INSTITUT FOR BÆRENDE KONSTRUKTIONER OG MATERIALER

# Shear Strength of Non-Shear Reinforced Concrete Elements

## Part 2. T-Beams

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DEPARTMENT OF STRUCTURAL ENGINEERING AND MATERIALS
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# Preface

The present work has been carried out at the Department of Structural Engineering and Materials, Technical University of Denmark, under the supervision of Professor, dr. techn. M.P. Nielsen.

I would like to thank my supervisor for giving valuable advice and inspiration as well as valuable criticism to the report.

Thanks are also due to my co-supervisors Assoc. Prof., lic.techn. Henrik Stang and lic.techn. Bent Feddersen.

Lyngby, July 1997.

Linh Cao Hoang

# Summary

This paper deals with the plastic shear strength of non shear reinforced T-beams.

The influence of an un-reinforced flange on the shear capacity is investigated by considering a failure mechanism involving crack sliding in the web and a kind of membrane action over an effective width of the flange.

The position of the crack in which sliding takes place is determined by the crack sliding model developed by Jin-Ping Zhang [94.1].

The theoretical calculations are compared with test results reported in the literature. A good agreement has been found.

A simplified method to calculate the shear capacity of T-beams is presented.

# Resumé

Denne rapport behandler forskydningsbæreevnen af T-bjælker uden forskydningsarmering bestemt v.h.a. plasticitetsteorien.

Indflydelsen på bæreevnen af en uarmeret trykflange er undersøgt ved at betragte en brudfigur, hvor glidningsbrud indtræffer langs en revne i kroppen, og en art membranvirkning udvikles over en effektiv bredde af flangen.

Beliggenheden af revnen, hvori glidningsbruddet indtræffer, bestemmes v.h.a. "crack sliding" modellen udviklet af Jin-Ping Zhang [94.1].

De teoretiske resultater sammenlignes med forsøgsresultater fra litteraturen. Der er fundet god overensstemmelse.

Der gives sidst i rapporten en simplificeret metode til at bestemme forskydningsbæreevnen af T- bjælker.

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# Notations

a : Shear span

a' : Distance from the external load to the starting point of a yield line

a/h : Shear span ratio

$A_c$ : Area of concrete cross section

$A_{cf}$ : Area of the flange ( = $tb_f$)

$A_{cf,ef}$ : Effective area of the flange ( = $tb_{fef}$)

$A_{cw}$ : Area of the web = $(h - t)\,b_w$

$A_s$ : Longitudinal reinforcement area

$b_f$ : Width of flange

$b_{fef}$ : Effective width of flange

$b_w$ : Width of web

e : Distance from the top face to the center of gravity of cross section

$f_c$ : Uniaxial compressive strength of concrete

$f_{tef}$ : Effective plastic tensile strength of concrete

h : Depth of beam

K : Parameter taking into account the influence of the flange

s(h) : Size effect parameter

t : Thickness of flange

u : Relative displacement in yield line

V : Reaction at support

$V_{cr}$ : Cracking load

$V_u$ : Ultimate load/reaction

$W_E$ : External work at failure

$W_I$ : Internal work at failure

$W_{I,flange}$ : Internal work in the flange

$W_{I,web}$ : Internal work in the web

x : Horizontal projection of yield line (in case of rectangular cross section) and horizontal projection of critical diagonal crack (in case of T- beams)

$x_w$ : Horizontal projection of yield line formed in the web of T- beams

$x_f$ : Geometrical quantity

$y_o$ : Geometrical quantity

z : Geometrical quantity

$\beta$ : $=A_{cf}/A_{cw}$

$\beta_{ef}$ : $=A_{cf,ef}/A_{cw}$

$\nu_0$ : Effectiveness factor for uncracked concrete

$\nu_m$ : Effectiveness factor for membrane action

$\theta$ : Rotation angle

$\rho$ : Reinforcement ratio

$\tau$ : Shear stress

$\tau_c$ : $=0.059\nu_0 f_c$

$\tau_o$ : Shear capacity corresponding to a yield line formed in uncracked concrete

$\tau_u$ : Shear capacity ($= V_u/b_w h$)

$\tau_{u,R}$ : Shear capacity of beams with rectangular cross section

$\tau_{u,T}$ : Shear capacity of T- beams

$\xi$ : $= \nu_m/\nu_0$

# Chapter 1

# Introduction

This paper deals with the shear capacity of T-beams without shear reinforcement. A study of the influence of an unreinforced flange on the shear capacity is carried out by considering a failure mechanism featuring crack sliding in the web and rotation in hinges in the flange.

Concerning the crack sliding failure in the web and the determination of the critical crack along which sliding takes place, we will employ the *crack sliding model* developed by Jin-Ping Zhang [94.1]. This model is originally developed for analysing the shear strength of non shear reinforced, simply supported beams with rectangular cross section.

It has turned out that the model is rather general and may also be applied when analysing statically indeterminate beams and prestressed hollow-core slabs, see [97.2] and [97.3]. In this paper, we shall demonstrate how this model may be applied to T-beams.

In what follows we shall briefly review the crack sliding model.

## 1.1 The crack sliding model

To determine the shear capacity of non shear reinforced concrete beams with rectangular cross section, the *crack sliding model* has been developed by Jin-Ping Zhang [94.1]. A detailed description of this model may be found in [94.1] and [97.2]. In this paper we shall only summarise the basic features of the model.

The crack sliding model is based upon the upper bound theorem of the theory of plasticity. According to the crack sliding model, the cracking of concrete introduces potential yield lines which, due to a reduced sliding resistance, may be more dangerous than the yield lines predicted by the usual plastic theory. In other words, shear failure in non shear reinforced concrete beams takes place as sliding in cracks. It has been demonstrated by Jin-Ping Zhang [97.4] that the sliding resistance of a crack transformed into a yield line is half of the sliding resistance of a similar yield line through uncracked concrete.

The diagonal crack which is transformed into a yield line is called *the critical diagonal crack*. For an overreinforced simply supported beam subjected to concentrated loading, the shear capacity and the position of the critical diagonal crack are determined as follows:

Consider the beam shown in figure 1.1. We assume that sliding takes place along a straight diagonal crack ending at the loading point and having the horizontal projection $x$. By a vertically directed motion of the part I, the internal work done in the diagonal crack transformed into a yield line may be determined by the following approximation to the correct dissipation formula, see [97.1] and [97.2],

$$W_I = 2\frac{\tau_c}{\frac{x}{h}} A_c \cdot u \tag{1.1}$$

Here $A_c = b \cdot h$ is the area of the cross section and $\tau_c$ is given as

$$\tau_c = 0.059 \nu_0 f_c$$

$f_c$ being the uniaxial compressive strength of concrete and $\nu_0$ the effectiveness factor which may be taken as, see [94.1] and [97.2],

$$\nu_0 = 0.88 \frac{1}{\sqrt{f_c}} \left(1 + \frac{1}{\sqrt{h}}\right)(1 + 26\rho) \tag{1.2}$$

Here $f_c$ must be inserted in MPa, $h$ in meter and $\rho$ is the reinforcement ratio. In [94.1] and [97.2] some restrictions have been set on the range of $f_c$, $h$ and $\rho$. Extrapolations may, however, be done without any substantial change of the formula.

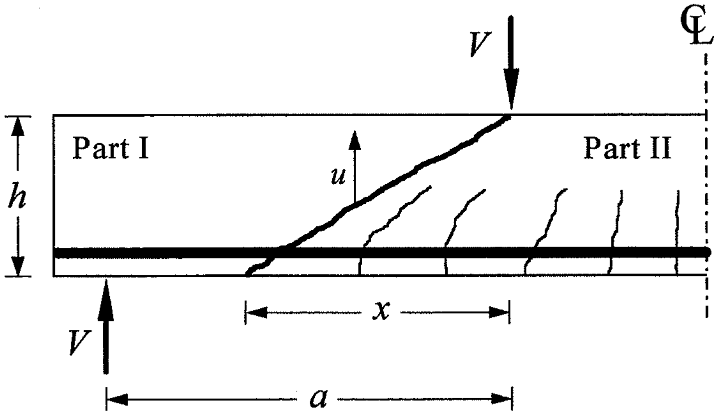


**Figure 1.1** *Simply supported beam with a critical diagonal crack.*

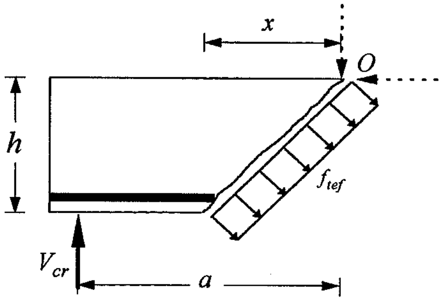


**Figure 1.2** *Stress distribution along a developing crack.*

By using the work equation with V·*u* as the external work done by the reaction, we find the following expression for the shear capacity

$$\tau_u = \frac{V_u}{A_c} = 2\frac{\tau_c}{\frac{x}{h}} \tag{1.3}$$

To form a diagonal crack with the horizontal projection $x$, a certain load level, here denoted as the cracking load $V_{cr}$, is needed. The cracking load may be found by assuming that the distribution of the normal stresses along the crack, while it develops, is constant and equal to the effective plastic tensile strength $f_{tef}$, see figure 1.2. By a rotation mechanism around point $O$, the cracking load is found to be

$$V_{cr} = \tfrac{1}{2} f_{tef} \frac{b}{a}(x^2 + h^2) \tag{1.4}$$

The effective plastic tensile strength of concrete may be taken as, see [94.1],

$$f_{tef} = 0156 \cdot f_c^{2/3} \cdot s(h) \tag{1.5}$$

s($h$) being a parameter taking into account the size effect

$$s(h)=\left(\frac{h}{0.1}\right)^{-0.3} \quad ,(h \text{ is the depth of beam in meter}) \tag{1.6}$$

After the formation of the crack, further rotation around point $O$ is prevented by the longitudinal reinforcement. Sliding in the crack may now only take place when the load needed to form the crack is equal to the sliding capacity of the crack, i.e. the horizontal projection of the critical diagonal crack must bring the following condition to fulfilment

$$V_u = V_{cr} \tag{1.7}$$

By inserting (1.3) and (1.4) into (1.7) we arrive at the following cubic equation to determine the horizontal projection of the critical diagonal crack

$$\left(\frac{x}{h}\right)^3+\frac{x}{h}-4\frac{\tau_c}{f_{tef}}\frac{a}{h}=0 \tag{1.8}$$

The shear capacity of the beam may now be found by inserting the solution of (1.8) into (1.3).

The basic content of the model is outlined in figure 1.3 where the variation of $V_u$ and $V_{cr}$ with $x$ is shown. The point of intersection of the two curves representing $V_u$ and $V_{cr}$ respectively determines both the position of the yield line and the load carrying capacity.

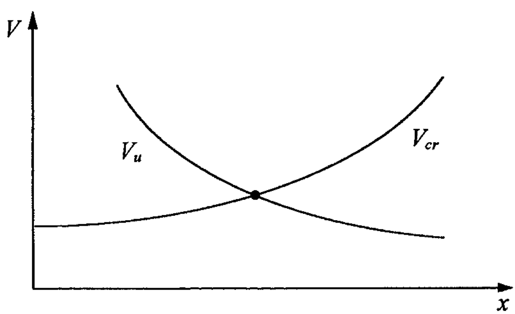


**Figure 1.3** *The variation of $V_u$ and $V_{cr}$ with x.*

# Chapter 2

# Shear Capacity of T-Beams

The crack sliding model, as it has been outlined in the previous chapter, has been successfully applied to beams with rectangular cross section. For such beams the assumption of a sliding failure along the hole length of the critical diagonal crack is reasonable and in good agreement with experimental observation, see [94.1].

If the model is used without modifications for T-beams it will lead to an overestimation of the shear capacity.

The overestimation is especially high in the cases with relatively high ratios of $t/h$ and $b_f/b_w$ or in cases where the concentrated loading is not distributed across the entire flange.

From experimental observations, it seems more likely that the sliding failure in the web is accompanied by a rotation mechanism in the flange, see figure 2.1.

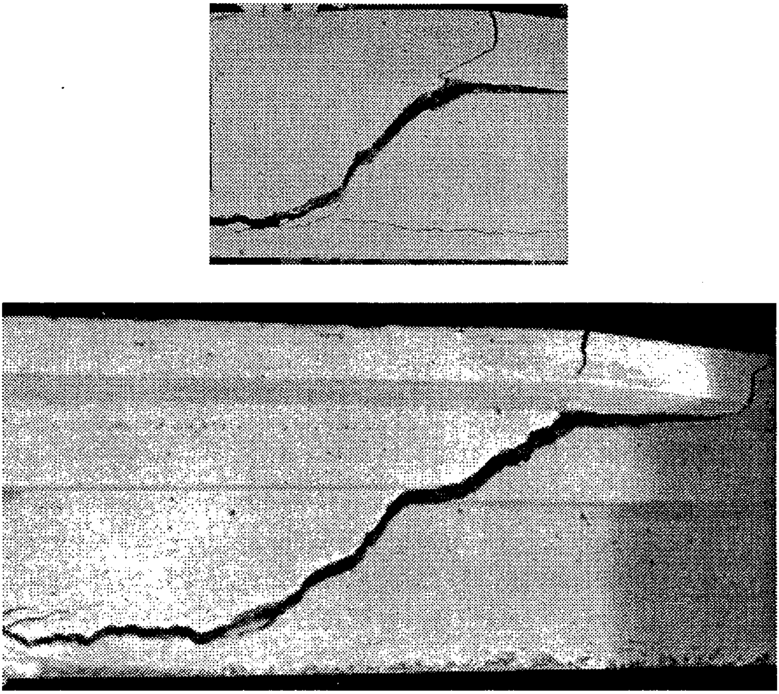

**Figure 2.1** *Shear failure mode in beams with compression flange* [*53.1*].

However, if the flange is provided with sufficiently strong longitudinal reinforcement, it may happen that crack sliding also takes place in the flange. In this case the sliding surface in the flange can by no means be considered as being plane, but must rather be characterised as a three-dimensional surface similar to what is met when punching failure takes place at the edge of a slab.

In the following we shall examine the shear capacity of T-beams without longitudinal reinforcement in the flange.

## 2.1 Failure mechanism for T-beams

We consider an overreinforced simply supported T-beam subjected to four point bending as shown in figure 2.2. The shear failure mechanism to be considered is shown in the figures 2.2 and 2.3. The mechanism consists of a sliding failure along a crack in the web and rotation in hinges in the flange. The mechanism in the flange is similar to the mechanism in J. F. Jensen's exact solution for a beam with rectangular cross section, see [78.1] and [78.2].

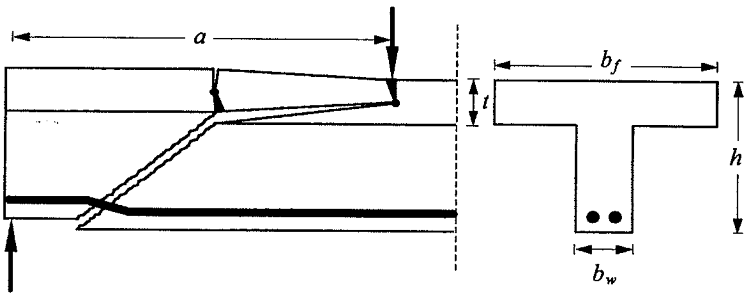


**Figure 2.2** *Simply supported T-beam.*

The crack transformed into a yield line is assumed to originate from point $A$ at the distance $a'$ from the loading point, see figure 2.3. The angles FBC and BCD are $\pi/2$, the length of FB and CD is denoted as $y_o$ and the distance from the top face to the hinges C and F is $\frac{1}{2}t$.

The failure mechanism may be described as follows:

Part III does not move, part II rotates around point C and the relative motion between part I and part II is a rotation around point F. The two rotation angles are equal and opposite, which results in a vertically directed motion of part I.

The yield lines BF and CD represent pure compression while BC and FE represent pure separation failure. Along the yield line AB with the horizontal projection $x_w$ we have both sliding and separation failure.

For beams provided with longitudinal reinforcement in the flange, the failure mechanism considered will only be geometrically possible if the reinforcement is yielding at failure or if the reinforcement is placed at the same level as the points F and C.

Reinforcement in the flange will not be treated in this paper.

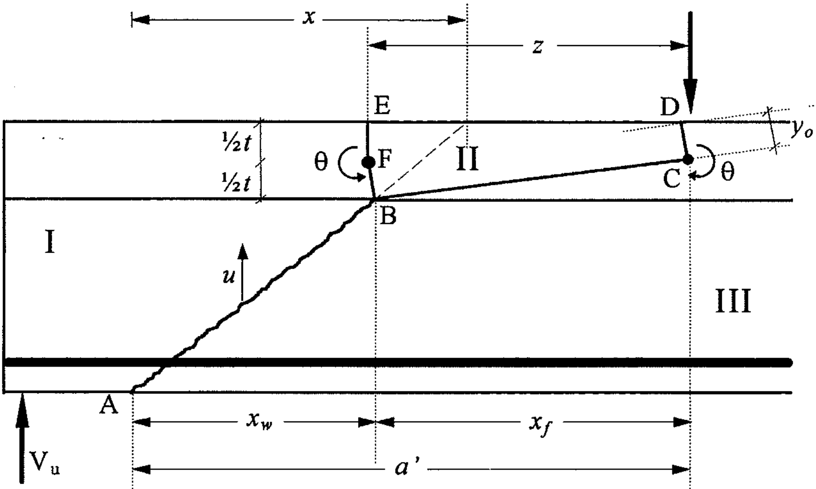


**Figure 2.3** *Failure mechanism.*

The geometrical quantities shown in figure 2.3 are found to be:

$$x_f = h\left(\frac{a'}{h} - \frac{x}{h}\left(1 - \frac{t}{h}\right)\right) \tag{2.1}$$

$$z = x_f + \frac{1}{4}\frac{t^2}{x_f} \tag{2.2}$$

$$y_o = \mathrm{FB} = \mathrm{CD} = \tfrac{1}{2} t \frac{\sqrt{{x_f}^2 + \tfrac{1}{4} t^2}}{x_f} \tag{2.3}$$

$$u = \theta \cdot z = \theta \cdot \left( x_f + \frac{1}{4} \frac{t^2}{x_f} \right) \tag{2.4}$$

In the formulas, $x$ defines the horizontal projection of a straight line running between the points A and B and ending at the top face. The relation between $x$ and $x_w$ is given as

$$\frac{x_w}{h-t} = \frac{x}{h} \tag{2.5}$$

The work equation may be written in the form

$$\mathrm{W_E} = \mathrm{W_{I,web}} + \mathrm{W_{I,flange}} \tag{2.6}$$

where $\mathrm{W_{I,web}}$ and $\mathrm{W_{I,flange}}$ are the contributions from the web and the flange, respectively. $\mathrm{W_E}$ is the external work given as

$$\begin{aligned} \mathrm{W_E} &= \mathrm{V_u} \cdot u \\ &= \mathrm{V_u} \cdot \theta \cdot \left( x_f + \frac{1}{4} \frac{t^2}{x_f} \right) \end{aligned} \tag{2.7}$$

The internal work done along the crack AB, which is transformed into a yield line, may be written as, see (1.1),

$$\begin{aligned} \mathrm{W_{I,web}} &= 2 \frac{\tau_c}{\frac{x_w}{h-t}} \mathrm{A_{cw}} \cdot u \\ &= 2 \frac{\tau_c}{\frac{x}{h}} \mathrm{A_{cw}} \cdot \theta \cdot \left( x_f + \frac{1}{4} \frac{t^2}{x_f} \right) \end{aligned} \tag{2.8}$$

$A_{cw}$ is the area of the web

$$A_{cw} = b_w(h - t) \tag{2.9}$$

To arrive at the contribution $W_{I,flange}$, an additional discussion on the mechanism in the flange is needed.

The mechanism in the flange simulates a kind of membrane action similar to what is met in slabs with restrained supports. However, it can not be expected that the part of the flange acting as a membrane slab, i.e. the region FBCD in figure 2.3, is sufficiently restrained across the whole width of the flange. The stress distribution is therefore very complicated and by no means uniaxial. The stresses are transferred from a narrow zone beneath the external load to a narrow zone at point B. The flow of the stresses is qualitatively illustrated in figure 2.4.

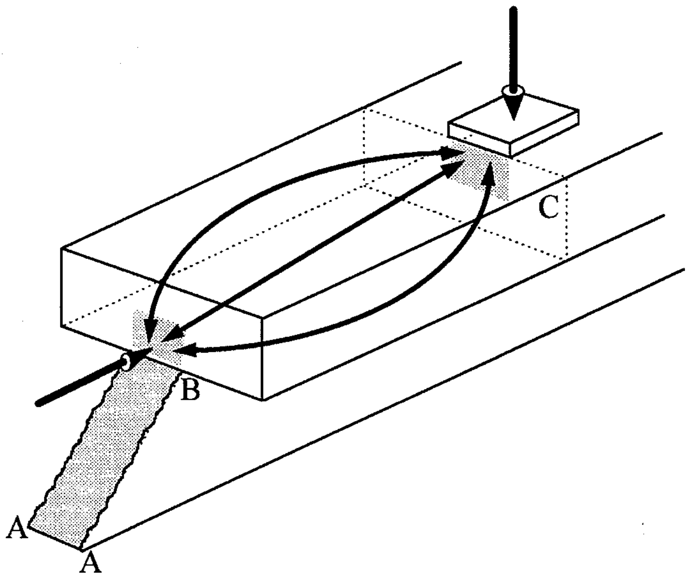


**Figure 2.4** *Transmission of stresses in the flange.*

To simplify the calculations, we will make the well-known approximation that the stresses are distributed along a length which is varying linearly from the narrow zones corresponding to an angle $\alpha$ given by $\tan\alpha = \frac{1}{2}$. How to simplify further the stress flow is visualised in a horizontal plane in figure 2.5.

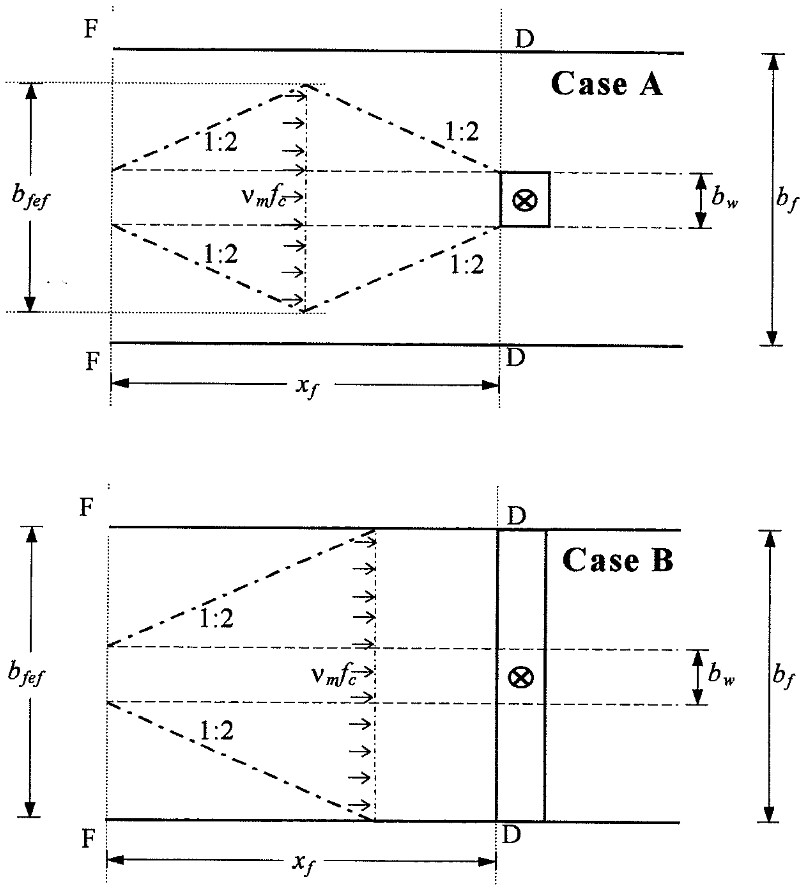


**Figure 2.5** *Assumed stress distribution in the flange.*

In case A, the external load is distributed across the web width $b_w$ (as indicated by the loading plate). The lines, within which we assume the stresses to be distributed, intersect at the distance $\frac{1}{2}x_f$. This cross section defines an effective flange width $b_{fef}$. We assume that the stresses at this cross section are equal to the effective compressive strength of concrete $\nu_m f_c$.

The effectiveness factor $\nu_m$ due to membrane action may be taken as, see [86.1] or [91.1],

$$\nu_m = \frac{2}{\sqrt{f_c}} \qquad (f_c \text{ in MPa}) \tag{2.10}$$

The stresses at the narrow zones must of course attain such values, that the resultants will be equal to $b_{fef}\nu_m f_c$.

In case B, the external load is distributed across the whole width of the flange. The corresponding approximation to the stress distribution is shown in the figure. For small values of $x_f$, the effective flange width may be less than $b_f$.

With the approximations introduced, we may now formulate the relation between the effective flange width $b_{fef}$ and the length $x_f$.

$$\left.\begin{array}{ll} b_{fef} = b_w + \tfrac{1}{2}x_f = b_w + \tfrac{1}{2}h\dfrac{a'}{h}\left(1-\dfrac{x}{a'}\left(1-\tfrac{t}{h}\right)\right) \text{ ,Case A} & \\ & b_{fef} \le b_f \\ b_{fef} = b_w + x_f = b_w + h\dfrac{a'}{h}\left(1-\dfrac{x}{a'}\left(1-\tfrac{t}{h}\right)\right) \text{ , Case B} & \end{array}\right\} \tag{2.11}$$

Now, by neglecting the tensile strength at failure, i.e. only the yield lines BF and CD are considered, we find the internal work in the flange to be

$$W_{I,\text{flange}} = 2\left(y_o \cdot b_{fef} \cdot \nu_m f_c \cdot \theta \cdot \tfrac{1}{2} y_o\right) \tag{2.12}$$

Inserting (2.3) into (2.12) we find

$$W_{I,\mathrm{flange}} = \frac{1}{4}\frac{t^2}{x_f}\left(x_f + \frac{1}{4}\frac{t^2}{x_f}\right)\cdot b_{fef}\cdot \nu_m f_c \cdot \theta \tag{2.13}$$

Finally, by inserting (2.1), (2.7), (2.8) and (2.13) into the work equation (2.6), the shear capacity, expressed by the reaction $V_u$ at failure, is found to be

$$V_u = \frac{A_{cw}\nu_0 f_c}{\frac{a'}{h}}\left(\frac{0.118}{\frac{x}{a'}} + \frac{0.25\xi\beta_{ef}\frac{t}{h}}{1-\frac{x}{a'}\left(1-\frac{t}{h}\right)}\right) \tag{2.14}$$

Here we have introduced the ratios $\xi$ and $\beta_{ef}$ defined as

$$\xi = \frac{\nu_m}{\nu_0} \tag{2.15}$$

$$\beta_{ef} = \frac{A_{cf,ef}}{A_{cw}} \tag{2.16}$$

$A_{cf,ef}$ is the effective flange area given as

$$A_{cf,ef} = t\cdot b_{fef} \tag{2.17}$$

Now, if we interpret the formula (2.11) as a condition that $b_{fef}$ must fulfill and not as a formula rendering $b_{fef}$, then we may for any assumed value of $a'$ determine a value of $x$ which minimizes the shear capacity given by (2.14).

By minimizing (2.14) with respect to $x/a'$, it turns out that the value of $x/a'$ rendering the minimum shear capacity is given as

$$\frac{x}{a'} = \frac{\sqrt{B^2+4A}-B}{2A} \tag{2.18}$$

The constants A and B are as follows

$$\left.\begin{aligned} A &= 2.119\,\xi\,\beta_{ef}\,\frac{t}{h}\left(1-\frac{t}{h}\right)-\left(1-\frac{t}{h}\right)^{2} \\ B &= 2\left(1-\frac{t}{h}\right) \end{aligned}\right\} \tag{2.19}$$

For any assumed value of $a'$ the shear capacity may be found by inserting (2.18) into (2.14). Any effective flange width $b_{fef}$ used in the calculation must fulfil the requirement given by (2.11). Since $x/a'$ depends upon $b_{ef}$, an iterative procedure is required.

The remaining problem is to determine the starting point of the critical diagonal crack, i.e. to determine the length $a'$. According to the crack sliding model, the position of the critical diagonal crack may be determined by equalizing the crack sliding capacity and the cracking load.

The cracking load may be found, as outlined in chapter 1, by assuming a constant distribution of the tensile stresses $f_{tef}$ along the crack. As in the case of a rectangular cross section, we will assume that the critical diagonal crack may run all the way to the top face, i.e. the horizontal projection of the crack is equal to the length $x$ shown in figure 2.3. It may be argued that this assumption is not safe and a bit illogical because visible cracks mainly are observed in the web and along the junction between the web and the flange. However, by assuming the crack to end at the top face, the determination of the cracking load may be related to a mechanism, which is consistent with the upper bound theorem. Furthermore, the contribution from the flange will not be significant due to the short lever arm.

By a rotation around the upper tip of the crack, see figure 2.6, the cracking load may be found from the following relation

$$V_{cr}\cdot(a-a'+x)=M_{cr}(x) \tag{2.20}$$

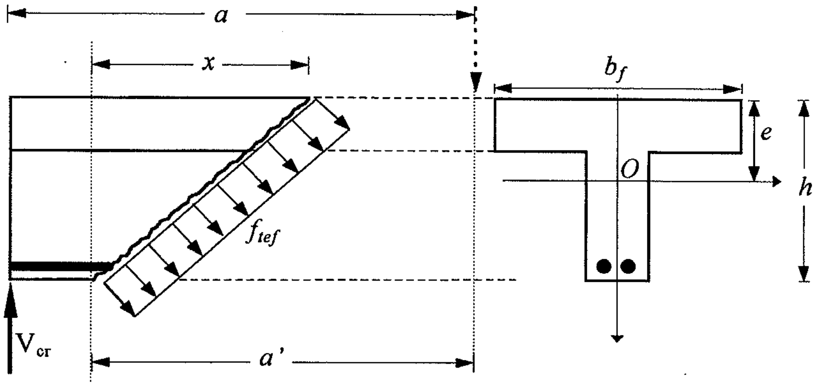


**Figure 2.6** *Stress distribution along the critical diagonal crack.*

$M_{cr}(x)$ is the cracking moment found by a moment equation around the upper tip of the crack for the stress distribution shown in figure 2.6. For any shapes of cross section $M_{cr}(x)$ may be determined by

$$M_{cr}(x) = f_{tef} \cdot A_c \cdot e \cdot \left( \left( \frac{x}{h} \right)^2 + 1 \right) \tag{2.21}$$

Here $A_c$ is the area of the cross section and $e$ is the distance from the top face to the centre of gravity of the concrete cross section. In the case of a T-section, $e$ is given as

$$e = \frac{1}{2} \frac{\left( b_w h^2 + \left( b_f - b_w \right) t^2 \right)}{A_c} \tag{2.22}$$

Now, inserting (2.21) and (2.22) into (2.20), the cracking load is found to be

$$V_{cr} = \frac{f_{tef} A_c \frac{e}{h}\left(\left(\frac{a'}{h}\right)^2 \left(\frac{x}{a'}\right)^2 + 1\right)}{\frac{a}{h} + \frac{a'}{h}\left(\frac{x}{a'} - 1\right)} \tag{2.23}$$

The ratio $x/a'$ is taken from (2.18).

Notice that the whole width of the flange is used to calculate $V_{cr}$.

Introducing the quantities $A_{cf}$ and $\beta$ given by

$$\left.\begin{aligned} A_{cf} &= t \cdot b_f \\ \beta &= \frac{A_{cf}}{A_{cw}} \end{aligned}\right\} \tag{2.24}$$

we may rewrite (2.23) as

$$V_{cr} = \frac{f_{tef} A_{cw}(1+\beta) \frac{e}{h}\left(\left(\frac{a'}{h}\right)^2 \left(\frac{x}{a'}\right)^2 + 1\right)}{\frac{a}{h} + \frac{a'}{h}\left(\frac{x}{a'} - 1\right)} \tag{2.25}$$

By equalizing (2.14) and (2.25) we arrive at the following equation rendering the starting point of the critical diagonal crack determined by $a'/h$

$$\frac{\nu_0 f_c}{f_{tef}}\left(\frac{0.118}{\frac{x}{a'}} + \frac{0.25\xi\beta_{ef}\frac{t}{h}}{1 - \frac{x}{a'}\left(1 - \frac{t}{h}\right)}\right) = \frac{(\beta+1)\frac{e}{h}\left(\left(\frac{a'}{h}\right)^3 \left(\frac{x}{a'}\right)^2 + \frac{a'}{h}\right)}{\left(\frac{a}{h} + \frac{a'}{h}\left(\frac{x}{a'} - 1\right)\right)} \tag{2.26}$$

The equation is solved iteratively. First a value of $b_{fef}$ is guessed, then (2.26) is solved with respect to $a'/h$ which is the only unknown quantity. Hereafter the solution together with (2.18) is inserted into (2.11). If the guessed value of $b_{fef}$ corresponds to the one calculated by (2.11), then the solution is valid. Otherwise another value of $b_{fef}$ must be chosen. When the iteration has succeeded the shear capacity may be found by inserting the solution $a'/h$ into (2.14).

It turns out that convergence is obtained very fast when the calculated value of $b_{fef}$ is used as the next guess.

When the correct solution to (2.26) is found, the shear capacity may of course be rewritten to an average shear stress as follows

$$\tau_u = \frac{V_u}{b_w h} = \frac{h-t}{h} \frac{\nu_0 f_c}{\frac{a'}{h}} \left( \frac{0.118}{\frac{x}{a'}} + \frac{0.25 \xi \beta_{ef} \frac{t}{h}}{1 - \frac{x}{a'}\left(1 - \frac{t}{h}\right)} \right) \quad (2.27)$$

## 2.2 Short shear spans

For beams with short shear spans, i.e. $a/h$ less than about 2, it may happen that the equation (2.26) only has solutions corresponding to $a'/h > a/h$. In such cases, $a'$ should be put equal to $a$.

Due to the fact that cracks are formed with finite distances, corresponding to the cracking distance, it may happen that a crack originating from the support can not be formed. This is indeed the case if the distance between an existing (not critical) crack and the support is less than the cracking distance. In this case, failure may take place along the existing crack or along a yield line formed in uncracked concrete and originating from the support. In both case, the shear capacity will be higher than the one found by solving (2.26) and inserting the result into (2.27).

The influence of finite crack distances is described in detail by Jin-Ping Zhang in [94.1].

The shear capacity corresponding to sliding in a yield line through uncracked concrete may be found by multiplying the web contribution $\dot{W}_{I,web}$

by a factor 2 and by setting $a'/h = a/h$. This solution, which is denoted as $\tau_o$, may be expressed as follows

$$\tau_o = \frac{h-t}{h}\frac{\nu_0 f_c}{\frac{a}{h}}\left(\frac{0.236}{\frac{x}{a}} + \frac{0.25\xi\beta_{ef}\frac{t}{h}}{1-\frac{x}{a}\left(1-\frac{t}{h}\right)}\right) \tag{2.28}$$

Here $x/a$ is given as

$$\frac{x}{a} = \frac{\sqrt{B^2+4A}-B}{2A} \tag{2.29}$$

The constants A and B are as follows

$$\left.\begin{aligned} A &= 4.238\xi\beta_{ef}\frac{t}{h}\left(1-\frac{t}{h}\right)-\left(1-\frac{t}{h}\right)^2 \\ B &= 2\left(1-\frac{t}{h}\right) \end{aligned}\right\} \tag{2.30}$$

Of course the effective flange width $b_{fef}$ must also fulfil the requirement (2.11).

# Chapter 3

# Comparison with Test Results

The shear capacity for a number of beams has been determined by the procedure outlined and the results will now be compared with test results found in the references [53.1], [57.1] and [69.1].

The material collected consists of 40 test results, see appendix A. The beams were simply supported and subjected to four point bending. All the tests included in this paper were reported to fail in shear without any yielding in the longitudinal reinforcement.

Only the tests done by Swamy [69.1] had a loading plate allowing the load to be distributed across the whole flange width. For this particular test series, the guessed effective flange width was compared with case B of formula (2.11). The compressive strength of concrete was about 30 MPa and the reinforcement ratio ($\rho = A_s/hb_w$) was kept constant and equal 2.54 %. The shear span ratio *a/h* varied from 0.96 to 5.69.

In figure 3.1 the test results are compared with the calculated shear capacities. The dark line represents equation (2.27) corresponding to sliding in cracks. The dotted line represents the shear capacity corresponding to the case with yield lines formed in uncracked concrete.

The agreement between theory and test results appears to be very good in the case of $a/h > 2$.

For the short shear spans, the test results are restricted since only two tests were done. The position of these test results comply well with the discussion in section 2.2.

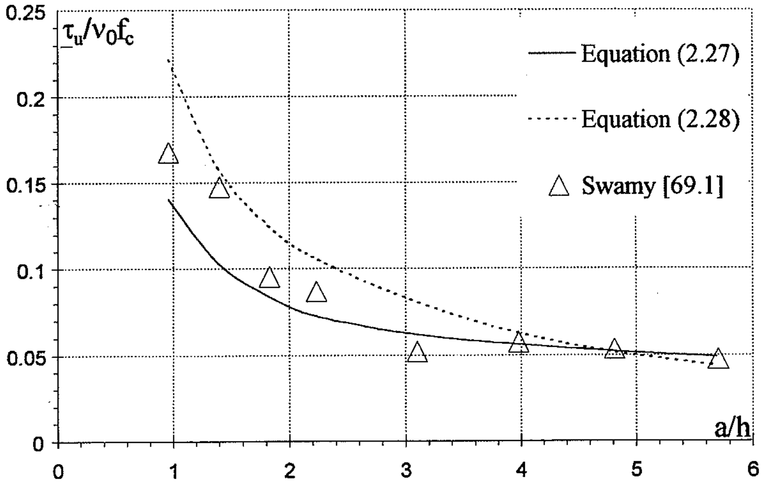


**Figure 3.1** *Comparison of calculations and tests by Swamy* [69.1].

Due to differences in the concrete strengths, in the reinforcement ratios and in the cross section dimensions, it is not possible to make a comparison similar to the one in figure 3.1 for the test series reported by Ferguson et al. [53.1] and Al-Alusi [57.1], respectively.

The comparison is done in figure 3.2 where the test results are depicted versus the shear capacity corresponding to equation (2.27). Also the tests done by Swamy [69.1] have been included.

The agreement appears to be good. The mean value of the ratio $\tau_{u,test}/\tau_{u,theory}$ is 1.03 and the standard deviation is 0.22. For a few test results, significant deviations are found (on the safe side though). Since these particular tests stem from beams with short shear spans, the deviations are probably due to the effect of the finite crack distances, see the discussion in section 2.2.

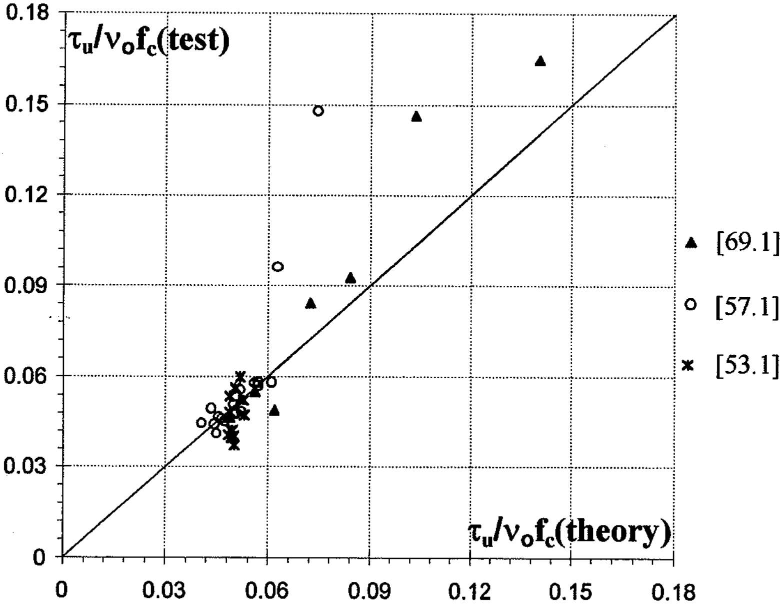


**Figure 3.2** *Comparison of shear capacity found by equation (2.27) and test results.*

By using an effective flange width $b_{fef}$ in the model, the actual flange width is of only little interest, especially for higher value of $b_f/b_w$. The main influence of the flange on the shear capacity is, according to the model, the thickness *t*.

The shear resistance contributed by the flange is equal to the last term of equation (2.27). Thus, when the valid values of $b_{fef}$ and a'/h are found, the contribution by the flange may be written as

$$\text{Contribution by the flange} = \frac{\dfrac{0.25\xi\beta_{ef}\dfrac{t}{h}}{1-\dfrac{x}{a'}\left(1-\dfrac{t}{h}\right)}}{\dfrac{0.118}{\dfrac{x}{a'}}+\dfrac{0.25\xi\beta_{ef}\dfrac{t}{h}}{1-\dfrac{x}{a'}\left(1-\dfrac{t}{h}\right)}} \tag{3.1}$$

In figure 3.3, the contribution to the shear capacity by the flange is shown for the 40 cases which have been calculated. It appears that the contribution of the flange approximately varies linearly with the ratio $t/h$, even though the expression (3.1) by no means is proportional to $t/h$. The point corresponding to $t/h$ =0.33 deviates a little from the straight line. The explanation is that for this particular test series, the external load was distributed across the whole flange width.

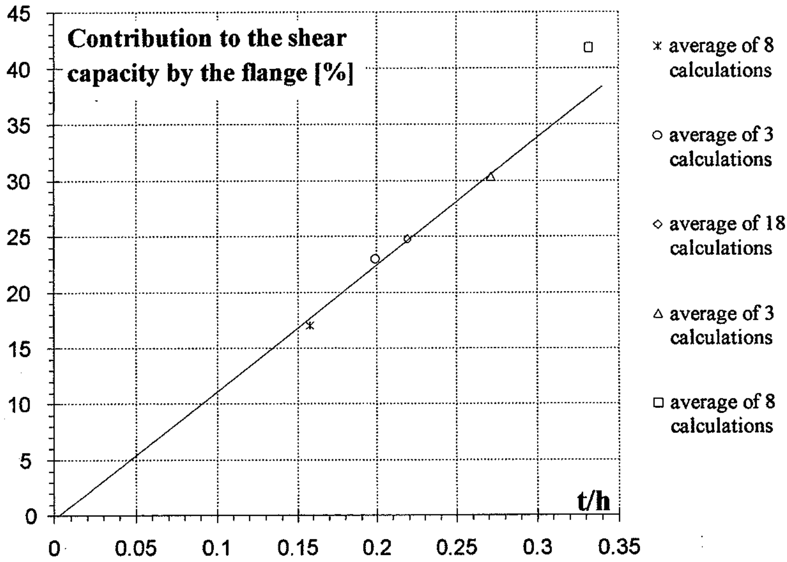


**Figure 3.3** *Contribution to the shear capacity by the flange.*

For the calculations corresponding to the tests in [69.1] and [57.1], the effective flange widths $b_{fef}$ which satisfy the condition (2.11) are shown in figure 3.4. It appears that $b_{fef}$ increases with increasing shear span.

Further, $b_{fef}$ increases when the external load is distributed across the whole flange width. In this case, we see from the figure that $b_{fef}/b_w$ is limited to 2.51. This is due to the fact that $b_{fef}$ can not exceed the true flange width, which in this particular case is equal to $2.51b_w$.

It is interesting to notice how little the flange is utilized when the external load is distributed only across the web width.

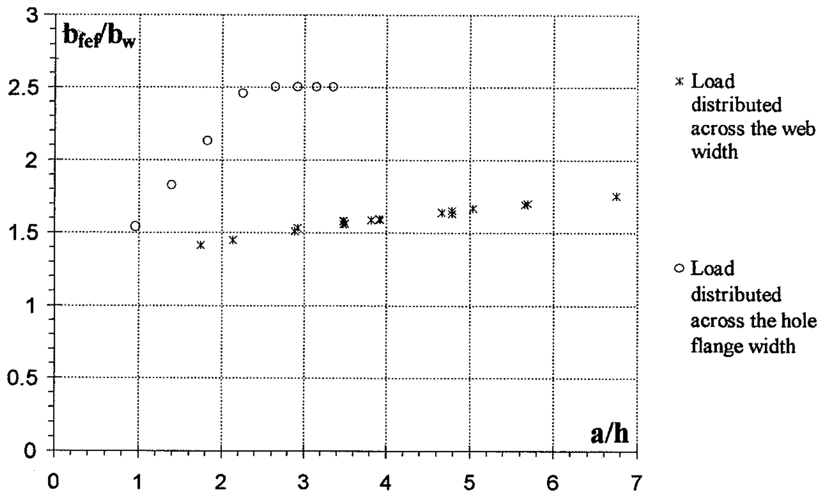


**Figure 3.4** *Ratio of effective flange width over web width versus a/h.*

# Chapter 4

# Simplified Calculation

It appears from the preceding chapters that it is a tedious task to determine the shear capacity of T-beams by the method outlined.

The investigations have shown that the effect of the compression flange is mainly governed by the flange thickness $t$ whereas the entire flange width $b_f$, according to the model, does not have any influence at all if it is larger than the effective flange width $b_{fef}$.

One may therefore expect, that the shear capacity of a T-beam in a simple way may be related to the shear capacity of a similar beam with rectangular cross section. This indeed turns out to be true.

If we disregard the flange and consider the cross section as being rectangular with the dimension $hb_w$, then the crack sliding model, as it is outlined in chapter 2, may be applied directly. The shear capacity obtained in this way may conveniently be denoted as $\tau_{u,R}$.

Similarly we denote the shear capacity obtained by taking into account the flange as $\tau_{u,T}$.

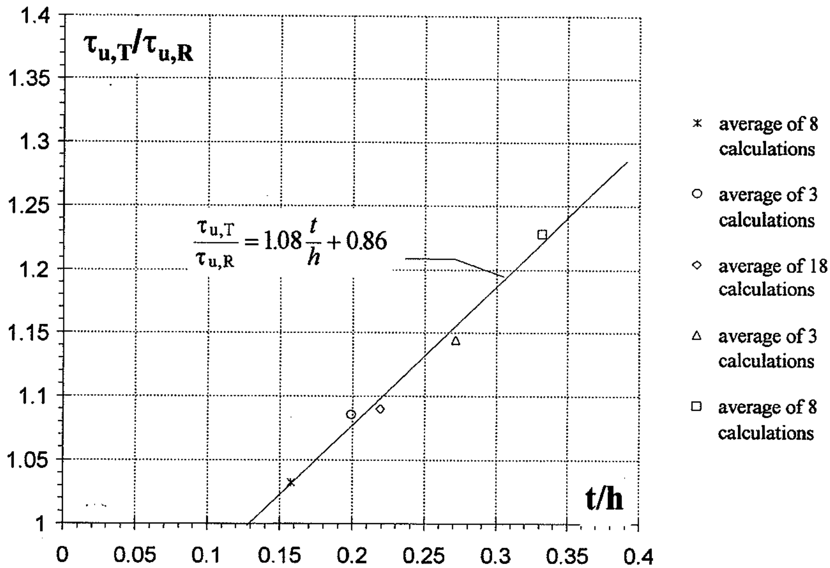


**Figure 4.1** *Ratio* $\tau_{u,T}/\tau_{u,R}$ *versus t/h.*

By depicting the ratio $\tau_{u,T}/\tau_{u,R}$ versus *t/h*, an approximately linear dependence is found, see figure 4.1.

By extrapolating the linear dependence, we find the interesting result that the flange does not increase the shear capacity if *t/h* is less than about 0.13.

The consequence of the relation shown in figure 4.1 is, that the shear capacity of T-beams may be determined by the method used for rectangular sections. The influence of the flange may then be taken into account by multiplying the solution with a factor which only depends upon the ratio *t/h*.

Schematically the analysis of T-beams therefore may be performed simply as follows:

♦ **Simplified calculation of the shear capacity of T-beams**

1) First, the following equation is solved with respect to *x/h*, see also (1.8),

$$\left(\frac{x}{h}\right)^3 + \frac{x}{h} - 4\frac{\tau_c}{f_{tef}}\frac{a}{h} = 0 \tag{4.1}$$

2) Then, the shear capacity $\tau_u = V_u/hb_w$, is determined as

$$\tau_u = \left(2\frac{\tau_c}{\frac{x}{h}}\right)\cdot K \tag{4.2}$$

The factor K, taking into account the effect of the flange, may be determined by the formula, see also figure 4.1,

$$K = 1.08\frac{t}{h} + 0.86 \qquad , K \geq 1 \tag{4.3}$$

## 4.1 Comparison of the simplified method with test results

The shear capacity obtained by solving the simplified equations (4.1) and (4.2) is compared with test results in figure 4.2.

The mean value of the ratio $\tau_{u,test}/\tau_{u,theory}$ is 1.03 and the standard deviation is 0.22. These results are, not surprising, the same as those obtained in chapter 3 by using the more tedious method.

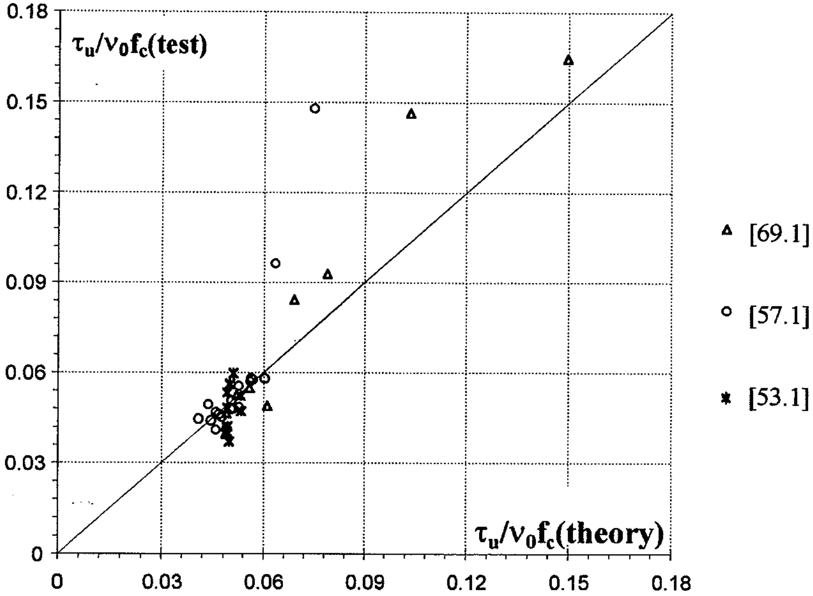


**Figure 4.2** *Comparison of the simplified calculation with test results.*

# Chapter 5
# Conclusion

In this paper an investigation concerning the shear capacity of T-beams without shear reinforcement has been carried out. The failure mechanism considered involves crack sliding in the web and rotation in hinges in the flange. Only beams without longitudinal reinforcement in the flange have been treated. For these beams the effect of the flange is mainly governed by the flange thickness.

The calculations were compared with 3 test series reported in the literature and the agreement was found to be good.

It has been shown, that the shear capacity of a T-beam may be found by multiplying a factor K on the shear capacity of a similar beam with a rectangular cross section $hb_w$. This factor, which only depends upon the ratio $t/h$, takes into account the effect of the flange.

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# Appendix

**Tests by Swamy [69.1]**

| No. | a/h | $b_w$ | $b_f$ | t | h | ρ (%) | $f_c$ | $\nu_0$ | $\tau_{u,test}/\nu_0 f_c$ |
|---|---|---|---|---|---|---|---|---|---|
| | | mm | mm | mm | m | $A_s/b_wh$ | MPa | | |
| TD 1.5 | 0.96 | 152 | 381 | 76 | 229 | 2.54 | 30 | 0.820 | 0.165 |
| TD 2 | 1.39 | 152 | 381 | 76 | 229 | 2.54 | 30 | 0.820 | 0.146 |
| TB10 | 1.82 | 152 | 381 | 76 | 229 | 2.54 | 30 | 0.820 | 0.093 |
| TD 3 | 2.25 | 152 | 381 | 76 | 229 | 2.54 | 30 | 0.820 | 0.084 |
| TD 4 | 3.11 | 152 | 381 | 76 | 229 | 2.54 | 30 | 0.820 | 0.049 |
| TD 5 | 3.97 | 152 | 381 | 76 | 229 | 2.54 | 30 | 0.820 | 0.055 |
| TD 6 | 4.83 | 152 | 381 | 76 | 229 | 2.54 | 30 | 0.820 | 0.052 |
| TD 7 | 5.69 | 152 | 381 | 76 | 229 | 2.54 | 30 | 0.820 | 0.046 |

**Tests by Al-Alusi [57.1]**

| No. | a/h | $b_w$ | $b_f$ | t | h | $\rho$ (%) | $f_c$ | $\nu_0$ | $\tau_{u,test}/\nu_0 f_c$ |
|---|---|---|---|---|---|---|---|---|---|
| | | mm | mm | mm | mm | $A_s/b_wh$ | MPa | | |
| 6 | 1.739 | 76 | 330 | 32 | 146 | 1.288 | 27.132 | 0.812 | 0.148 |
| 12 | 2.130 | 76 | 330 | 32 | 146 | 2.288 | 25.167 | 1.000 | 0.096 |
| 11 | 2.887 | 76 | 330 | 32 | 146 | 2.288 | 28.614 | 0.944 | 0.058 |
| 2 | 2.922 | 76 | 330 | 32 | 146 | 1.279 | 27.876 | 0.800 | 0.058 |
| 3 | 3.478 | 76 | 330 | 32 | 146 | 1.296 | 27.201 | 0.812 | 0.058 |
| 10 | 3.478 | 76 | 330 | 32 | 146 | 2.349 | 28.614 | 0.953 | 0.048 |
| 4 | 3.487 | 76 | 330 | 32 | 146 | 1.305 | 26.546 | 0.823 | 0.057 |
| 13 | 3.496 | 76 | 330 | 32 | 146 | 2.366 | 28.718 | 0.954 | 0.056 |
| 18 | 3.817 | 76 | 330 | 32 | 146 | 2.306 | 26.891 | 0.976 | 0.048 |
| 7 | 3.913 | 76 | 330 | 32 | 146 | 2.358 | 25.443 | 1.000 | 0.048 |
| 24 | 3.922 | 76 | 330 | 32 | 146 | 2.358 | 28.476 | 0.957 | 0.051 |
| 17 | 4.661 | 76 | 330 | 32 | 146 | 2.306 | 29.580 | 0.931 | 0.045 |
| 8 | 4.783 | 76 | 330 | 32 | 146 | 2.358 | 26.270 | 0.996 | 0.046 |
| 19 | 4.783 | 76 | 330 | 32 | 146 | 3.663 | 30.614 | 1.000 | 0.041 |
| 25 | 5.035 | 76 | 330 | 32 | 146 | 2.480 | 25.994 | 1.000 | 0.047 |
| 9 | 5.652 | 76 | 330 | 32 | 146 | 2.358 | 31.717 | 0.907 | 0.044 |
| 20 | 5.687 | 76 | 330 | 32 | 146 | 3.663 | 27.270 | 1.000 | 0.049 |
| 23 | 6.748 | 76 | 330 | 32 | 146 | 3.828 | 28.201 | 1.000 | 0.045 |

**Tests by Ferguson et al [53.1]**

| No. | a/h | $b_w$ | $b_f$ | t | h | $\rho$ (%) | $f_c$ | $\nu_0$ | $\tau_{u,test}/\nu_0 f_c$ |
|---|---|---|---|---|---|---|---|---|---|
| | | mm | mm | mm | mm | $A_s/b_w h$ | MPa | | |
| A1 | 2.95 | 102 | 432 | 38 | 241 | 4.15 | 29.717 | 1.000 | 0.040 |
| A2 | 2.95 | 102 | 432 | 38 | 241 | 4.15 | 27.304 | 1.000 | 0.040 |
| A3 | 2.95 | 102 | 432 | 38 | 241 | 4.15 | 35.096 | 0.932 | 0.042 |
| A4 | 2.95 | 102 | 432 | 38 | 241 | 4.15 | 34.958 | 0.934 | 0.039 |
| A5 | 2.95 | 102 | 432 | 38 | 241 | 4.15 | 45.369 | 0.820 | 0.037 |
| A6 | 2.95 | 102 | 432 | 38 | 241 | 4.15 | 38.681 | 0.888 | 0.042 |
| D1 | 2.95 | 178 | 330 | 38 | 241 | 2.38 | 31.300 | 0.769 | 0.047 |
| D2 | 2.95 | 178 | 330 | 38 | 241 | 2.38 | 29.580 | 0.791 | 0.052 |
| N1 | 3.73 | 108 | 483 | 38 | 191 | 2.76 | 20.685 | 1.000 | 0.056 |
| N2 | 3.73 | 108 | 483 | 38 | 191 | 2.76 | 20.616 | 1.000 | 0.056 |
| N3 | 3.73 | 108 | 483 | 38 | 191 | 2.76 | 17.513 | 1.000 | 0.060 |
| G1 | 5.09 | 108 | 559 | 38 | 140 | 3.76 | 22.890 | 1.000 | 0.041 |
| G2 | 5.09 | 108 | 559 | 38 | 140 | 3.76 | 21.720 | 1.000 | 0.048 |
| G3 | 5.09 | 108 | 559 | 38 | 140 | 3.76 | 21.860 | 1.000 | 0.053 |

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