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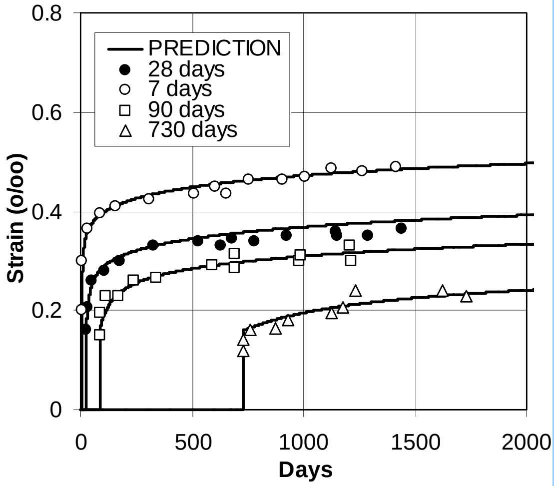
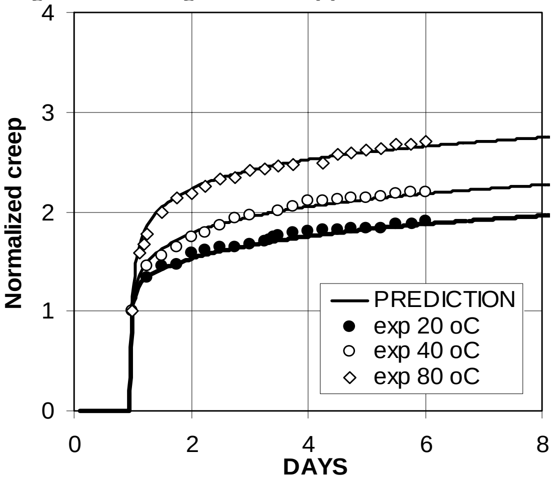
Composite creep analysis of concrete

a rational, incremental stress-strain approach

Lauge Fuglsang Nielsen



L'Hermite: Creep of concrete at different ages at loadings. Loads applied: 6.9 MPa.



Umehara: Creep of concrete at various temperatures.

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Abstract: The author has previously presented a so-called incremental stress-strain model convenient for FEM-analysis of concrete structures (1). Concrete composition is considered by this model as well as age at loading and curing conditions. No other incremental models presented in the literature offer these features in one approach.

The present paper is an operational summary and simplification of this method with respect to applicability. As in the original paper it is concluded that cement paste and concrete can be considered linear-viscoelastic materials from an age of approximately 1 day. This observation is a significant age extension relative to earlier studies in the literature where linear-viscoelastic behavior is only demonstrated from ages of a 'few days'.

An additional advantage of considering concrete as a composite material is that the number of calibration tests needed for property determination can be held at a minimum. Material properties can be calculated from properties, typical for basic concrete components. A software 'Fem-Creep' has been developed for any analysis made in this paper. On request (lfn@byg.dtu.dk) this software is available for interested readers.

Key words: Incremental stress-strain analysis, concrete composition, composite behavior, curing conditions.

1. INTRODUCTION

A viscoelastic composite analysis of concrete has previously been presented by the author in (2). The basic stress-strain equation used in this study is the classical one with strains given as integral functions of stress and creep functions. Creep- and relaxation problems, eigenstrain/stress problems (such as internal concrete stress states caused by shrinkage and temperature) are solved in (2).

Such detailed materials analysis is very time consuming in structural analysis. In this respect a more rational analysis is presented in this paper, which is designed for fast FEM-analysis of concrete structures. It is emphasized, however, that timesavings are obtained on the expense of knowing how local materials stress-strains develop in concrete members. Such information needed for damage evaluation must be obtained by selective combinations of the method presented in this paper and the one presented in (2). Both methods consider in one approach concrete composition, age at loading, and curing conditions.

Symbols used in this paper are summarized in a list at the end of the paper. In general Young's moduli and viscosities are denoted by E and η respectively. Stress is denoted by σ and strain by ε. Time and age are denoted by t.

The final results of the analysis presented apply in general for any water-cement ratio (W/C) and for any temperature curing (T). Results presented (up to Chapter 4) without specified temperatures apply for $T \equiv 20°C$.

1.1 Concrete as a homogeneous material

1.1.1 Observations on concrete creep

Stress-strain relations for concrete can be established from experimental observations already known about 40 years ago. This statement will be verified in this paper where the mechanical behavior of concrete is modeled by viscoelastic models the properties of which are determined from the following list of key-observations identified in, and adapted from (3,4): Concrete can be considered practically to be a linear-viscoelastic material (5,6) with the following creep function components:

- A momentary elastic deformation, which decreases with age.
- A delayed elastic deformation (reversible strain). This has very convincingly been shown already by Illston (7) and Glucklich et al. (8). Delayed elasticity of concrete develops substantially more rapidly (in days/weeks) than irreversible, viscous creep does (in months/years). The rate of development decreases with age. The size of reversible creep is approximately 0.4 of the elastic strain associated. These statements are based on observations made by Illston (7) for example. Subsequent research (9) on creep of hardening cement paste (HCP) shows that the elastic strain and the delayed elastic strain of this material are of similar magnitudes.
- A viscous (irreversible) deformation. This has been known since the unique and original works of Dischinger (10,11). Creep continues to increase - at least for a lifetime. The rate, however, is always decreasing (12,13). It has often been observed that creep from some time after load application is well described by logarithmic functions of time (13,14,15) - and also (14,15) that creep functions for different ages of loading tend to become parallel as time proceeds to long times. A special viscous deformation (consolidation) has been identified in (3) to develop shortly after load applications.

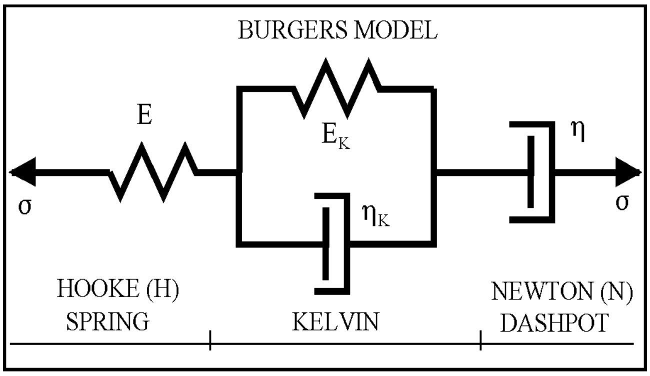

Figure 1. *Burgers model. Properties are age dependent.*

These observations are enough to suggest that the mechanical behavior of concrete can be modeled linear viscoelastically by the Burgers model illustrated in Figure 1: Momentary elastic strain is explained by the free spring (Hooke), delayed elastic strain by the Kelvin model, and the irreversible creep strain by the free dashpot (Newton) plus consolidation strain as "frozen" recovery strain from the former two mechanisms (due to aging).

Rheological models such as Burgers, Kelvin, and Maxwell models can be studied in details in (16,17).

1.1.2 Stress-strain relations

An incremental stress-strain relation of a Burgers model can be formulated as explained in Equation 1 reproduced from (1). Other incremental formulations are suggested in (18,19).

Incremental stress-strain representation of the Burgers model

$$\frac{d\varepsilon}{dt} = \frac{d\varepsilon_H}{dt} + \frac{d\varepsilon_K}{dt} + \frac{d\varepsilon_N}{dt} \quad \text{with} \quad \begin{cases} \dfrac{d\varepsilon_H}{dt} = \dfrac{1}{E(t)}\dfrac{d\sigma}{dt} & \text{(Hooke)} \\ \dfrac{d\varepsilon_K}{dt} = \dfrac{\sigma - E_K(t)\varepsilon_K}{\eta_K(t)} & \text{(Kelvin)} \\ \dfrac{d\varepsilon_N}{dt} = \dfrac{\sigma}{\eta(t)} & \text{(Newton)} \end{cases} \qquad (1)$$

1.1.3 Creep function and relaxation function

The so-called creep function c(t) is defined as strain caused by a constant stress $\sigma(t) = 1$ applied at the age of $t = \theta$. Thus, the creep function corresponds to ε determined from Equation 1 with $d\sigma/dt \equiv 0$ and $c(\theta) = 1/E(\theta)$. The so-called relaxation function r(t) is defined as stress caused by a constant strain $\varepsilon(t) = 1$ applied at the age of $t = \theta$. Thus, this function corresponds to σ determined by Equation 1 with $d\varepsilon/dt \equiv 0$ and $r(\theta) = E(\theta)$. The procedures to follow when calculating these two special functions are presented in Equation 2 and Equation 3 respectively reproduced from (1).

CREEP FUNCTION (strain for unit stress applied at t = θ): $\sigma \equiv \begin{cases} 0 & \text{for } t < \theta \\ 1 & \text{for } t \geq \theta \end{cases}$ (2)

$$c(\theta) = \frac{1}{E(\theta)} \quad \text{and} \quad \frac{dc(t)}{dt} = \sigma\left(\frac{1}{\eta(t)} + \frac{1}{\eta_K(t)}\right) - \frac{E_K(t)\varepsilon_K}{\eta_K(t)} \quad \text{with } \varepsilon_K \text{ from } \frac{d\varepsilon_K}{dt} = \frac{\sigma - E_K(t)\varepsilon_K}{\eta_K(t)}$$

RELAXATION FUNCTION (stress for unit strain applied at t = θ): $\varepsilon \equiv \begin{cases} 0 & \text{for } t < \theta \\ 1 & \text{for } t \geq \theta \end{cases}$ (3)

$$r(\theta) = E(\theta) \quad \text{and} \quad \frac{dr(t)}{dt} = E(t)\left[-r(t)\left(\frac{1}{\eta(t)} + \frac{1}{\eta_K(t)}\right) + \frac{E_K(t)\varepsilon_K}{\eta_K(t)}\right] \quad \text{with } \varepsilon_K \text{ from } \frac{d\varepsilon_K}{dt} = \frac{r(t) - E_K(t)\varepsilon_K}{\eta_K(t)}$$

Remarks: Creep- and relaxation functions are each others inverse viscoelastic quantities, see (16,17) for example. This means that Equation 2 will predict $c(t) \equiv 1$ when $\sigma = r(t)$ is introduced. This feature makes a fine check on programs developed for the stress-strain analysis of concrete. An example is demonstrated in Appendix B at the end of this paper.

1.1.4 Material parameters

The material parameters in Equation 1, Young's modulus E (of free spring), Young's modulus E_K of the Kelvin element, viscosity η_K for the Kelvin element, and η for the viscosity of the Newton element, are thought of as calibrated from experiments. Such a procedure used in (18,19), however, is a very time consuming and expensive task – and the parameters obtained are valid only for the specific concrete considered. In other words, they apply only for concretes, which have the same proportioning and curing conditions as the concrete in question – and for a period of time which is the same as time used for calibration.

In the subsequent chapters we will overcome these limitations developing a procedure, which applies for any concrete (including proportioning, curing conditions and time in use). A first step in this procedure is to respect more detailed the key observations previously made in Section 1.1.1 on creep of concrete – such as summarized in Equation 4 reproduced from (1,2). A next step is to utilize the obvious fact that concrete is a composite material.

Deductions from the thee key-observations made in Section 1.1.1

$$
\begin{array}{lll}
E = E(t) & \text{Hooke Young's modulus} & \\
E_K = E_K(t) = \dfrac{E(t)}{\alpha} & \text{Kelvin Young's modulus with } \underline{\text{constant delayed elasticity factor } \alpha} & \\
\eta_K = \eta_K(t) = \dfrac{E_K(t)}{Q}\,t & \text{Kelvin viscosity with } \underline{\text{constant rate power, } Q} & (4) \\
\eta = \eta(t) = F\,\dfrac{t}{1 + C/t} & \text{Newton viscosity with } \begin{cases} \underline{\text{constant flow-modulus, } F} \\ \underline{\text{constant consolidation factor, } C} \end{cases} &
\end{array}
$$

2. CONCRETE AS A COMPOSITE MATERIAL

As just indicated, it is not very satisfying to perform calibration tests every time a new concrete is suggested to be used. An alternative method to determine the material sub-parameters introduced in Equation 2 is offered by composite theory, by which these parameters can be calculated from knowing about the concrete composition as presented in Table 1.

CONCRETE COMPOSITION	
Water/Cement weight ratio	W/C
Silica/Cement weight ratio	S/C
Aggregate/Cement weight ratio	A/C

Table 1. *Composition of concrete.*

2.1 Composite model (CSA)

It is well known that concrete can be considered approximately as a particulate composite with spherical particles in a continuous matrix of hardened cement paste. The geometry normally used (also in this paper) is the CSA-geometry described by Hashin (20) and illustrated in Figure 3. Volume concentration of particles (particle volume/total composite volume) is denoted by c.

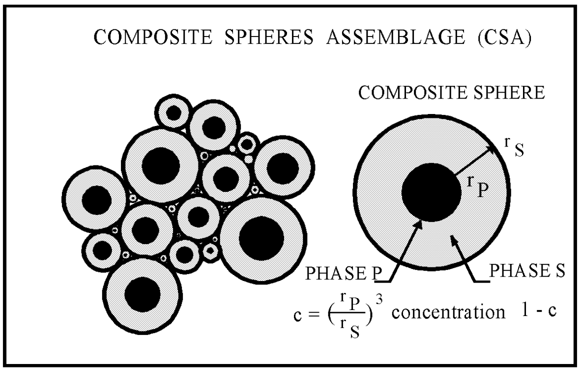

***Figure 3**. Composite spheres assemblage (CSA) with phase P particles in a continuous phase S.*

2.1.1 Elastic property of CSA

The elastic behavior of a CSA composite can be calculated from Equation 3 from knowing the volume concentrations and elastic properties of the two phases, see (17, for example).

$$
E = E_S \frac{A + n}{1 + An} \quad \text{with stiffness ratio } n = \frac{E_P}{E_S} \quad \text{and volume parameter } A = \frac{1 - c}{1 + c} \qquad (5)
$$

2.1.2 Viscoelastic properties of CSA

Without significant loss of accuracy the Burgers model hitherto considered can be replaced by the Pseudo Maxwell model shown in Figure 2. This is justified by the observation previously made in Section 1.1.1 that delayed elastic strain develops very fast. In the present context we assume that it develops just as fast as the elastic strain such that the composite creep function, c(t), becomes as presented in Equation 6.

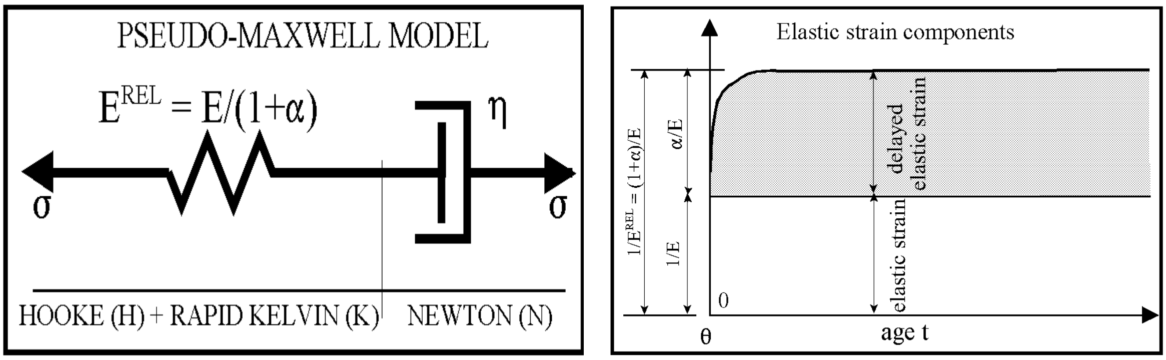

Figure 2. *Concrete as a Pseudo Maxwell material. Delayed elastic strain corresponds to Kelvin strain. Relaxed elastic strain is Hooke + max. Kelvin strain. The relaxed Young's modulus, E^{REL}, refers to the sum of these strains.*

$$c(t) \simeq \frac{1}{E^{REL}} + \frac{t}{\eta} = \frac{1+\alpha}{E} + \frac{t}{\eta} \qquad \text{(Pseudo-Maxwell model)} \qquad (6)$$

NOTE: α/E is a rapidly developing Kelvin strain with delayed elasticity factor, α

The pseudo Maxwell creep function is now used to predict *viscoelastic composite material properties from the viscoelastic matrix properties.* The viscoelastic behavior of a CSA composite with very soft or very stiff particles can be related to the viscoelastic behavior of the matrix phase (S) by the following expression developed in (17), where c(t) and $c_S(t)$ denote creep functions of composite and matrix respectively.

$$c(t) = \begin{cases} c_S(t)/A & \text{when particles (phase P) are very soft } (n = 0) \\ c_S(t)*A & \text{when particles (phase P) are very stiff } (n = \infty) \end{cases} \qquad (7)$$

This expression tells us that the viscoelastic behavior of such extreme composites is very much the same as the viscoelastic behavior of the matrix (phase S): For concrete, for example: The viscoelastic behavior of concrete, with very soft or very stiff aggregates, reflects very much the viscoelastic behavior of the HCP used. The behaviors differ only by a factor (A) reflecting the volume concentration of aggregates.

A more detailed study of moderately extreme CSA-composites, introducing the Pseudo Maxwell approximation of the Burgers creep function, see Figure 2, reveals that Equation 7 can be re-written as follows in Equation 8.

For the subsequent analysis of concrete we notice that this expression can be used successively in a two step analysis of a 'two layer' composite (a CSA material with a matrix, which is in itself a CSA material). In both steps the conditions must be kept on stiffness ratios (n) as indicated. The stiffness restriction, n > 1.5, in Equation 8 causes no problems in the analysis of *ordinary concrete,* where particles always are much stiffer than the matrix.

COMPOSITE PROPERTIES FROM PHASE S PROPERTIES

$$
\begin{aligned}
&E = E_S \frac{A+n}{1+An} && \textit{Hooke Young's modulus} \\
&\alpha = (1+\alpha_S)\frac{A+n}{1+An}\frac{1+An(1+\alpha_S)}{A+n(1+\alpha_S)} - 1 && \textit{delayed elasticity factor} \quad \left(n = \frac{E_P}{E_S} \; ; \; A = \frac{1-c}{1+c} \right) \\
&E_K = E/\alpha \quad ; && \textit{Kelvin Young's modulus} \\
&\eta = \begin{cases} A\eta_S & \text{when } n < 0.005 \\ \eta_S/A & \text{when } n > 1.5 \end{cases} \quad ; && \textit{Newton viscosity} \\
&c(t) = \frac{1+\alpha}{E} + \frac{t}{\eta} && \textit{Creep function}
\end{aligned} \qquad (8)
$$

3. COMPOSITE ANALYSIS OF CONCRETE

The information hitherto gathered on concrete composite behavior is used in this chapter to determine the material properties applying for this material with any composition such that a stress-strain analysis can be made by Equation 1.

The theory presented below (*italic letters*) is reproduced from (9,21) where concretes with no silica fumes were considered, meaning S/C = 0. The present formulation is modified to consider concrete containing silica fume. The modification has been made using the volume analysis (22) by Mejlhede of HCP with micro Silica.

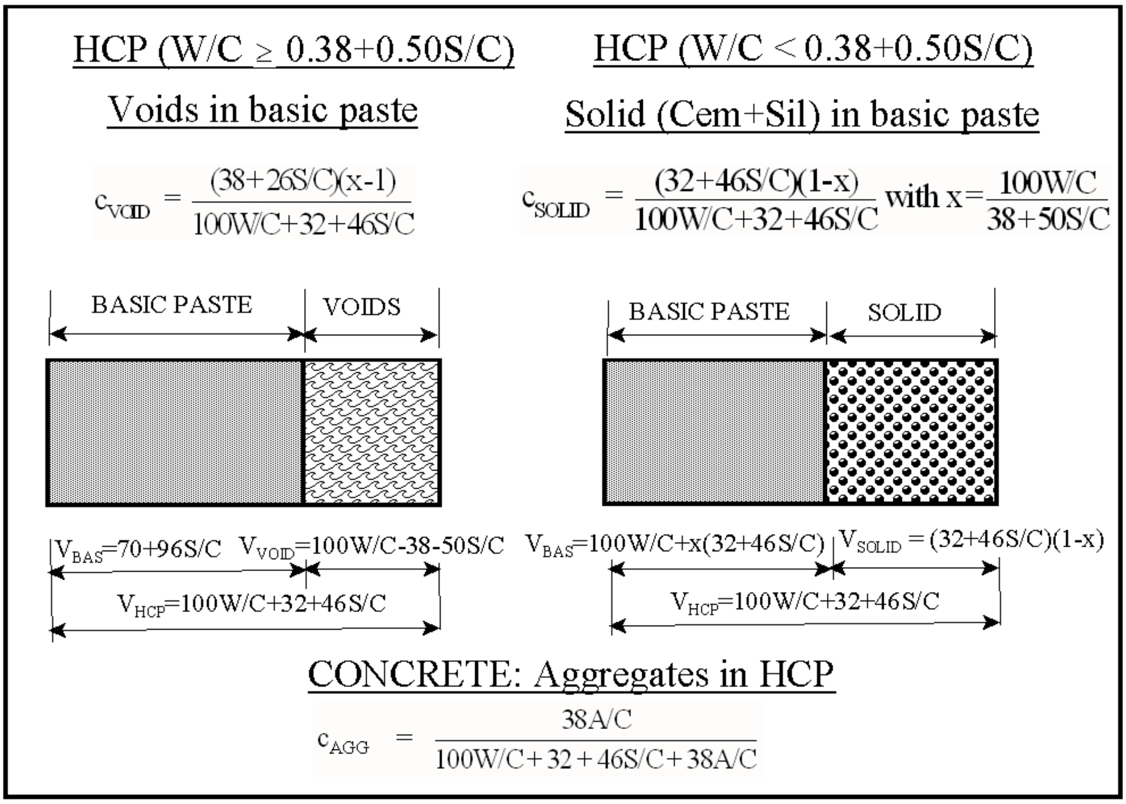

Figure 3. *Sub-components in concrete. The meaning of 'basic paste' is explained in the main text of this section*

Prior to considering concrete as a composite material two problems have to be solved: How can we model hardening cement paste (HCP) which is made of varying amounts of cement, silica fume, water and voids to appear as a homogeneous material. And how can *we solve the problem in a way such that the solutions obtained can be used for any W/C and S/C, recognizing that HCP changes its appearance at W/C ≈ 0.38 + 0.5S/C, see Figure 3.*

3.1 Basic Paste

The problems are solved introducing the concept of basic paste: Basic paste is that part of the water-cement-silica system which will hydrate 100% with an effective W/C = 0.38 + 0.5S/C. Basic paste appears as a viscoelastic solid the properties of which vary with age as explained in Table 2.

BASIC PASTE PROPERTIES	
Young's modulus of Basic paste	$E^o \approx 32000*g(t)$ MPa
Flow modulus	$F^o \approx 25000$ MPa
Delayed elasticity factor	$\alpha^o \approx 1$
Rate power	$Q \approx 10$
Consolidation factor	$C \approx 5$ days
Derived	
Kelvin stiffness	$E_K{}^o = E^o/\alpha^o$
Kelvin viscosity	$\eta_K{}^o = (E_K{}^o/Q)*t$
Newton viscosity	$\eta^o = F^o*t/(1+C/t)$

Table 2. *Basic paste properties. Numerical quantities indicated by ≈ are orders of magnitudes. The degree of hydration, g(t), is calculated by Equation 9 suggested in (23).*

$$g(t) = exp\left(-\left(\frac{\tau_{HYD}}{t}\right)^{\beta}\right) \quad degree\ of\ hydration \quad \left(\begin{array}{l} relaxation\ time\ \tau_{HYD} \\ hydration\ power\ \beta \end{array}\right) \qquad (9)$$

Now, concrete can be described as a CSA composite with aggregates of concentration c_{AGG} in the following HCPs, see Figure 3:

For W/C ≥ 0.38 + 0.50S/C the HCP is a porous basic paste with a void concentration of c_{VOID}

For W/C < 0.38 + 0.50S/C the HCP is a basic paste mixed with particles (un-hydrated grains of cement and silica fume) of concentration c_{SOLID}.

3.2 Hardening cement paste properties

The viscoelastic material parameters for the two HCPs just identified are determined as for the CSA composite explained in Chapter 2. The matrix is basic paste with properties from Table 2. The particle phase is voids or solids (not hydrated cement and silica fume) respectively. An algorithm for such analysis is presented in Tables A1 and A2 in Appendix A at the end of this paper.

The complexity of void geometry (capillary pores) is considered in Table A2 with a power of 1.8 on the particle concentration parameter A_{VOID}. This modification is introduced due to arguments presented in (9).

3.3 Concrete properties

As explained in Chapter 2 the analysis of concrete properties can now be determined repeating the analysis of hardening cement paste: The matrix properties, however, are replaced by the HCP properties just determined, and the particle phase properties are replaced with aggregate properties. An algorithm for this step is presented in Table A3, Appendix A.

3.4 Stress-strain analysis

Now, a stress-strain analysis (of both HCP and concrete) can be made with a numerical application of Equation 1. We re-call that the expressions hitherto presented apply for temperatures of T ≡ 20°C. Other conditions must be considered modifying the material properties according to Chapter 4. Examples of concrete analysis are presented in Chapter 5.

4. INFLUENCE OF TEMPERATURE

4.1 Maturity

As previously indicated, the material properties hitherto presented apply at T ≡ 20°C. With respect to the influence of temperature on the cement paste microstructure they can, however, be converted to apply also for other temperatures. We only have to replace age (t) with maturity age (t_M) determined by Equation 10, see Freisleben et.al. (23,24). The composition of a cement paste at real age t when cured at a temperature history T(t) equals the composition at age t_M (maturity age) when cured at a temperature history of T ≡ 20°C. It is noticed that the temperature function in Equation 10 has been simplified relative to the one presented in (23).

$$t_M = \int_0^t L(\tau)d\tau \qquad \text{Maturity age}$$
$$L(t) = \left(\frac{T(t)°C + 15}{35}\right)^{2.4} \quad (\equiv 0 \ \text{ if } T < -15°C) \ \text{Temperature function} \qquad (10)$$

4.2 Additional effect of temperature

According to (21) the influence of temperature on creep of concrete (and HCP) at a fixed composition can be considered by introducing a climate sensitive flow modulus F^o as explained in Equation 11 where f_C is a so-called curing factor.

In Table 2 (Basic paste):
$$F^O \text{ becomes } F^O/f_C \ \text{ or } \eta^o \text{ becomes } \eta^o/f_C \text{ where } f_C = L(t) \text{ is curing factor} \qquad (11)$$

The curing factor was deduced in (9) to depend on temperature in a similar way as the temperature function does. The simple expression f_C = L(t) has been tested with good results in (2). It is noticed that rate of creep is predicted to increase in general with increasing temperature.

4.3 Total effect of temperature

In itself the analysis of concrete structures by Equation 1 (with Appendix A) is invariable with respect to curing temperatures. The concrete properties, however, are not. The influence of temperature on concrete properties is considered modifying the viscoelastic properties presented in Table 2 as explained in this section, and summarized in Equation 12.

In Table 2 (Basic paste) :   *Replace ages, t, in material properties with maturity ages t_M and,* (12)   *additionally, divide Newton viscosity η^o with the temperature function L(t)*

5. EXAMPLES

The method of concrete analysis developed in this paper is tested in this section against experimental results reported in the literature. The algorithms presented in Appendix A are used for this purpose – together with *temperature modification according to Equation 12.* The concretes and curing conditions considered are described in Tables 3 - 5. The degrees of hydration for modern cements (HETEK and Umehara) are estimated from observations made in (18). For older cement types (L'Hermite and Glucklich) the degree of hydration is estimated from (25,26,27). The basic paste properties, delayed elasticity factor α^o, flow modulus F^o, consolidation factor C, and the rate power Q presented in Table 4 are properties calibrated from the experiments considered.

The parameters differ slightly from such presented in (1,2). The reasons are: The modification of the basic paste concept considered in Chapter 3, and a broader definition of 'delayed elastic strain' such that the theory becomes invariant with respect to the definition of Young's modulus (static, dynamic). The Figures presented are self explaining, except for the relaxation Figures 9 and 10 where inaccurate experimental data had to be modified. For curiosity some strain components (others than total strain) are presented in the graphs.

CONCRETE COMPOSITION AND PARTICLES STIFFNESS					
Author/experiment	**W/C**	**A/C**	**S/C**	**E_{SOLID} (MPa)**	**E_{AGG} (MPa)**
HETEK (18,2,28)	0.45	6.5	0.25	45000	45000
Glucklich et.al. (8)	0.32	0 (HCP)	0	40000	-
L'Hermite et.al. (15)	0.49	4.8	0	70000	70000
Umehara et.al. (29)	0.56	6.57	0	55000	55000

Table 3. *Concrete composition and particles stiffness. 'SOLID' indicates unhydrated grains of cement and silica fume.*

BASIC PASTE PROPERTIES					
Author/experiment	**E^o (MPa)**	**α^o**	**F^o (MPa)**	**C (days)**	**Q**
HETEK (18,2,28)	32000*g(t)	1.2	20000	7	10
Glucklich et.al. (8)	32000*g(t)	0.75	42000	5	10
L'Hermite et.al. (15)	32000*g(t)	0.75	57000	4.	2
Umehara et.al. (29)	32000*g(t)	0.5	32000	1	10

Table 4. *Basic paste properties. g(t) denotes degree of hydration.*

HYDRATION AND CURING			
Author/experiment	**β**	**τ_{HYD} (days)**	**CURING ^{o}C**
HETEK (18,2,28)	0.95	0.63	≡ 20
Glucklich et.al. (8)	0.95	3.0	≡ 16
L'Hermite et.al. (15)	0.5	3.0	≡ 20
Umehara et.al. (29)	0.95	0.63	20 up to 1 day, then 20, 40, and 80

Table 5*. Hydration and curing*

5.1 Variable loads

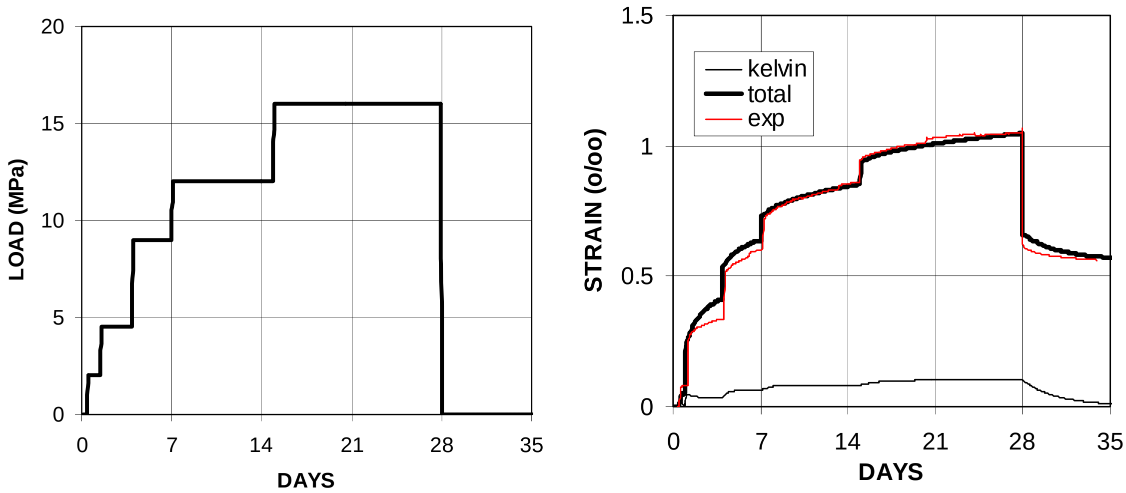

Figure 4. *HETEK: Strain due to variable load. Heavy line and thin line are predicted and measured strain respectively. First loaded at 0.6 days.*

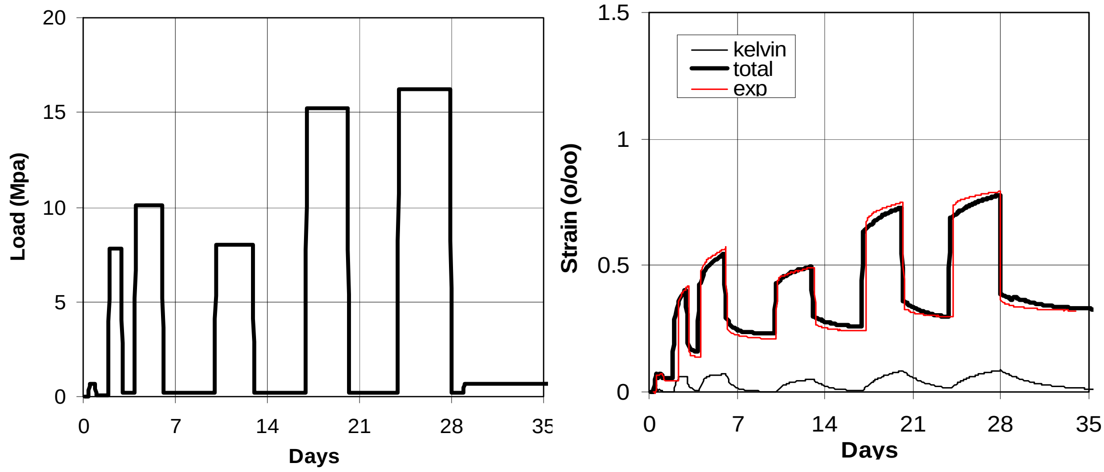

Figure 5. *HETEK: Strain due to variable load. Heavy line and thin line are predicted and measured strain respectively. First loaded at 0.6 days.*

5.2 Unloading

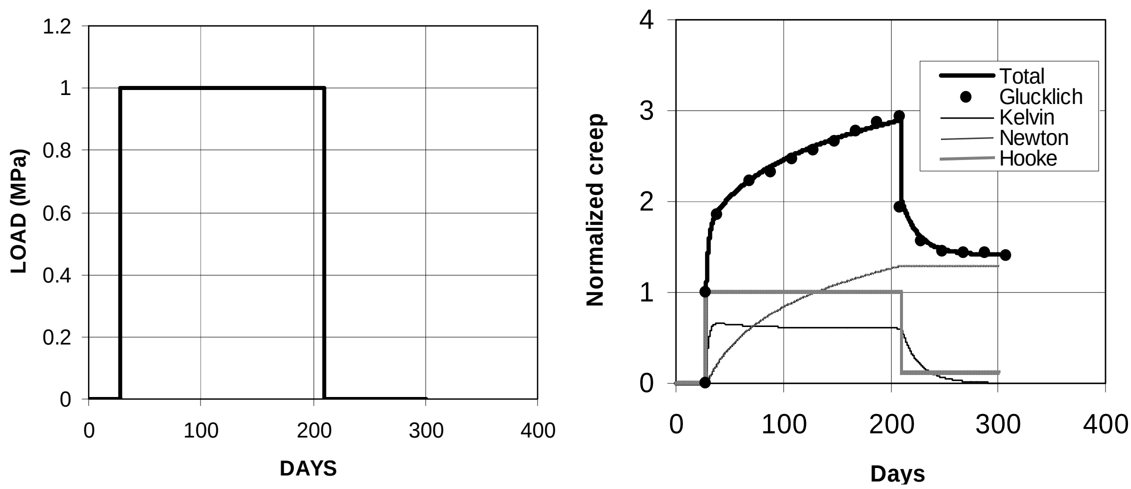

Figure 6. *Glucklich: Creep for loading at 28 days and unloading at 210 days. Normalized with respect to value at t = 28 days.*

5.3 Various ages at loading and various curing histories

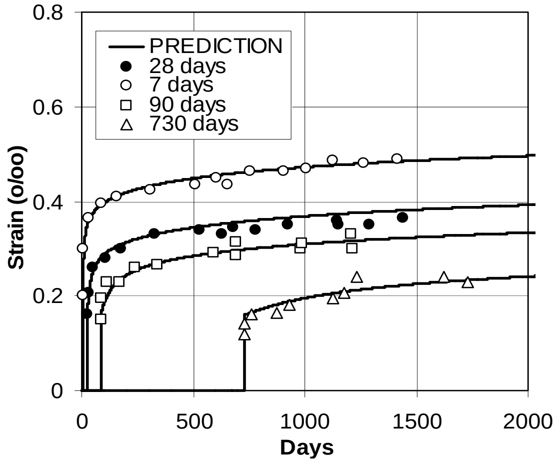

Figure 7. L'Hermite: Creep of concrete at different ages at loadings. Loads applied: 6.9 MPa.

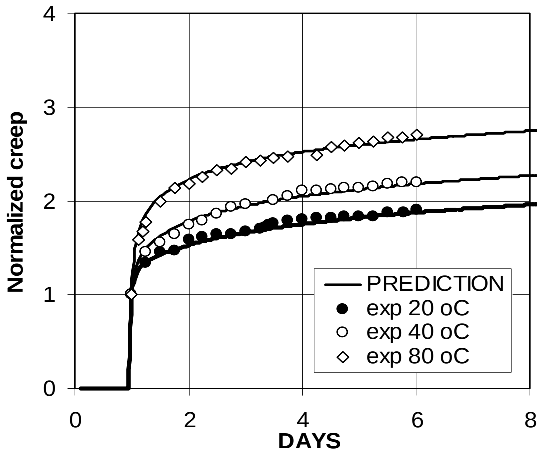

Figure 8. Umehara: Creep of concrete at different temperatures. Normalized with respect to value at t = 1 day.

5.4 Relaxation function

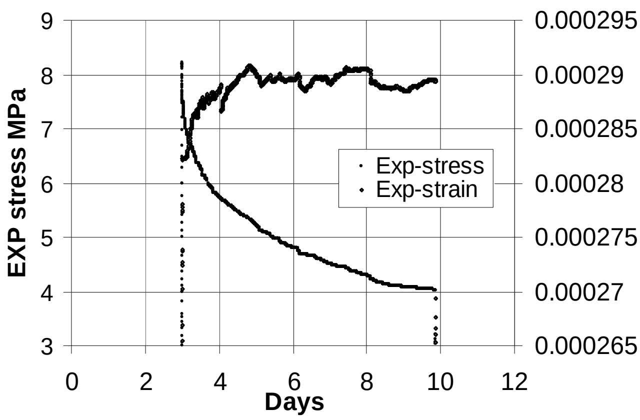

Figure 9. HETEK: Associated strain- and stress data in relaxation test. Start at t = 72 h = 3 days

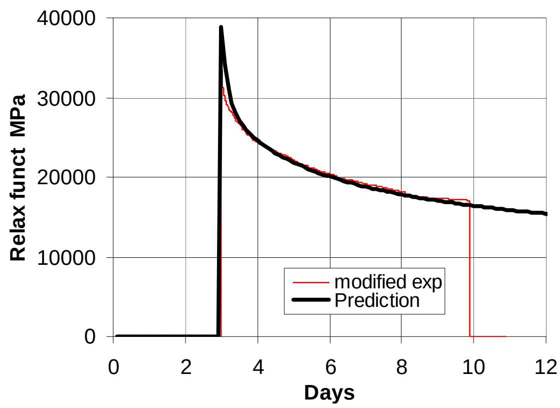

Figure 10. HETEK: Heavy line: Predicted relax function. Thin line: Relax function based on modified experimental data from Figure 9.

The data presented in Figure 9 are reproduced from relaxation experiments reported in (28), (the intension was to measure stress caused by a constant strain). Although the tests were made very carefully with the finest technologies available, it was not possible, within seconds, to produce a precise constant strain. The experimental relaxation function presented in Figure 9 (exp. stress) is therefore somewhat imprecise.

The experimental data can be 'improved' somewhat introducing the modification procedure presented in Equation 13 where the initial stiffness is realistically reflected by $E_{MODIFIED}$. That is to introduce a more reliable Young's modulus. At the same time this procedure considers the traditional concept of relaxation functions always to start with a stress equal to the Young's modulus.

The experimental relaxation function such obtained is presented in Figure 10 together with the relaxation function predicted by the present method, Equation 3, with concrete parameters calculated according to Section 3.3 with Tables 3 – 5.

$$E_{MODIFIED} = max\left(\frac{exp\,erimental(\,stress\,)}{exp\,erimental(\,strain\,)}\right) \;\Rightarrow\; Re\,laxation\;\; function\;\; becomes \qquad (13)$$

$$mod\,ified(\,stress\,) = \frac{E_{MODIFIED}}{max(exp\,erimental(\,stress\,))} * exp\,erimental(\,stress\,)$$

5.5 Discussion

Various concretes have been considered in this section involving various load variations, various temperature conditions, and various ages at loadings. It is concluded that all these features are well considered by the method developed in this paper.

As expected viscoelastic parameters (E^o,α^o,F^o,Q,C) vary somewhat from concrete to concrete. This is not very surprising: Cement, Silica fume, and aggregates will vary from location to location (from 'author to author').

What, however, is worthwhile noticing is, that these parameters keep constant within each type ('author') of experiment. This observation is a strong justification of the underlying principles of the theory presented.

6. CALIBRATION OF MATERIAL PARAMETERS

From Section 5.5 follows that the basic paste parameters (E^o,α^o,F^o,Q,C) are the significant quantities to know prior to a concrete analysis. It seems that calibration tests such as the Glucklich experiment (Figure 6) with various ages at loading and un-loading will be appropriate in this respect.

6.1 First estimate analysis

For first estimate analysis of concrete structures we suggest the basic paste parameters presented in Table 6 to be used. They represent tentative "average estimates" from Table 4 together with the author's experience obtained in creep analysis elsewhere.

Basic paste properties				
E^o (MPa)	α^o	F^o (MPa)	Q	C (days)
32000*g(t)	1	30000	10	6

Table 6. *Basic paste parameters for first estimate stress-strain analysis of concrete.*

7. CONCLUSIONS AND FINAL REMARKS

An incremental stress-strain relation convenient for FEM-analysis of concrete structures has been developed. Concrete composition, age at loading, and climatic conditions are considered simultaneously in this relation. No other incremental models presented in the literature offer these features in one approach.

The number of calibration tests needed for property determination can be held at a minimum. Material properties can be calculated from properties, typical for a basic concrete component – in this paper, the so-called basic paste. The advantage of respecting 'old' observations on concrete behavior, summarized in Section 1.1.1, means that properties calibrated are not bound to apply only for a period of time equal to time of calibration. This aspect has been further discussed in (1) where predictions similar to those predicted by the present composite method are compared with such made by the methods presented in (18,19) which are based the concept of concrete as a homogeneous material.

The simplicity of the method presented is due to the basic paste composite concept introduced: Concrete with any water-silica-cement ratio can always be considered as a two-phase composite of invariable geometry. The results of applying the incremental stress-strain model presented are successfully compared with experimental data reported in the concrete literature.

The analysis made in this paper is based on assuming that cement paste and concrete behave as linear-viscoelastic materials. The very satisfactory agreements between theoretical and experimental data justify this assumption. This means that cement paste and concrete in fact behave linear-viscoelastically from an age of approximately 1 day, see Figures 4 and 5. This observation is a significant age extension relative to earlier studies in the literature where linear-viscoelastic behavior is only demonstrated from ages of a 'few days'.

It is emphasized that any experimental observations used to develop the method presented in this paper are from basic research on concrete creep reported before 1965. The positive conclusions made above, with respect to the method developed, indicate that "old" information on basic concrete behavior apply also when modern concretes are considered. It is important to recognize this feature to avoid "re-inventions" when future research projects are planned on creep of concrete.

8. LIST OF NOTATIONS

The symbols most frequently used in this paper are listed below. Local symbols used only in intermediate results or closed sections are not listed.

Subscripts and Superscripts

P	Phase P (particles)
S	Phase S (matrix)
HCP	Hardening cement paste
o	Basic paste
M	Maturity age
K, N	Kelvin and Newton respectively

Concrete

W/C	Water-cement weight ratio
A/C	Aggregate-cement weight ratio
S/C	Silica fume-cement weight ratio
c	Volume concentration of particles in composite
A = (1-c)/1+c)	Particle concentration parameter

Age and time in general

t	Age of concrete

Maturity, hydration, and curing

L(t)	Temperature function
g=g(t)	Degree of hydration: hydrated amount of cement+silica/total amount
τ_{HYD},ß	Relaxation time and power factor in g(t) expression
T	Temperature [°C]
f_C	Curing factor

Elasticity and viscoelasticity

E = E(t)	Young's modulus
n= n(t)	Stiffness ratio
η	Viscosity
Q	Rate power
C	Consolidation factor
F	Flow modulus
α	Delayed elasticity factor

Stress and strain

σ	Stress
ε	Strain

Appendix A: Algorithms for property determination

Algorithms for the prediction of concrete properties are presented in the following tree tables. Tables A1 and A2 consider HCP. Table A3 considers concrete (= HCP + aggregates). Curing (temperature histories) is considered with basic paste properties modified as explained in Equation 12, Chapter 4.

HCP PROPERTIES when W/C < 0.38+0.5S/C	
Young' modulus	$E_{HCP} = E^{o} \dfrac{A_{SOLID} + n_{HCP}}{1 + A_{SOLID} n_{HCP}} \quad (n_{HCP} = \dfrac{E_{SOLID}}{E^{o}})$
Flow modulus	$F_{HCP} = F^{o}/A_{SOLID}$
Delayed elasticity factor	$\alpha_{HCP} = (1+\alpha^{o}) \dfrac{A_{SOLID}+n_{HCP}}{1+A_{SOLID}n_{HCP}} \dfrac{1+A_{SOLID}n_{HCP}(1+\alpha^{o})}{A_{SOLID}+n_{HCP}(1+\alpha^{o})} - 1$
Derived	
Kelvin stiffness	$E_{K,HCP} = E_{HCP}/\alpha_{HCP}$
Kelvin viscosity	$\eta_{K,HCP} = (E_{K,HCP}/Q)*t$
Newton viscosity	$\eta_{HCP} = F_{HCP}*t/(1+C/t)$
$c_{SOLID} = \dfrac{(32+46S/C)(1-x)}{100W/C+32+46S/C}$; $\quad A_{SOLID} = \dfrac{1-c_{SOLID}}{1+c_{SOLID}}$; $\quad x = \dfrac{100W/C}{38+50S/C}$	

***Table A1**. Properties of hardening cement paste with cement and silica particles (solid).*

HCP PROPERTIES when W/C > 0.38+0.5S/C	
Young' modulus	$E_{HCP} = A_{VOID}*E^{o}$
Flow modulus	$F_{HCP} = A_{VOID}*F^{o}$
Delayed elasticity factor	$\alpha_{HCP} = \alpha^{o}$
Derived	
Kelvin stiffness	$E_{K,HCP} = E_{HCP}/\alpha_{HCP}$
Kelvin viscosity	$\eta_{K,HCP} = (E_{K,HCP}/Q)*t$
Newton viscosity	$\eta_{HCP} = F_{HCP}*t/(1+C/t)$
$c_{VOID} = \dfrac{(38+26S/C)(x-1)}{100W/C+32+46S/C}$; $\quad A_{VOID} = \left(\dfrac{1-c_{VOID}}{1+c_{VOID}}\right)^{1.8}$	

***Table A2**. Properties of hardening cement paste with voids*

CONCRETE PROPERTIES	
Young' modulus	$E = E_{HCP} \dfrac{A_{AGG}+n_{CON}}{1+A_{AGG}n_{CON}} \quad (n_{CON} = \dfrac{E_{AGG}}{E_{HCP}})$
Flow modulus	$F = F_{HCP}/A_{AGG}$
Delayed elasticity factor	$\alpha = (1+\alpha_{HCP}) \dfrac{A_{AGG}+n_{CON}}{1+A_{AGG}n_{CON}} \dfrac{1+A_{AGG}n_{CON}(1+\alpha_{HCP})}{A_{AGG}+n_{CON}(1+\alpha_{HCP})} - 1$
Derived	
Kelvin stiffness	$E_K = E/\alpha$
Kelvin viscosity	$\eta_K = (E_K/Q)*t$
Newton viscosity	$\eta = F*t/(1+C/t)$
$c_{AGG} = \dfrac{38A/C}{100W/C+46S/C+38A/C+32}$; $\quad A_{AGG} = \dfrac{1-c_{AGG}}{1+c_{AGG}}$	

***Table A3**. Properties of concrete to be used for stress-strain analysis by Equation 1.*

Appendix B: Test of calculation procedures

As mentioned in Section 1.1.3: Creep- and relaxation functions are each others inverse viscoelastic quantities. This means that Equation 2 (in main text) will predict c(t) ≡ 1 when σ = r(t) is introduced. An example of this feature is demonstrated in this Appendix, Figures B1 – B3. The concrete is the HETEK described in Chapter 5. Curing is described in Figure B4. Loading is at t = 50 days. As in Chapter 5 some strain components are shown as the are predicted by Equation 1.

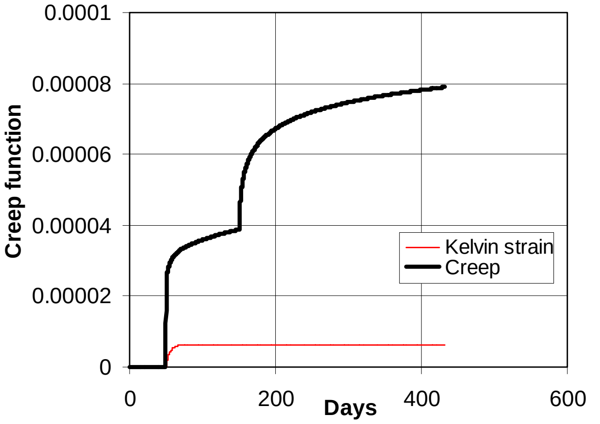

Figure B1. *Creep function. The concrete is loaded at t = 50 days.*

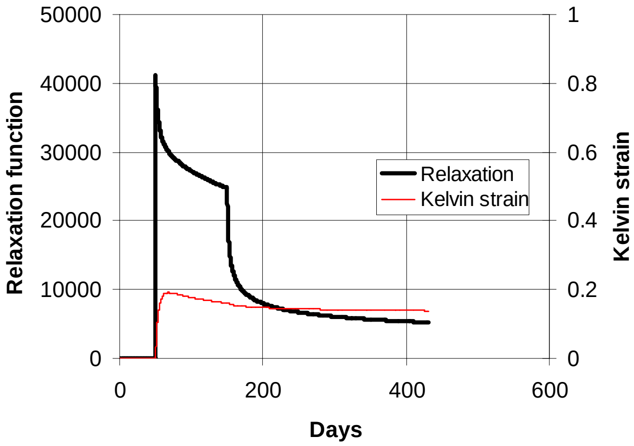

Figure B2. *Relaxation function. The concrete is loaded at t = 50 days.*

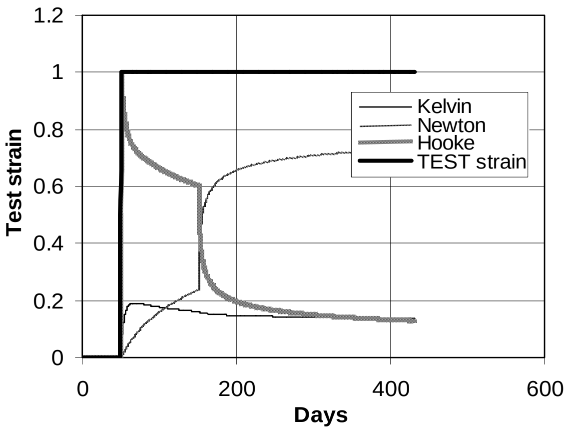

Figure B3. *Test of calculation procedures. Strain must become unity if concrete is 'loaded' with relaxation function.*

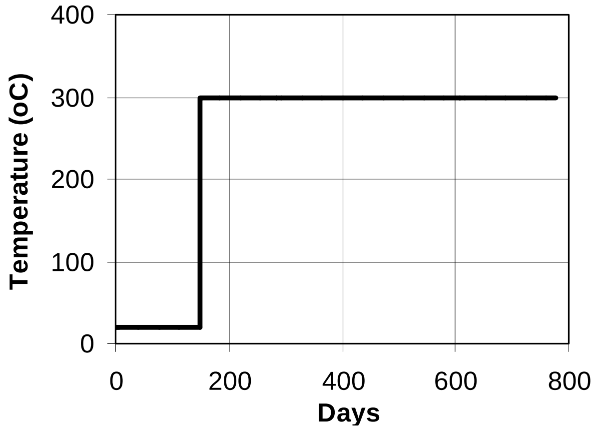

Figure B4. *Curing temperature. Shift from 20°C to 300 °C at 150 days.*

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