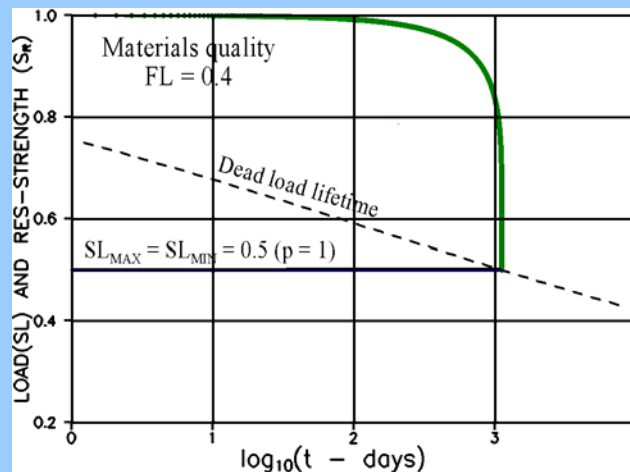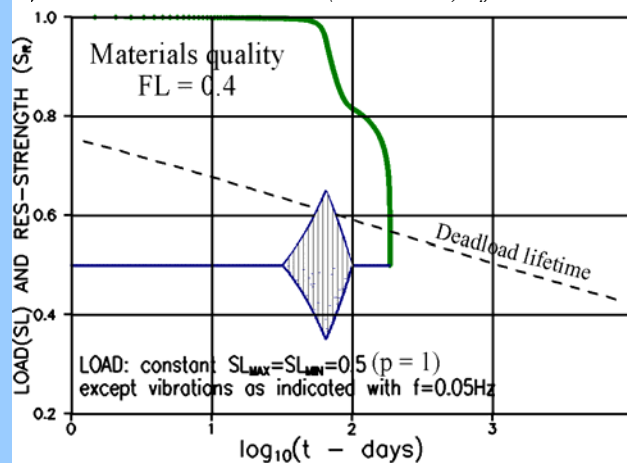# Fatigue of viscoelastic materials
such as wood with overload

Lauge Fuglsang Nielsen

Lifetime and residual strength of a material subjected to a constant load (dead load) of SL = 0.5.

The influence of a vibration overload on lifetime and residual strength of a material subjected to a constant load of SL = 0.5 as in preceding figure.

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Fatigue of viscoelastic materials
such as wood with overload

Lauge Fuglsang Nielsen

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Fatigue of viscoelastic materials

such as wood with overload

Lauge Fuglsang Nielsen[1]

Abstract: A method is presented in this paper by which lifetime and residual strength (re-cycle strength) can be predicted for viscoelastic materials such as wood subjected to variable loads. Elastic materials are included as special materials with no creep.

The method is shown to predict creep- and frequency dependent Wöhler-graphs, which compare favorably with such curves determined experimentally. It is demonstrated that considerable strength loss and lifetime reduction are the results of extra loads on top of design loads - so-called overloads caused by earthquakes or sudden changes of wind power.

Results predicted by the method developed are compared with results obtained by the so-called Miner's rule often used in practice to estimate materials fatigue behavior. It is concluded that this rule can only be justified when based on appropriate creep- and frequency dependent Wöhler curves determined experimentally or as they can be derived – less expensive - by the theory presented in this paper.

Keywords: Load variation, Power-Law creep, lifetime, fatigue, residual strength (re-cycle strength), Wöhler-curves, Smith-graphs, Miner's rule. Overloads (earthquakes, sudden changes in wind power).

1. INTRODUCTION

An operational summary of a fatigue lifetime theory presented in [1, based on 2,3,4,5,6] is presented in this paper. According to this theory materials strength is due to the propagation of a major damage (crack) toward a catastrophic state. The text of the paper is rather brief. A full description of the theory must be studied in the original papers just mentioned. The subsequent list of symbols should be frequently consulted.

Software was developed in [1] for any calculations needed to predict residual strength and lifetime of viscoelastic materials subjected to harmonically varying loads. A modified version of this software is presented in this paper such that other loads can also be considered.

1.1 List of symbols

The notations most frequently used in this note are listed below. Among the more significant notations are: Stress level, $SL = \sigma/\sigma_{CR}$, with stress (σ) relative to short time strength (σ_{CR}). The strength level $FL = \sigma_{CR}/\sigma_L$ with short time strength relative to the materials theoretical (un-damaged) strength (σ_L). In general the size of a major damage is denoted by l. The initial size (at time t = 0) is l_o. A damage ratio is defined as $\kappa = l/l_o$.

The viscoelastic materials behavior is defined by the so-called Power-Law creep function explained in Equation 1 with relaxation time, τ, and creep power, b. The fatigue parameters C (damage rate constant) and M (damage rate power) are material properties as explained in [1].

Load and strength

Load in general	σ
Strength (reference)	σ_{CR}
Theoretical strength	σ_L

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1. Department of Civil Engineering, Technical University of Denmark, DK-2800 Lyngby, Denmark

Strength level (materials quality)	$FL = \sigma_{CR}/\sigma_L$
Load level	$SL = \sigma/\sigma_{CR}$
Minimum load	σ_{MIN}
Minimum load level	$SL_{MIN} = \sigma_{MIN}/\sigma_{CR}$
Maximum load	σ_{MAX}
Maximum load level	$SL_{MAX} = \sigma_{MAX}/\sigma_{CR}$
Load ratio	$p = \sigma_{MIN}/\sigma_{MAX} = SL_{MIN}/SL_{MAX}$
Residual strength (re-cycle strength)	$S_R = \sigma_{CR}(t)/\sigma_{CR}$

Damage

Major damage size	l
Initial damage size (at t = 0)	l_o
Contact parameter and efficiency factor	Z, U
Damage ratio (or just damage)	$\kappa = l/l_o$

Fatigue parameters

Damage rate constant	$C \approx 3$
Damage rate power	$M \approx 9$
Critical load ratio	$p_{CR} \approx -0.6$

Time and creep

Normalized creep function	$C = 1 + (t/\tau)^b$
Time in general	t
Creep power	$b \approx 0.25$
Relaxation time (or doubling time)	$\tau \approx 1\ day$
Time shift parameter[2]	$q = (0.5(1 + b)(2 + b))^{1/b}$

Lifetime and load cycles

Cycling time	T
Frequency	$f = 1/T$
Number of load cycles	n
Lifetime	t_{CAT}
Number of load cycles to failure	$N = t_{CAT}/T = f*t_{CAT}$ (for harmonic loads)

Remark: The numerical indications of creep- and fatigue parameters, (τ, b, C, M), in the above list of symbols are quantities deduced in [1] for a number of wood related materials. A so-called critical load ratio, p_{CR}, is estimated as shown in Section 3.3 of this paper.

In numerical evaluations of the theory presented these parameters are used as 'standard' material properties - meaning (τ, b, C, M, p_{CR}) = (1 day, 0.25, 3, 9, -0.6). In general these parameters are expected to change from material to material.

2. MATERIALS MODEL

As previously indicated in Section 1, materials fatigue has previously been considered by the author. The materials model used in his analysis is an opening mode Dugdale crack [7] considering energy dissipation at the crack front, see Figure 1 - modified by the Rice model [8] of crack closure considering energy dissipation from alternating openings of the crack front, as outlined in Figure 2.

The analysis is not very sensitive to the specific damage model chosen: Formally the expressions developed for an opening mode Dugdale model (such as opening, displacement, stress distribution) apply also for other damage models such as sliding and tearing mode models. Only symbols change their meanings in an obvious and logical way. In a sliding mode model, for example, strength and load relate to shear strength and shear load respectively. Damages propagate along directions defined by the sliding crack. In non-dimensional formulations (dama-

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2. A second time shift parameter $h \approx 1$ defined in [1] has safely been ignored in the present study.

ge ratio, κ, in stead of damage size, strength level, FL, in stead of strength, and load level, SL, in stead of load) the damage relations become identical.

The non-dimensional formulation of the theory forms a generalization, which 'loosens' it from only to apply for the Dugdale damage concept [1,2]. 'Real' damage modes are well 'fingerprinted' (replaced) by relative quantities of load (FL) and strength (SL).
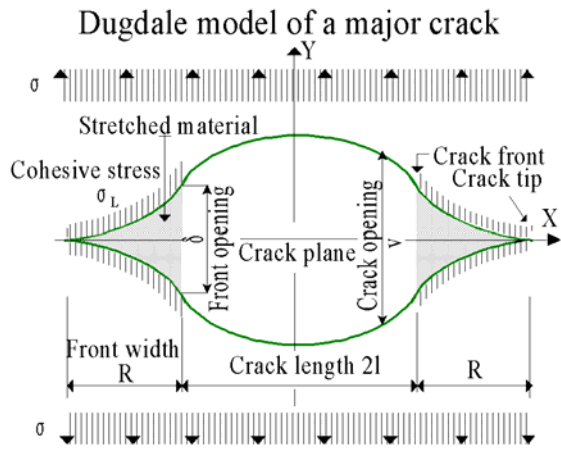

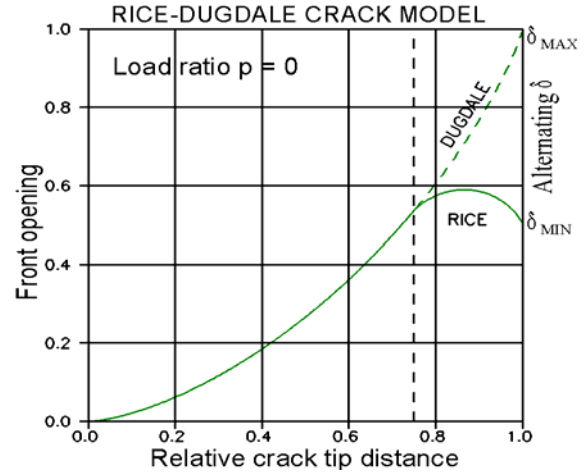**Figure 1.** *The plain Dugdale crack model in opening mode.*

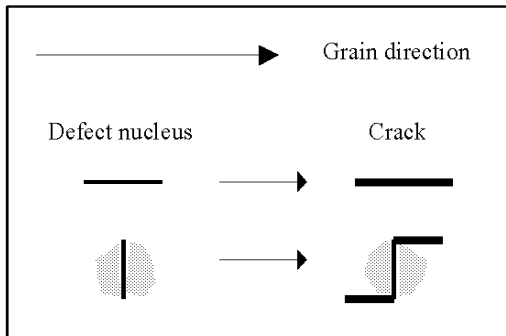**Figure 2.** *Front opening of a Dugdale model – before (Dugdale) and after (Rice) the load drops to 0.*

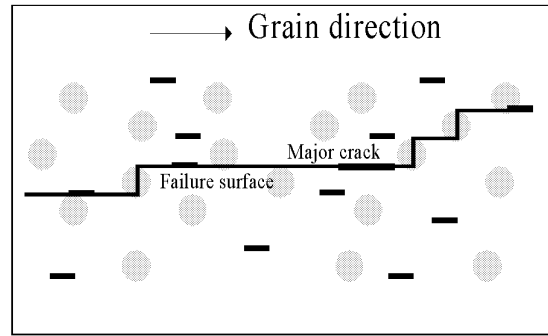**Figure 3.** *Basic elements of a failure model for wood. Shadings indicate weak areas.*

Figure 4. *Failure surface develops from a major crack along the grain. It changes 'level' through weak areas without resistance.*

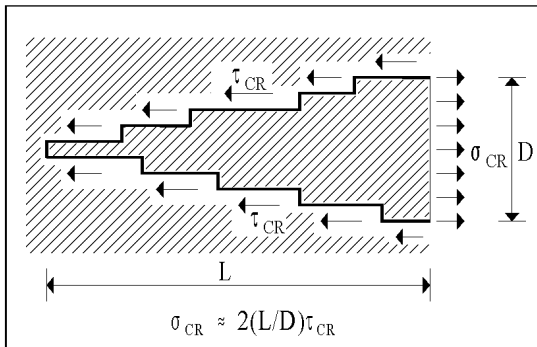$$\sigma_{CR} \approx 2(L/D)\tau_{CR}$$

Figure 5. *Finger-joint failure surface for load along the grain.*

Other arguments for generalizing the Dugdale solutions can be found in the fact that the results presented are not limited to a defect system, which literally consists of cracks. Dislocations may also be the defect source. It is known [9] that the behaviour of dislocations is described by the same equations that govern the crack problem.

In the non-dimensional (or generalized) formulation the analysis presented takes the form of a so-called theory of "damage accumulation" where damages range from large cracks to very small

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defects not visible to the naked eye. Although several analytical damage models may be thought of, some are, physically, more realistic than others. The finger joint model outlined in Figures 3-5 is a model, which seems to have the capacity of describing most failure processes considered in this paper.

Summary: The fatigue analysis made in [1] applies equally well when loads, material properties, and failure mechanisms refer to the same modes of action. Symbols in this paper therefore keep their appearance from [1] (and Section 1.1) irrespective of loading mode. Simultaneously, normalized creep C(t) should also relate to this mode. Such as shear creep strains relative to associated elastic strain (reciprocal shear modulus 1/G).

2.1 Materials quality

The materials quality is defined as the strength level FL previously introduced. It can [3] be expressed by Equation 1 consistent with the Dugdale crack model shown in Figure 1.

$$FL = \frac{\sigma_{CR}}{\sigma_L} \approx \sqrt{1 - exp\left(-\frac{d}{l_o}\right)} \rightarrow \sqrt{d/l_o} \; for \; l_o/d > 10 \; \begin{cases} d \text{ is characteristic microstruct. dimension} \\ l_o \text{ is initial size of major damage} \end{cases} \quad (1)$$

$$Example \; wood : d \approx \left(fiber \; length * fiber \; diameter\right)^{1/2} \approx 0.3 \; mm$$

MATERIALS QUALITY (Strength level)

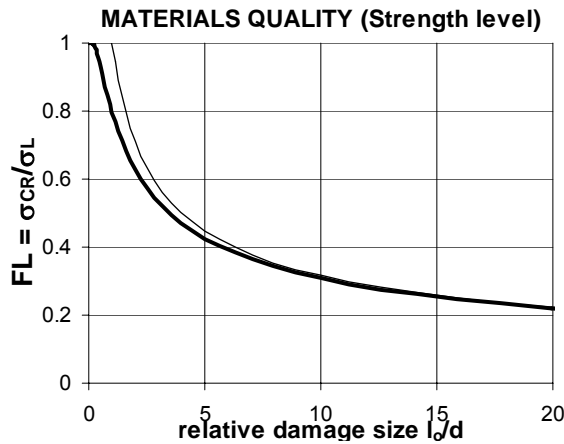*Figure 6. Strength level (or materials quality), FL = σ_{CR}/σ_L is traditional strength (at t = 0) relative to theoretical strength (cohesive stress). Characteristic micro structural dimension is denoted by, d. Actual damage size is l_o.*
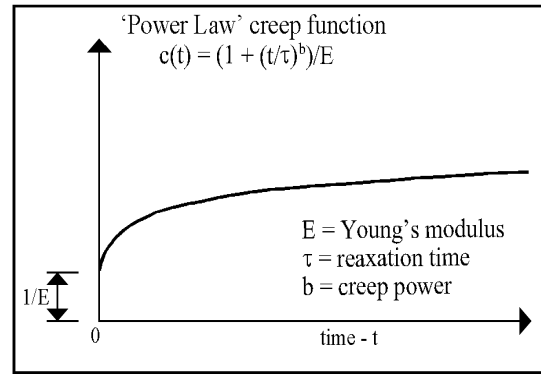

Figure 7. Power Law creep function in the front area of damage. The so-called normalized creep function is creep function multiplied by Young's modulus.

2.2 Viscoelasticity

The viscoelastic materials behavior is assumed to be well described by the so-called Power-Law creep expression presented in Equation 2 and Figure 7, which is quantified by the creep power b - and the relaxation time τ. Time in general, is denoted by t. It is emphasized that Equation 2 describes creep in front of material damages. For bulk creep (large amounts of material) the relaxation time is much larger than indicated. It is noticed that elastic materials are included with relaxation times, $\tau \rightarrow \infty$. Both creep parameters, b and τ, are constants defining a so-called non-ageing material which is the prime material considered in this paper. Ageing materials, however, with time (age) dependent parameters can be considered as indicated in the final Section 7 of this paper.

$$C(t) = 1 + \left(\frac{t}{\tau}\right)^b \qquad Normalized \; Power \; Law \; creep \; function \; in \; damage \; area \qquad (2)$$

Quite a number of building materials experience Power Law creep. This has been convincingly shown by Nielsen in [10].

3. FATIGUE ANALYSIS

3.1 Domain of application

Basically the fatigue analysis presented in this section of viscoelastic materials subjected to harmonic loads, such as outlined in Figure 8, is reproduced from [1] which again is based on the 'low-stress' version of the so-called DVM theory (Damaged Viscoelastic Material) presented in [2]. Practically this means [3] that it applies for strength levels FL < 0.45 corresponding to damage sizes l_o greater than approximately four times a micro structural dimension of the material considered, see Figure 6. Such damages are very likely to appear in most building materials.
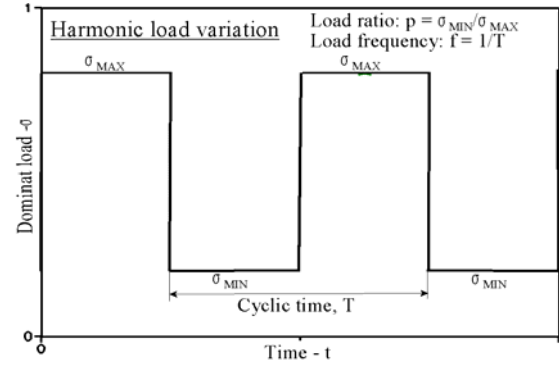

Figure 8. *Harmonic square load variation considered in this paper (and in [1]). Traditional frequency, f = 1/T.*

3.2 Load ratio p > 0

The theory developed in [1] to determine fatigue properties (lifetime and residual strength) of viscoelastic materials subjected to harmonic loads with load ratios p > 0 is summarized in the algorithm presented in Equation 3.

Harmonic loads
At a fixed damage ratio, κ, damage rate is determined by :

$$Y = \left(A_1 X\right)^b + A_2 X - A_3 = 0 \quad where \quad X = FL^2 \frac{\Delta t}{\Delta \kappa} = \frac{FL^2}{f} \frac{\Delta n}{\Delta \kappa} \quad (and \; A_i \; from \; Equation \; 4)$$

$$\frac{dY}{dX} = b(A_1 X)^{b-1} A_1 + A_2 \; \Rightarrow \; X_{NEW} = \frac{Y}{dY / dX} \; (Newton-iteration)$$

repeat until X_{NEW} is very close to X − then for progressing damage : $\Delta t = \dfrac{X \Delta \kappa}{FL^2}$ *(or $\Delta n = f \Delta t$) (3)*

with appropriate κ–steps : $\Delta \kappa = \dfrac{1/SL_{MAX}^2 - 1}{1000}$ *and start values :* $(t,n,\kappa) = (0,0,1) \Rightarrow$
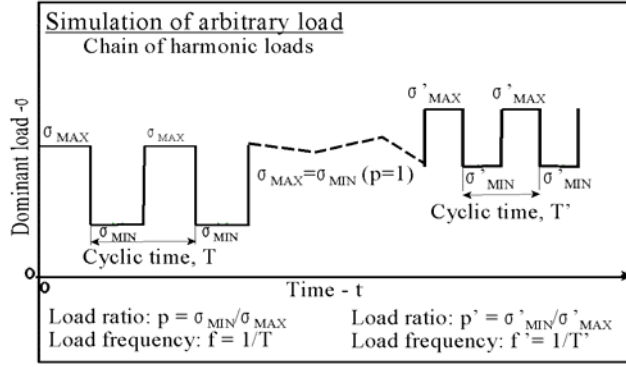
Intermediate results : $t = t + \Delta t$; $n = n + \Delta n$; $\kappa = \kappa + \Delta \kappa$; *Residual strength is* $S_R = 1/\sqrt{\kappa}$
Lifetime : t_{CAT} *is t at* $\kappa = 1 / SL_{MAX}^2$;

$$\left. \begin{array}{l} A_1 = \dfrac{\phi}{q\tau} \\ A_2 = \phi Z f \\ A_3 = \dfrac{1 - \kappa SL_{MAX}^2}{\kappa SL_{MAX}^2} \end{array} \right\} \; with \; \left\{ \begin{array}{l} q = \left[\dfrac{(1+b)(2+b)}{2}\right]^{1/b} \\ \phi = \dfrac{\pi^2}{8} \kappa SL_{MAX}^2 \\ Z = \dfrac{C}{8} [U(1-p)]^M \left[\kappa SL_{MAX}^2\right]^{M/2-2} \; with \; efficiency \; factor \; U = \dfrac{1+p}{2} \end{array} \right. \quad (4)$$

The above algorithm can be generalized to apply also for arbitrary load variations as they are described in Figure 9 if we assume:

1) Damage propagation, $\Delta\kappa/\Delta t$, at a certain time, t, depends only on the damage properties at that particular time. Damage steps, $\Delta\kappa$, chosen for integration must be refined such that local load sections of different loading types are sufficiently well covered – and not 'overlooked'. This feature is obtained by modifying $\Delta\kappa$ such that time steps, Δt, become sufficiently small.

2) A subroutine provides at any time, t, the analysis with the immediate load parameters $(\sigma_{MAX}(t),f(t),p(t))$ and the immediate damage size, κ, at that particular time.

With these additional features included Equation 3 becomes the algorithm developed in Appendix A, at the end of this paper, for the lifetime analysis of materials subjected to load histories as outlined in Figure 9.

Figure 9. *Arbitrary load simulated by chain of harmonic loads bound together with continuous loads (simulated as a harmonic load with load ratio p = 1).*

3.3 Any load ratio p

The efficiency factor U suggested in Equation 5, reproduced from [11], ensures that negative load ratios (p) can also be considered in Equation 3 (and Appendix A). The critical load ratio p_{CR} introduced is considered as a materials constant. A suggested value of $p_{CR} \approx 0.6$ has been calibrated from reversed bending results (p = -1) from tests on Pine heartwood reported by Kraemer [12] (as presented in [13]), see Figure 10. The calibration follows the procedure presented in Section 4.1.

$$Efficiency\ factor\ U = \frac{1}{2}\begin{cases} Max[1,1+p] & when\ p \geq p_{CR} \\ Min\left[1,\dfrac{1-p_{CR}}{1-p}\right] & when\ p < p_{CR} \end{cases} \tag{5}$$
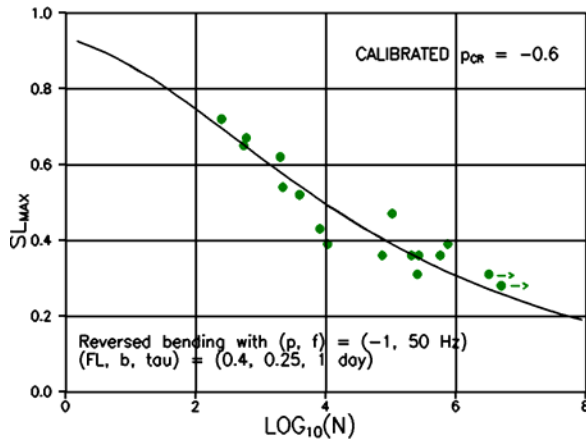$$with\ critical\ load\ ratio\ p_{CR} \approx -0.6\ (a\ materials\ cons\tan t)$$

Figure 10. *Wöhler curve for reversed bending (p = -1) of Pine heartwood. Exp. data from [12] as presented in [13]. Estimated frequency, f = 50 Hz.*

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3.4 Special solutions

For p = 1 Equation 3 (and Appendix A) includes lifetime predictions of viscoelastic materials subjected to a continuous load history. The results are presented in Equation 6. Elastic fatigue solutions are obtained from Equation 3 (and Appendix A) with $\tau \to \infty$ as presented in Equation 7.

$$\textbf{Continuous loads:} \; \underline{SL_{MAX} = SL_{MIN} \; (p = 1)}$$

$$\frac{d\kappa}{dt} = \frac{(\pi FL)^2}{8q\tau} \frac{\kappa SL^2}{[1/(\kappa SL^2)-1]^{1/b}} \quad with \; q = \left[\frac{(1+b)(2+b)}{2}\right]^{1/b} \; ; \; S_R = \frac{1}{\sqrt{\kappa}} \tag{6}$$

$$t_{CAT} = \frac{8q\tau}{\pi^2 (FL*SL)^2} \int_0^{1/SL^2 -1} \frac{x^{1/b}}{1+x} dx \quad (constant\;load,\;SL\;;\;so{-}called\;'deadload')$$

$$\textbf{Elastic fatigue:} \; \tau \to \infty$$

$$\frac{d\kappa}{dn} = \frac{C\pi^2 FL^2}{64} [U(1-p)]^M \frac{(\kappa SL_{MAX}^2)^{M/2}}{1 - \kappa SL_{MAX}^2} \qquad ; \; S_R = \frac{1}{\sqrt{\kappa}} \tag{7}$$

$$N = \frac{13}{C(FL*SL_{MAX})^2} \left[\frac{1}{U(1-p)}\right]^M \left[\frac{1-SL_{MAX}^{M-2}}{(M-2)SL_{MAX}^{M-2}} - \frac{1-SL_{MAX}^{M-4}}{(M-4)SL_{MAX}^{M-4}}\right] \begin{cases} constant\;SL_{MAX}\,,\,p \\ t_{CAT} = N*T = N/f \end{cases}$$

4. DESIGN GRAPHS

4.1 Predicted Wöhler-curves

Lifetime of materials is often estimated from so-called Wöhler curves [14] (also known as S-N curves), which express number of load cycles to failure of materials subjected to harmonic loads. The Wöhler curves presented in Figures 11 and 12 are predicted by the present theory (Appendix A) for an elastic material ($\tau = \infty$) and a viscoelastic material respectively. The material properties used are the standard parameters defined in Section 1.1. A materials quality of FL = 0.4 is estimated.

Remark: Through the so-called Miner's rule (subsequently considered in Section 6) Wöhler-curves are used in practice to estimate lifetime of materials subjected to loads composed of harmonic loads of various load levels.
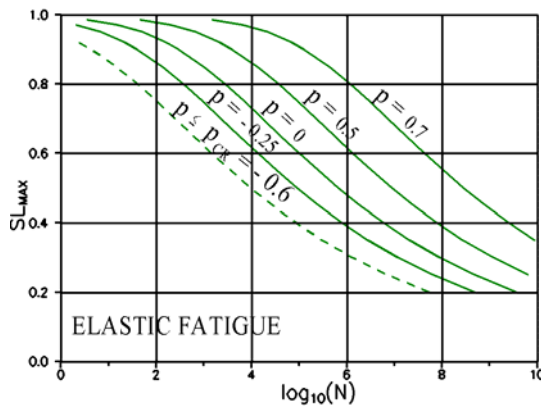

Figure 11. *Predicted Wöhler curves for an elastic material subjected to harmonic load. Apply for any load frequency.*
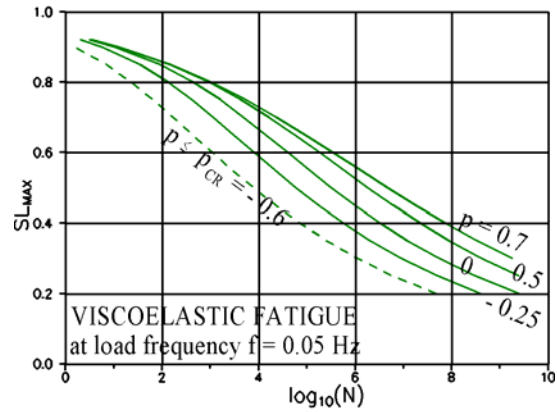
Figure 12. *Predicted Wöhler curves for a viscoelastic material subjected to harmonic load with frequency, f = 0.005 Hz.*

4.2 Theory versus experiments

The statements previously made in Chapter 2 that the theory developed applies en general for various modes of loading are justified by the following experimentally determined Wöhler curves: Figure 13 with experimental data from tests by Bach [15] on clear Douglas Fir subjected to harmonic *compression* loads, in Figure 14 with experimental tensile data from Hashin & Rotem [16] on fiber reinforced epoxy, and in Figure 15 with experimental data reported by McNatt [17] from *interlaminar shear* tests on particleboards and hardboards - and in Figure 16 with experimental *tensile* data from Bohannan and Kanvik [18] on Dough-Fir finger joints − and in Figure 17 with experimental *tensile* data from Nielsen & Madsen [19] on artificially damaged Doug-Fir loaded ⊥ to the grain.

Unless otherwise indicated the solid line data in all Figures 13-17 are predicted by the present theory with standard parameters also used in the preceding Section 4.1.
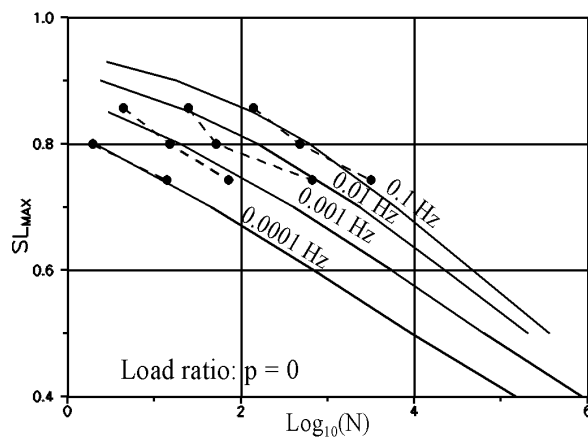

Figure 13. *Wöhler graphs for clear spruce subjected to compression parallel to grain (p = 0). Experimental data from [15].*
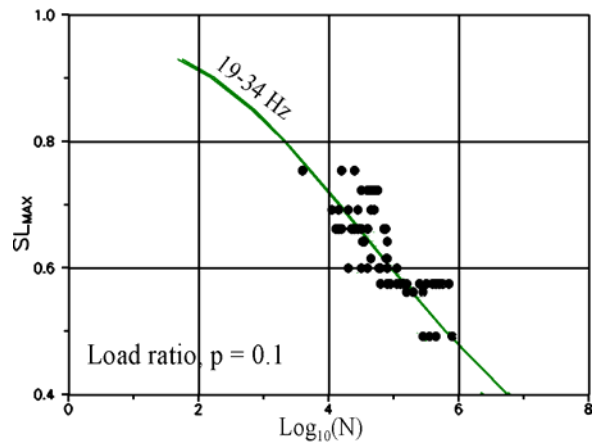

Figure 14. *Wöhler graph for glass fiber reinforced epoxy loaded parallel to fibers with (p,f) = (0.1, 19-34 Hz). Experimental data from [16].*
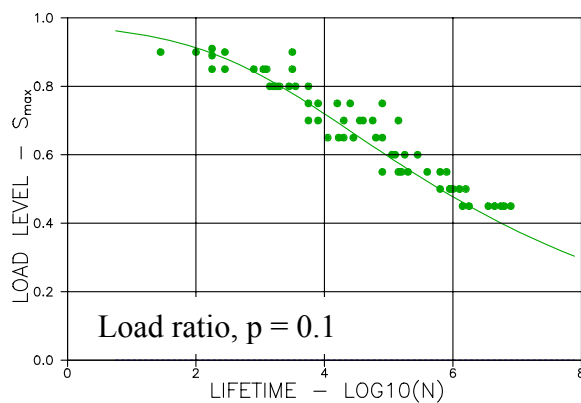

Figure 15. *Wöhler curve for particleboards and hardboards loaded in tension and interlaminar shear with (p,f) = (0.1,15 Hz). Experimental data from [17].*
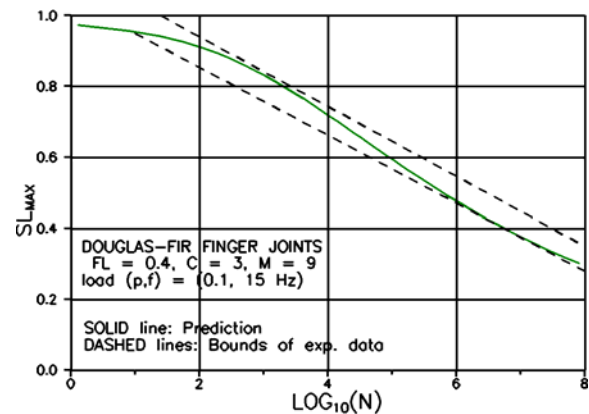

Figure 16. *Bounds for Wöhler graph for Doug-Fir finger-joints in tension ≠ to grain with (p, f) = (0.1, 15 Hz). Experiments from [18].*
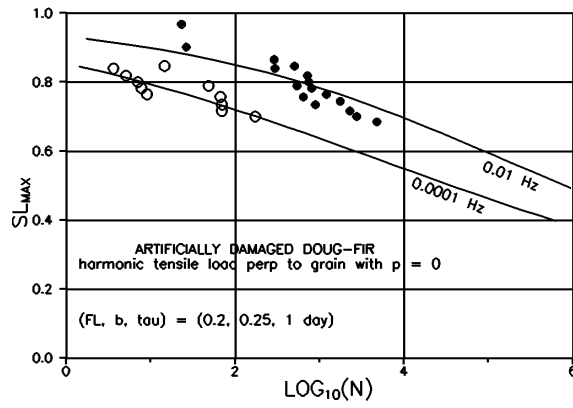
Figure 17. Wöhler graphs for artificially damaged Doug-Fir loaded ⊥ to grain. Experimental data from [19]. Notice that a strength level of Fl = 0.2 has been estimated.

4.3 Smith-graphs

A so-called Smith-graph is another way of presenting lifetime of materials subjected to harmonic loads. A Smith graph quantifies harmonic load variations, which provoke failure after a certain fixed number of load cycles, N_{TARGET}. The basic information needed for establishing a Smith graph is the same as for constructing a Wöhler curve. Two examples of Smith-graphs predicted by the present theory for a viscoelastic material are shown in Figure 18. The basic prediction parameters used are the same as used in Sections 4.1 and 4.2. A materials quality of FL = 0.4 is assumed.
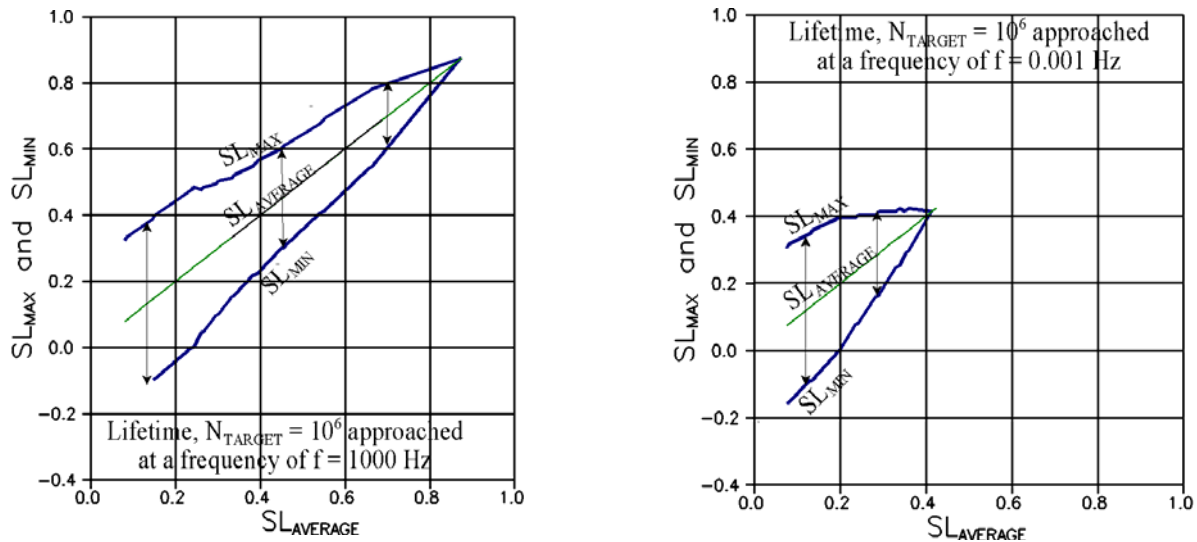

Figure 18. Smith graphs for a viscoelastic material subjected to harmonic loads of f = 1000 Hz and f = 0 .001 Hz respectively. $N_{TARGET} = 10^6$.

5. OVERLOADS (illustrative examples)

Some more general applications of Appendix A are demonstrated in Figures 19-24. Unless otherwise indicated it is assumed that the basic material properties are as the standard properties presented in Section 1.1. Materials quality FL varies as indicated in the figures.

The examples are constructed such that the effects of overloads on materials lifetime and residual strength can be studied. Overloads can be caused by phenomena such as earthquakes or sudden changes in wind power on windmills.

It is demonstrated that considerable strength loss and lifetime reduction may be the results of extra loads on top of design loads.
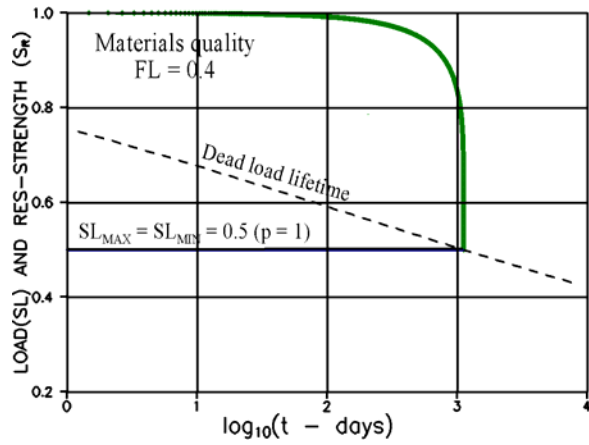
Figure 19. *Lifetime and residual strength of a material subjected to a constant load (dead load) of SL = 0.5.*
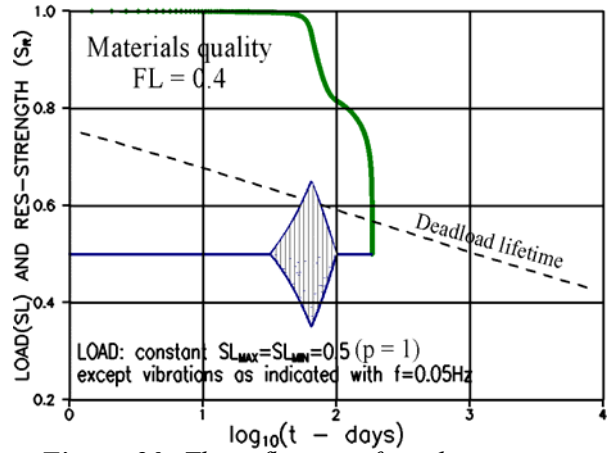

Figure 20. *The influence of a vibration overload on the residual strength and residual strength of a material subjected to a constant load of SL = 0.5 as in Figure 19.*
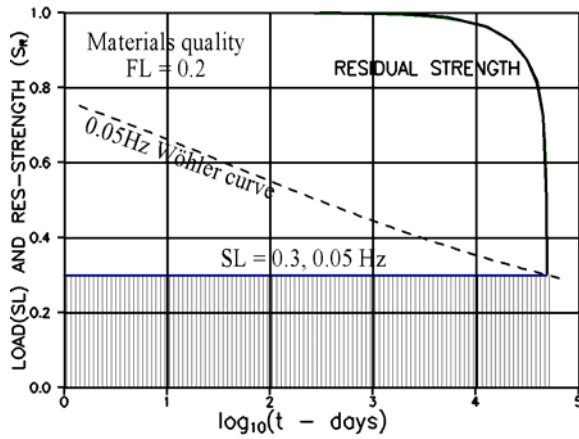

Figure 21. *Lifetime and residual strength of a viscoelastic material subjected to a harmonic load of $SL_{MAX} = 0.3$ with load ratio p = 0.*
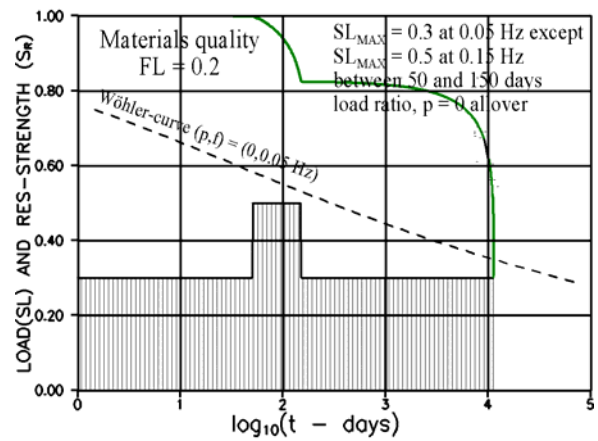

Figure 22. *The influence of a vibration overload as indicated on lifetime and residual strength of a viscoelastic material subjected to a harmonic load as described in Figure 21.*
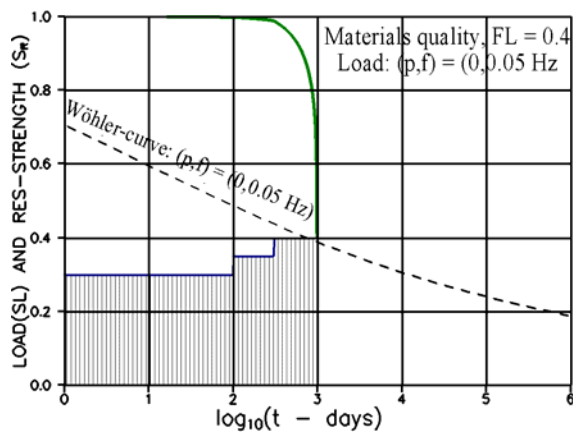

Figure 23. *Lifetime and residual strength of a viscoelastic material subjected to an increasing harmonic load and a decreasing harmonic load respectively. Load ratios p = 0.*
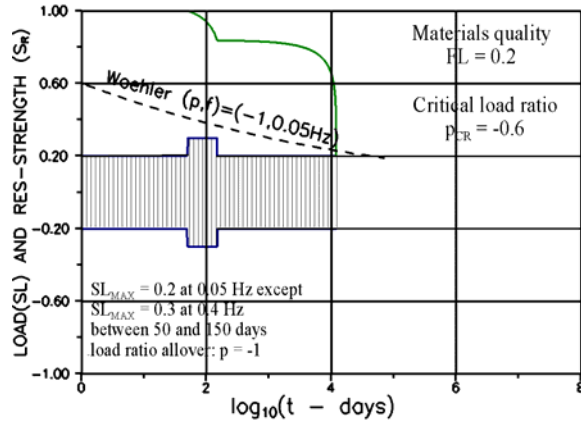
10

Figure 24. The influence of overload on lifetime of a structure with reversed loading. Lifetime without overload is approximately 10^5 days.

6. MINER's RULE

Very often the so-called Miner's rule[3] [20,21], summarized in Equation 8 and Figure 25, is used to estimate lifetime of materials subjected to a number of harmonic load levels. Miner's rule is basically a semi-theoretical expression, which is applied, mainly because of the lack of alternatives [22]. The following presentations apply to combinations of loads with load ratios p = 0. Materials quality, FL = 0.4. Creep- and fatigue parameters are the same as previously applied, namely (b,τ,C,M) = (0.25,1 day,3,9).

$$\sum_{i=1}^{k} \frac{n_i}{N_i} = 1 \; where \begin{cases} k & = number\ of\ load\ levels \\ n_i & = number\ of\ cycles\ at\ the\ i^{th}\ load\ level \\ N_i & = number\ of\ cycles\ to\ failure\ corresponding\ to\ the\ i^{th}\ load\ level\ (W\ddot{o}hler) \\ n_i\,/\,N_i & = damage\ ratio\ at\ the\ i^{th}\ load\ level \end{cases}$$

Example with two load levels $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (8)$

n_1 *cycles with* SL_1 *(* N_1 *cycles to failure for cons tan t* SL_1 *)*
n_2 *cycles with* SL_2 *(* N_2 *cycles to failure for cons tan t* SL_2 *)*

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} = 1 \; \Rightarrow \; n_2 = N_2\left(1 - \frac{n_1}{N_1}\right) \; \Rightarrow \; N = n_1 + n_2$$
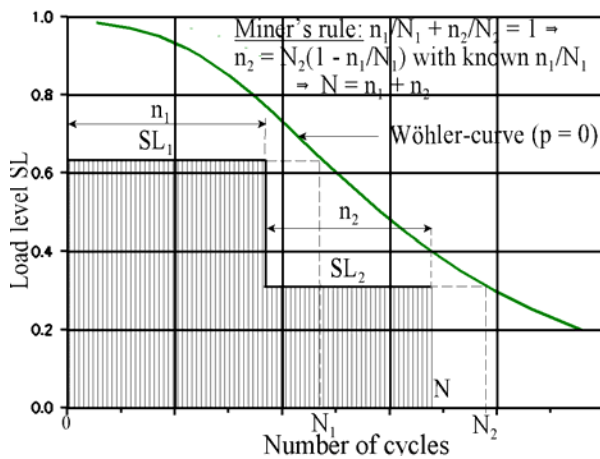

Figure 25. Principles in Miner's rule.

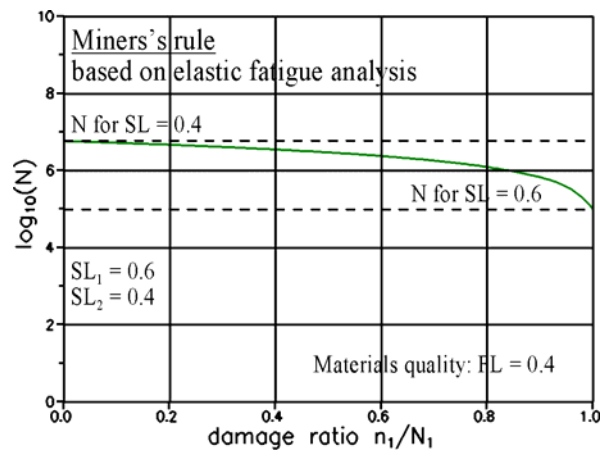*Figure 26. Application of Miner's rule to determine lifetime of an elastic material subjected to a stepwise decreasing load level (SL₁,SL₂) = (0.6, 0.4) with p = 0.*

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3. Also named the Palmgren-Miner's rule because of similar methods developed by Miner and Palmgren [21].

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Figure 27. *Application of Miners rule to estimates of the lifetime of a viscoelastic material subjected to a decreasing harmonic load level. $(SL_1, SL_2) = (0.6, 0.4)$ with $p = 0$.*
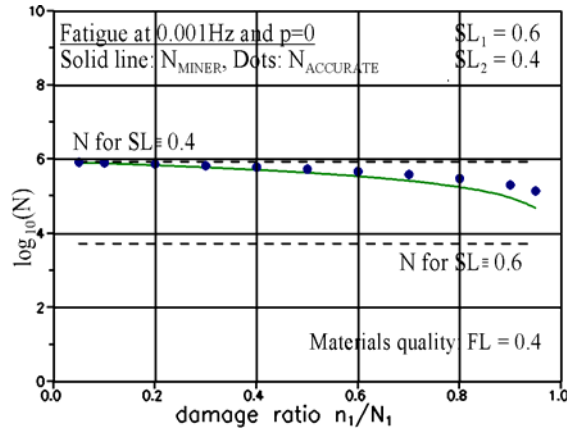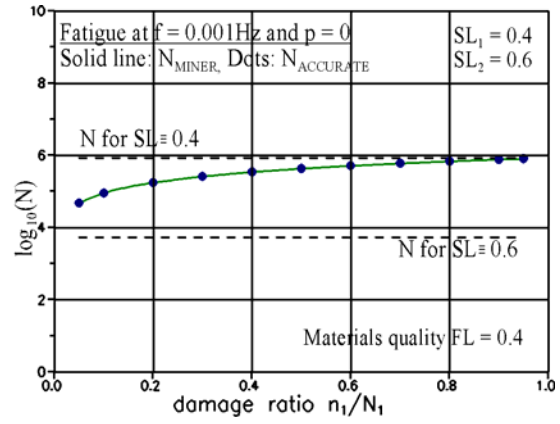
Figure 28. *Application of Miners rule to estimates of the lifetime of a viscoelastic material subjected to an increasing harmonic load level. $(SL_1, SL_2) = (0.4, 0.6)$ with $p = 0$.*

Comments: Two features are noticed when comparing results obtained by Miner's rule with predictions made by the present theory: 1) From Figures 26 and 27 is observed that lifetime of a viscoelastic material is overestimated by the Miner's rule when this rule is based on elastic Wöhler curves. 2) From Figures 27 and 28 is observed that predictions by the two methods are surprisingly close when the basic Wöhler curves are viscoelastic – especially close when increasing load histories are considered.

7. FINAL REMARKS

A method is presented in this paper by which lifetime and residual strength (re-cycle strength) can be predicted for materials subjected to variable loads. The method applies to viscoelastic as well as for elastic materials.

The method is shown to predict Wöhler-graphs, which compare favorably with such curves determined experimentally. It is demonstrated that considerable strength loss and lifetime reduction are the results of extra loads on top of design loads - so-called overloads caused by earthquakes or sudden changes of wind power.

Miner's rule will overestimate lifetime of viscoelastic materials when based on elastic Wöhler curves. Reliable results require viscoelastic, frequency dependent Wöhler curves. In general these observations rule out the applicability of Wöhler curves based on high frequency experiments. Thus, for most practice it can only be justified to estimate lifetime by Miner's rule combined with appropriate 'viscoelastic' Wöhler curves determined experimentally or as they can be derived – less expensive - by the theory presented in this paper.

Finally, the method developed in this paper for prediction of lifetime behavior of materials subjected to variable loads has been further developed in [23] to apply also for concrete with ageing creep. Such generalization is obtained introducing an extra subroutine into the lifetime algorithm presented in Appendix A such that concrete creep, stepwise is approximated to follow a Power-Law description. Some features of this generalization have been demonstrated in [24].

APPENDIX A: Lifetime algorithm

FATIGUE ANALYSIS : PART 1
(Time introduced in days)

MATERIALS DATA

FL : Quality

b : Creep power

τ : Creep relaxation time

C : Damage rate constant

M : Damage rate power

PREPARATION OF ANALYSIS

$\max \Delta t \approx 3$ days, Maximum time step in numerical analysis. To ensure a proper integration of sections with different loading modes.

$X \approx 1$ day   (not critical) First extimate of X at $t = 0$;

$p_{CR} = -0.6$ (critical load ratio - material dependent)

START : $n = 0,\ t = 0,\ \kappa = 1$

1 : Call SUBROUTINE load(t, SL_{MAX}, p, f)

$$\Delta\kappa = \frac{1/SL_{MAX}^2 - 1}{1000} \quad \text{(First estimate)}$$

$$\left.\begin{array}{l} A_1 = \dfrac{\varphi}{q\tau} \\[2mm] A_2 = Z\varphi f \\[2mm] A_3 = \dfrac{1 - \kappa SL_{MAX}^2}{\kappa SL_{MAX}^2} \end{array}\right\} \quad \text{with} \quad \left\{\begin{array}{l} q = \left(\dfrac{(1+b)(2+b)}{2}\right)^{1/b} \\[3mm] \varphi = \dfrac{\pi^2}{8}\kappa SL_{MAX}^2 \\[3mm] Z = \dfrac{C}{8}\left[U(1-p)\right]^M \left[\kappa SL_{MAX}^2\right]^{M/2-2} \quad \text{(contact parameter)} \\[2mm] \quad \text{with efficiency factor, U, from equation 6} \end{array}\right.$$

ANALYSIS

2 : $Y = (A_1 X)^b + A_2 X - A_3$; $\qquad \dfrac{dY}{dX} = bA_1(A_1 X)^{b-1} + A_2$

$$X_{NEW} = X - \frac{Y}{dY/dX} \qquad\qquad \text{(Newton iteration)}$$

if $\left|\dfrac{X_{NEW}}{X} - 1\right| \leq 0.00001$ goto 3 :

if $X_{NEW} < 0$; $X = X/100$

if $X_{NEW} \geq 0$; $X = X_{NEW}$

go to 2 :

3 : $X = X_{NEW}$

4 : $\Delta t = \dfrac{X\Delta\kappa}{FL^2}$;   if(p.ne.1.) $\Delta n = f\Delta t$; if(p.eq.1.) $\Delta n = 0$

if $\Delta t \leq \max\Delta t$ goto 5 :

$\Delta\kappa = \Delta\kappa/2$

goto 4 :

$$
\boxed{
\begin{array}{l}
\textit{\textbf{FATIGUE ANALYSIS : PART 2}} \\
\textbf{RESULTS} \\
\textbf{5:}\ \kappa = \kappa + \Delta\kappa \ ; \ \ n = n + \Delta n \ ; \ \ t = t + \Delta t \ ; \ \ S_R = \dfrac{1}{\sqrt{\kappa}} \\
\underline{\text{WRITE}(n, t, S_R)} \\
\underline{\text{if}\ \kappa > \kappa_{CR} = \dfrac{1}{SL_{MAX}^2}\ \ \underline{\text{goto 6:}}} \\
\underline{\text{go to 1:}} \\
\\
\textbf{6:}\ \text{Stop; End} \\
\textbf{SUBROUTINE LOAD}(t,\ SL_{MAX}(t),\ p(t),\ f(t))
\end{array}
}
$$

8. LITERATURE

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