-\mu_{90})S\}\left(\frac{r}{R}\right)^n\right]*D_2 \qquad (4.30)$$

$$\frac{\sigma_R}{E_{90}s_T} = \frac{1-S}{2}\log_e\left(\frac{r}{R}\right)*D_1 - \frac{1+n-S}{n(2+n)}\left\{1-\left(\frac{r}{R}\right)^n\right\}*D_2 \qquad (4.31)$$

$$\frac{\sigma_T}{E_{90}s_T} = \frac{1-S}{2}\left(1+\log_e\left(\frac{r}{R}\right)\right)*D_1$$

$$-\frac{1+n-S}{n(2+n)}\left\{1-(1+n)\left(\frac{r}{R}\right)^n\right\}*D_2 \qquad (4.32)$$

4.2 PLANE STRAIN (CIRCULAR POLE)

Plane strain modifications of the deformation and stress solutions obtained in the preceding section will now be given on the basis of wood modelled as a plane isotropic material.

The general constitutive conditions relevant for the problem considered are expressed by

$$\epsilon_R + Ms_R = \frac{1}{E_{90}}(\sigma_R - \mu_{90}\sigma_T - \mu'_0\sigma_L) \qquad (4.33)$$

$$\epsilon_T + Ms_T = \frac{1}{E_{90}}(\sigma_T - \mu_{90}\sigma_R - \mu'_0\sigma_L) \qquad (4.34)$$

$$\epsilon_L + Ms_L = \frac{1}{E_0}[\sigma_L - \mu'_0(\sigma_R + \sigma_T)] \qquad (4.35)$$

where the Poisson's ratio μ'_0 defines strain, $-\mu'_0/E_{90}$, in the 0-direction (L) for extension in the 90-direction (R = T). According to Equation 4.5 we have

$$\frac{\mu_\Theta}{E_\Theta} = \frac{\mu'_\Theta}{E_{\varphi\Theta}} \tag{4.36}$$

where the Poisson's ratio, μ_Θ defines strain, $-\mu_\Theta/E_\Theta$, in the 90-direction for extension in the 0-direction.

Fixed ends:

When plane strain is considered with fixed ends of the pole we have $\epsilon_L = 0$ which introduced into Equation 4.33 gives us

$$\sigma_{L,FIXED} = \mu'_\Theta(\sigma_R + \sigma_T) + E_\Theta M s_L \tag{4.37}$$

$$\epsilon_R + M(s_R + \mu_\Theta s_L) = \frac{1-(\mu'_\Theta)^2}{E_{\varphi\Theta}}\left(\sigma_R - \frac{\mu_{\varphi\Theta}+(\mu'_\Theta)^2}{1-(\mu'_\Theta)^2}\sigma_T\right) \tag{4.38}$$

$$\epsilon_T + M(s_T + \mu_\Theta s_L) = \frac{1-(\mu'_\Theta)^2}{E_{\varphi\Theta}}\left(\sigma_T - \frac{\mu_{\varphi\Theta}+(\mu'_\Theta)^2}{1-(\mu'_\Theta)^2}\sigma_R\right) \tag{4.39}$$

The subscript, "fixed", on σ_L indicates that another plane strain axial stress will be introduced in a following section where ϵ_L is not 0.

The plane stress expressions corresponding to Equations 4.37 - 4.39 are $\sigma_L = 0$ and

$$\epsilon_R + M s_R = \frac{1}{E_{\varphi\Theta}}(\sigma_R - \mu_{\varphi\Theta}\sigma_T) \tag{4.40}$$

$$\epsilon_T + M s_T = \frac{1}{E_{\varphi\Theta}}(\sigma_T - \mu_{\varphi\Theta}\sigma_R) \tag{4.41}$$

When Equations 4.38 - 4.41 are comparred we notice that plane strain solutions simply can be obtained from the plane stress counterparts just by replacing the coefficients of elasticity and shrinkage according to

$$E_{\varphi\Theta} \Rightarrow \frac{E_{\varphi\Theta}}{1-(\mu'_\Theta)^2} \tag{4.42}$$

$$\mu_{\varphi\Theta} \Rightarrow \frac{\mu_{\varphi\Theta}+(\mu'_\Theta)^2}{1-(\mu'_\Theta)^2} \tag{4.43}$$

$$s_R \Rightarrow s_R + \mu_\Theta s_L \quad \text{and} \quad s_T \Rightarrow s_T + \mu_\Theta s_L \tag{4.44}$$

When this has been made we determine the axial stress, σ_L from Equation 4.37.

For wood we have approximately

$$\mu_\Theta = \mu_{\varphi\Theta} = 0.5 \quad \text{and} \quad E_{\varphi\Theta} = E_\Theta/15 \tag{4.45}$$

meaning that

$$\mu'_{\Theta} = \frac{E_{T\Theta}}{E_{\Theta}}\mu_{\Theta} = 0.03 \qquad (4.46)$$

which is too small to have any practical influence on the substitutions given by Equations 4.42 and 4.43. Adding the information from Equation 3.2 that the longitudinal shrinkage coefficient, s_L, is much smaller than s_R and s_T applying to the T- and R-directions it is evident that the deformation, u_R, and stresses, σ_R and σ_T, do not change practically going from plane stress to fixed end plane strain.

Free ends:

The axial stress, $\sigma_{L,FIXED}$, expressed by Equation 4.37 requires at the pole ends a reaction force, P, of size

$$P = \int_A \sigma_{L,FIXED}dA \qquad (4.47)$$

in order to keep $\varepsilon_L \equiv 0$ all over the pole. $A = \pi R^2$ is the area of the pole cross-section.

Poles normally have free ends with $\sigma_L \equiv 0$. We can obtain the effect of free ends neutralizing the reactions. At a distance approximately three times the pole diameter from the ends this means that the free end stress, $\sigma_{L,FREE}$, can be calculated by

$$\sigma_{L,FREE} = \sigma_{L,FIXED} + \sigma_{L,COR} \qquad (4.48)$$

where the correction stress, $\sigma_{L,COR}$, is given by

$$\sigma_{L,COR} = -\frac{P}{A} = -\frac{1}{A}\int_A \sigma_{L,FIXED}dA \qquad (4.49)$$

The radial and tangential stresses, σ_R and σ_T, do not change going from the fixed end to the free end situation.

We get from Equations 4.49, 4.31, 4.32 and 4.37

$$\sigma_{L,COR} = -\frac{2E_{\Theta}s_L}{R^2}\int_0^R Mrdr = -E_{\Theta}s_L(D_1 + \frac{2}{2+n}D_2) \qquad (4.50)$$

where it has been used that the integral of $(\sigma_R + \sigma_T)$ between 0 and R is 0.

Now combining Equations 4.48, 4.37 and 4.50 we get the <u>free end axial stress</u> expressed by

$$\sigma_{L,free} = \mu'_{0}(\sigma_{R}+\sigma_{T}) + E_{0}s_{L}[(\frac{r}{R})^{n} - \frac{2}{2+n}]*D_{2} \qquad (4.51)$$

where σ_R and σ_T are the plane strain stresses. Practically, however, we may apply the plane stress solutions as given by Equations 4.31 and 4.32. This has been justified in the preceding section.

The axial strain, is given by $\epsilon_L = \sigma_{L,cor}/E_0$; that is

$$\epsilon_L = \; = - s_L(D_1 + \frac{2}{2+n}D_2) \qquad (4.52)$$

The radial deformation of the pole surface is determined by

$$u_R(R) = u_{R,FIXED}(R) - \mu_0 R\epsilon_L \qquad (4.53)$$

where $u_{R,FIXED}(R)$ is the fixed end plane strain deformation which is derived from Equation 4.25 introducing N = 1 and the shrinkage coefficients transformed according to Equation 4.44. With ϵ_L from Equation 4.52 we can now rewrite Equation 4.53 as follows,

$$\frac{u_R(R)}{Rs_T} = - \frac{1}{2}(1+S)(D_1 + \frac{2}{2+n}D_2) \qquad (4.54)$$

which is identical to the plane stress solution (Equation 4.25 with N = 1).

At uniform moisture distribution ($D_2 = 0$) the average shrinkage coefficient, $s_{AV} = u_R(R)/(-RD_1)$, of the pole cross-section is given by

$$s_{AV} = \frac{1}{2}(s_T + s_R) \qquad (4.55)$$

4.3 CONCLUSIONS

From the results obtained in Section 4.2 we may suggest the following results to apply when normal wood poles are considered.

Radial and tangential stresses can be predicted by the plane stress solutions given by Equations 4.23 and 4.24.

The axial stress can be predicted by Equation 4.51 with stresses from Equations 4.23 and 4.24 - and $E_\Theta = E_L$ and $\mu'_\Theta = 1/30$ as given by Equation 4.46.

Longitudinal strain is predicted by Equation 4.52 and surface deformation by the plane stress solution in Equation 4.25.

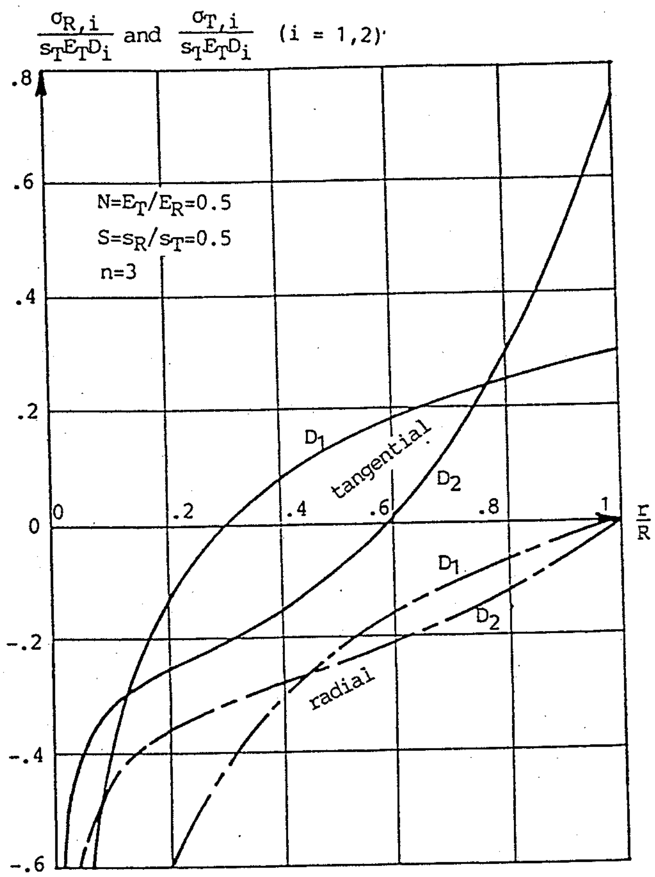

Figure 4.1. Radial and tangential elastic stresses in a drying wood pole. The stresses are separated in two parts. One influenced by moisture loss, D_1, and one influenced by moisture loss, D_2. The final stress is the sum of these components.

The results are summarized as follows:

Radial stress:

$$\frac{\sigma_R}{s_T E_T} = \frac{1-S}{1-N}\left[1-\left(\frac{r}{R}\right)^{\sqrt{N}-1}\right]*D_1 + \frac{1+n-S}{(1+n)^2-N}\left[\left(\frac{r}{R}\right)^n - \left(\frac{r}{R}\right)^{\sqrt{N}-1}\right]*D_2 \qquad (4.56)$$

Tangential stress:

$$\frac{\sigma_T}{s_T E_T} = \frac{1-S}{1-N}\left[1-\sqrt{N}\left(\frac{r}{R}\right)^{\sqrt{N}-1}\right]*D_1$$

$$+\ \frac{1+n-S}{(1+n)^2-N}\left[(1+n)\left(\frac{r}{R}\right)^n-\ \sqrt{N}\left(\frac{r}{R}\right)^{\sqrt{N}-1}\right]*D_2 \qquad (4.57)$$

Axial stress:

$$\sigma_L = \frac{1}{30}(\sigma_R+\sigma_T) + E_L s_L\left[\left(\frac{r}{R}\right)^n - \frac{2}{2+n}\right]*D_2 \qquad (4.58)$$

Radial strain (average) and surface deformation:

$$\epsilon_{R,AV} = \frac{u_R(R)}{R} = -(s_R+s_T\sqrt{N})\left[\frac{D_1}{1+\sqrt{N}} + \frac{D_2}{1+n+\sqrt{N}}\right] \qquad (4.59)$$

Axial strain:

$$\epsilon_L = \ = -s_L\left(D_1 + \frac{2}{2+n}D_2\right) \qquad (4.60)$$

Some examples of stress prediction by expressions 4.56, 4.57 and 4.58 are presented graphically in Figures 4.1, 4.2 and 4.3.

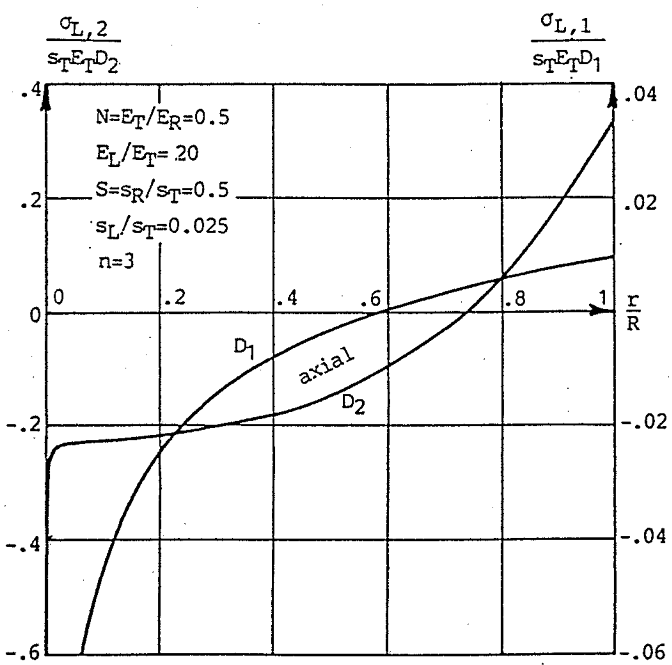

Figure 4.2. Axial elastic stress of a drying wood pole. The stress is separated in two parts. One influenced by moisture loss, D_1, and one influenced by moisture loss, D_2. The final stress is the sum of these components.

5. VISCOELASTIC STRESS ANALYSIS

The elastic stress analysis made in the preceding section does not reflect the true behavior of wood because this material is a viscoelastic material with time dependent mechanical properties. However, the solutions obtained are very useful when predicting the real behavior of wood. This section will demonstrate how.

As an illustrative example we will consider only the tangential stress at the surface of a pole. The elastic solution is given by Equation 4.57 with r = R. We get

$$\frac{\sigma_{T,EL}}{s_T E_T} = \frac{1-S}{1+\sqrt{N}} * D_1 + \frac{1+n-S}{1+n+\sqrt{N}} * D_2 \qquad (5.1)$$

where subscript, el, means elastic solution. When we introduce the moisture history described by Equation 2.3 we get

$$\frac{\sigma_{T,EL}}{s_T E_T D} = \frac{1-S}{1+\sqrt{N}}(1 - e^{-t/\alpha_C})$$

$$+ \frac{1+n-S}{1+n+\sqrt{N}}(e^{-t/\alpha_C} - e^{-t/\alpha_D}) \qquad (5.2)$$

Assuming a third degree parabolic moisture distribution (n = 3) and parameter ratios, S = 0.5 and N = 0.5, according to Equations 3.3 and 4.9 we may reduce further the $\sigma_{T,EL}$ expression. We get

$$\frac{\sigma_{T,EL}}{s_T E_T D} = 0.29 + 0.45e^{-t/\alpha_C} - 0.74e^{-t/\alpha_D} \qquad (5.3)$$

which is the elastic point of departure in our example on, how to to determine the real stress state in a wood pole.

We will rewrite Equation 5.3 introducing the so-called load level, $SL_{T,EL}$, and the critical moisture loss, D_{CR}.

The load level is defined by

$$SL_{T,EL} = \frac{\sigma_{T,EL}}{\sigma_{T,CR}} \qquad (5.4)$$

where $\sigma_{T,CR}$ is the tangential strength of wood. The critical moisture loss is the instantaneously acting moisture loss which will produce a tangential stress equal to the tangential strength. This specific quantity is obtained by Equation 5.3 introducing $\alpha_D = 0$ and $t = 0$. We get

$$D_{CR} = \frac{\sigma_{T,CR}}{0.74 s_T E_T} \qquad (5.5)$$

Now Equation 5.3 can be written

$$SL_{T,EL} = \frac{D}{D_{CR}} * [0.39 + 0.61 e^{-t/\alpha_C} - e^{-t/\alpha_D}] \qquad (5.6)$$

As previously mentioned we consider wood to be a linear viscoelastic material behaving approximately according to the Power Law creep function,

$$c_i(t) = \frac{1}{E_i}[1 + (\frac{t}{\tau_i})^{.25}] \quad ; \quad (i = R,T,L) \qquad (5.7)$$

where the power, .25, has been suggested in (1) to be in general the best value. The "doubling time", τ, is the time, $t = \tau$, at which deformation is twice its initial value.

The doubling time is shown in (1) to be very dependent of both direction and climate. A value of $\tau_R \approx \tau_T \approx 50$ days was suggested when perpendicular to grain creep is considered at a moisture content of u = 15 % and a temperature of T = 20 °C. (This is about 100 - 1000 times less than what applies to parallel to grain creep). At constant climate, τ decreases at increasing moisture content and/or increasing temperature. However, climatic variations, up or down, will always decrease the doubling time.

The following factor, a, on τ (suggested in (1)) may be used when estimating the influence on creep of equilibrium climatic conditions different from (u,T) = (15%, 20 °C)

$$a \approx 10^{(15-u)/10 + (20-T)/15} \quad ; \quad (u(\%) \le u_F) \qquad (5.8)$$

Another result obtained in (1) is that the following relaxation function, r(t), applies for wood,

$$r_i(t) \approx c_i(t)^{-1} = E_i[1 + (\frac{t}{\tau_i})^{.25}]^{-1} \quad ; \quad (i=R,T,L) \qquad (5.9)$$

(The simple result, $r(t) \approx 1/c(t)$, applies practically for any Power Law material as long as the power is less than 1/3).

Knowing the viscoelastic properties of wood we can now determine the actual drying stresses in a pole applying the elastic-viscoelastic analogy (e.g 4,5) to the elastic stress solutions. The actual tangential stress, σ_T, for example, is obtained from Equation 5.3 simply by replacing the elastic modulus, E_T, with the so-called relaxation-integral-operator such that

$$\sigma_T = \frac{1}{E_T}\int_{-\infty}^{t} r_T(t-\theta)\frac{d\sigma_{T,EL}}{d\theta}d\theta \tag{5.10}$$

It should be noticed that we have here utilized the suggestion made above that $\tau_R \approx \tau_T$ which implies creep to be isotropic in the RT plane ($E_T c_T(t) \equiv E_R c_R(t)$ or $r_T(t)/E_T \equiv r_R(t)/E_R$). A consequence of this concept is that the stiffness ratio, $N = E_T/E_R$, appearing in the elastic stress expressions can be considered as a real constant without any time influence on the viscoelastic solution.

Equation 5.10 may be rewritten in terms of load levels. We only have to divide on both sides with the strength, $\sigma_{T,CR}$, given by Equation 5.4. We get

$$SL_T = \frac{1}{E_T}\int_{-\infty}^{t} r_T(t-\theta)\frac{dSL_{T,EL}}{d\theta}d\theta \tag{5.11}$$

where SL_T is the viscoelastic load level. In our example we introduce $SL_{T,EL}$ and r_T as given by Equations 5.6 and 5.9 respectively. We get

$$SL_T = \frac{D}{D_{CR}}\int_{0}^{t} \left[1+\left(\frac{t-\theta}{\tau}\right)^{.25}\right]^{-1}\left[\frac{1}{\alpha_D}e^{-\theta/\alpha_D} - \frac{.61}{\alpha_C}e^{-\theta/\alpha_C}\right]d\theta \tag{5.12}$$

When this expression is evaluated numerically it is noticed that a sufficiently good and simple approximation of the load level considered is given by

$$SL_T \approx SL_{T,EL} * r_T(t)/E_T \approx \frac{SL_{T,EL}}{1+(t/\tau)^{.25}}$$

$$= \frac{D}{D_{CR}} \, \frac{0.39 + 0.61 e^{-t/\alpha_C} - e^{-t/\alpha_O}}{1 + (t/\tau)^{0.25}} \qquad (5.13)$$

where again $SL_{T,EL}$ is the elastic load level as expressed by Equation 5.6. Theoretical arguments given in (6) support the validity of the approximation given by Equation 5.13 as long as the exponential functions in Equation 5.12 can be considered practically congruent with the time dependent part of the relaxation function.

Some examples of the viscoelastic stress analysis are demonstrated graphically in Figure 5.1. The results shown are based on the approximate description in Equation 5.13.

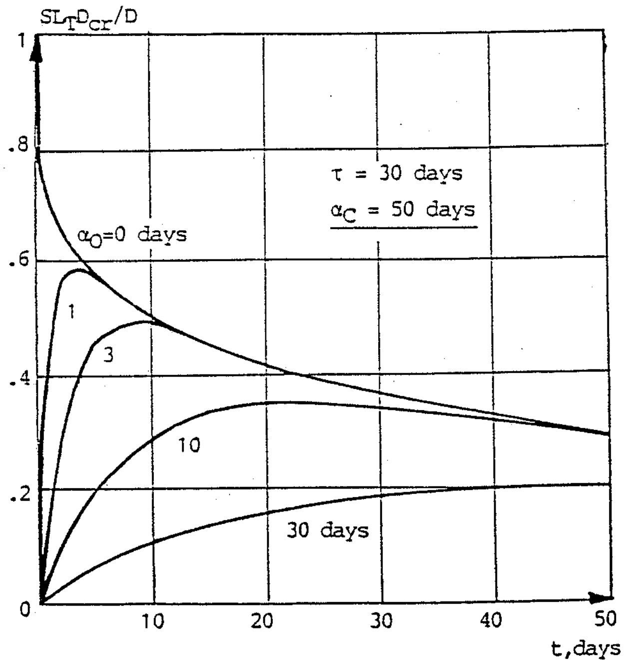

Figure 5.1. Tangential surface stress of a drying wood pole as related to rate of surface drying.

6. FINAL REMARKS

A value of $SL_T \geq 1$ is of course not possible. This would predict a stress greater than or equal to the tangential strength. Thus, one might think that the results presented in this article can be used to predict maximum rate of drying which will not destroy the wood structure. For example by applying the approximate description 5.13 with $SL_T = 1$, to determine an upper bound for moisture loss,

$$\frac{D_{UPPER}}{D_{CR}} = \mathrm{MIN.}\left[\frac{1 + (t/\tau)^{0.25}}{0.39 + 0.61e^{-t/\alpha_C} - e^{-t/\alpha_D}}\right] \qquad (5.14)$$

where Min. means minimum with respect to time.

This conclusion, however, is not correct. An upper bound on moisture loss is given by Equation 5.14 (or a more accurate expression based on Equation 5.12). It is, however, not the best bound. Due to the viscoelastic behavior of wood, defects will start propagating - and become critical some time before $SL_T = 1$ has been reached.

Much research has still to be made in order to determine the true upper bound for drying rates which will not decrease dramatically the strength of wood.

However, some important results can be deduced from the analysis made in this paper: The results presented are valid as long as the wood structure is stable. This means that no defects are expanding, weakening the materials stiffness. Thus, knowing from crack mechanics when defects start propagating, the results can be used predicting "safe" rates of drying where the original wood structure and strength is maintained.

These concluding remarks indicate the topic of a subsequent article on the drying effects on the strength behavior of wood.

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