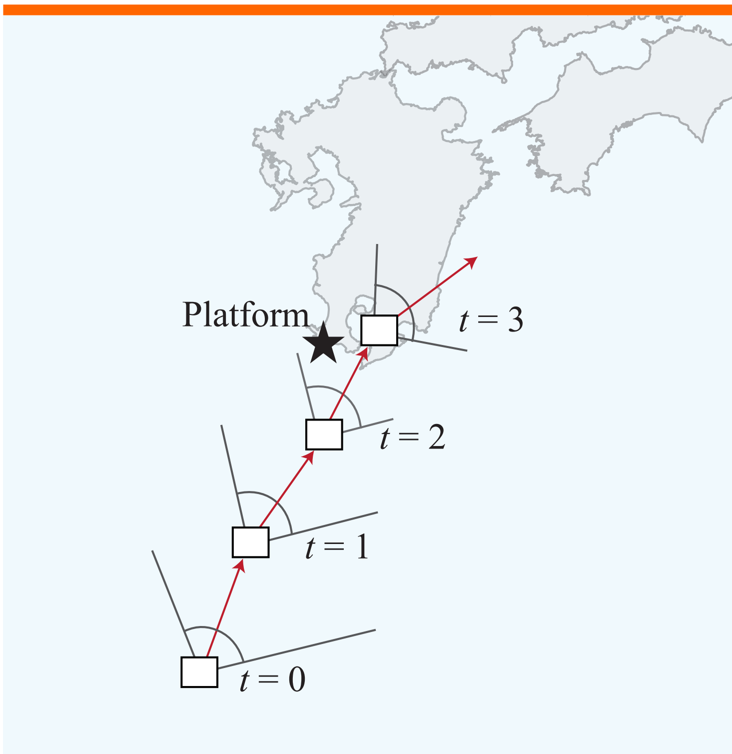
Real-time decision support in the face of emerging natural hazard events



Annett Anders

PhD Thesis

Department of Civil Engineering

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Preface

This thesis is a partial fulfillment of the requirements for the degree of Doctor of Philosophy at the Technical University of Denmark (DTU). The work was completed at DTU Byg, Department of Civil Engineering, Section of Structural Engineering. The principal supervisor of the Ph.D. project was Associate Professor Kazuyoshi Nishijima with co-supervisor Professor Michael Havbro Faber. Both supervisors were affiliated with DTU Byg at the time writing the thesis.

Originally this project was started at ETH Zurich, Switzerland, in July 2009. At ETH Zurich, the principal supervisor was Professor Michael Havbro Faber. Due to the change of affiliation of Professor Michael Havbro Faber and Associate Professor Kazuyoshi Nishijima from ETH Zurich to DTU, the Ph.D. program was transferred as well on March 1, 2011. Therefore part of the work was carried out at ETH Zurich.

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Lyngby, the 18th March 2014

Annett Anders

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I would like to thank my colleagues and fellow Ph.D. students of the research groups I have joint during my stay at ETH Zurich and DTU for the fantastic time at and outside the university over the past years.

Finally, I would like to thank my family and friends close to me and in particular Gerhard for the constant support and encouragement during the past years. This work is dedicated to them.

Abstract

Engineering structures are designed to resist a certain range of intensities of natural hazards. However, they are not designed to resist the entire range of possible intensities due to technical and economic constraints. Instead, in cases where they are most likely to fail as a result of emerging hazard events, several actions are undertaken to minimize possible consequences in real-time. For example, a dike is built to protect inhabitants and properties against flood events up to the intensity level that has a certain return period. In extreme rain storm events where dike failure is likely to occur, persons and movable property can be evacuated or temporary physical protections can be built. Such measures, when deemed prudent or necessary, are recommended or ordered by public authorities but also voluntarily undertaken by individuals. Other examples where private sector agents are in charge include engineering facilities such as wind turbines, agricultural facilities and offshore platforms. Operators of these facilities are often required to make decisions regarding the continued operations of their facilities in extreme storm events. These decisions, which in the present thesis are called real-time decisions, are often made by a small number of people in extremely stressful situations, ad-hoc relying on personal experiences of decision makers.

On the other hand, recent advancements of information technology potentially make it possible for decision makers to access various types of information in real-time. Remarkable examples that facilitate real-time decision making in emerging natural hazard events are weather observation systems at the global scale, observation data processing systems, provision of best estimates of current atmospheric states and weather forecasts. However, the information provided is in most cases limited to the estimate of the current intensity of the emerging hazard event and the forecast thereof, and includes, in very limited cases, the prediction of risks. Yet, none of the cases seem to systematically utilize such information for the decision optimization of the choice and commencement of risk reduction measures in real-time. Consequently, unnecessary costs and losses may occur. However, systematic use

of such information on a decision support system would not only alleviate the stress of decision makers but also facilitate the identification of optimal decisions, thereby avoiding unnecessary costs and losses. Motivated by these factors, the present thesis aims at developing a framework for the decision support system for real-time decision making in emerging natural hazard events. The thesis also demonstrates the implementation of the developed framework to illustrate its use and advantages.

The developed framework is based on the work by Nishijima et al. (2009). They formulate the general framework concept; however, it lacks an algorithm that solves the optimization problem with sufficient speed so that it can be utilized in practice. The difficulty lies in the sequential nature of the optimization problem, which requires backward induction. Respecting the analogy between the considered decision problem and the American option pricing, the present work proposes a very efficient algorithm on the basis of the Least Squares Monte Carlo method (LSM), which has been developed as an algorithm for pricing American options. The main contribution of the present work is the development of the efficient algorithm based on LSM, which is called enhanced LSM (eLSM). As shown in the examples the efficiency of the proposed algorithm is up to the order of 100 compared to other algorithms applied. Due to its efficiency it becomes possible to utilize decision support systems for a variety of real-time decision problems. Moreover, whereas the algorithm is developed primarily aiming at applications to the real-time decision making in emerging natural hazard events, the algorithm can be straightforwardly applied for other types of decision problems that share the same decision problem characteristics. These include decision problems in quality control and structural health monitoring.

Resumé

Konstruktioner er designet for naturkatastrofer op til en vis intensitet, men de er ikke designet til at modstå hele spektret af mulige intensiteter på grund af tekniske og økonomiske begrænsninger. I stedet bliver der iværksat foranstaltninger i realtid for at minimere mulige konsekvenser for de steder og situationer hvor konstruktionerne har størst sandsynlighed for at bryde sammen. For eksempel er diger bygget for at beskytte indbyggerne og ejendomme mod oversvømmelser op til en vis gentagelsesperiode. I ekstremt vejr med store mængder nedbør, hvor det er sandsynligt at digerne bliver oversvømmet, kan personer blive evakueret eller der kan bygges en midlertidig fysisk beskyttelse. I situationer hvor foranstaltninger synes nødvendige, bør disse foranstaltninger beordres af myndighederne, men i tilfælde hvor myndighederne ikke er ansvarlige, kan individer frivilligt organisere sig og foretage disse foranstaltninger. Andre eksempler hvor den private sektor er ansvarlige omfatter tekniske faciliteter, såsom som vindmøller, landbrugs-faciliteter og offshore platforme. Operatører af disse faciliteter er ofte forpligtet til at træffe beslutninger vedrørende den fortsatte operation af deres faciliteter i ekstreme storme. Disse beslutninger, som i nærværende afhandling kaldes realtid beslutninger er ofte lavet af et lille antal mennesker i ekstremt stressede situationer, og er baseret på personlig erfaringer af beslutningstagerne.

På den anden side, har de seneste fremskridt indenfor informationsteknologi gjort det potentielt muligt for beslutningstagere at få adgang til forskellige typer af information i realtid. Bemærkelsesværdige eksempler, der indeholder realtid beslutningstagen for naturkatastrofer er vejrobservationer på globalt plan, observation databehandlingssystemer, tilvejebringelsen af bedste skøn over de nuværende atmosfæriske tilstande og vejrudsigter. Imidlertid, er oplysninger i de fleste tilfælde begrænset til estimatet af det nuværende intensitet af den spirende fare begivenhed og prognosen deraf, og omfatter i meget begrænsede tilfælde, forudsigelse af risici. Ingen af tilfældene synes endnu systematisk at udnytte sådanne oplysninger for beslutningsoptimeringen af valg og påbegyndelse af foranstaltninger for risikoreducering i realtid. Følgelig kan unødvendige omkostninger og tab forekomme. Systema-

tisk anvendelse af sådanne beslutningsstøttesystemer vil ikke blot afhjælpe den stressede situation for beslutningstagerne, men også lette identifikationen af optimal beslutninger, og derved undgå unødvendige omkostninger og tab. Motiveret af disse faktorer, sigter denne afhandling på at udvikle en ramme for beslutningsstøtte i realtid i forbindelse med naturkatastrofer. Afhandlingen viser også implementeringen af den udviklede metode for at illustrere dens anvendelse og fordele.

Den udviklede metode er baseret på arbejdet af Nishijima et al. (2009). De formulerer det generelle koncept, men det mangler en algoritme det løser optimeringsproblemet med tilstrækkelig hastighed, så det kan udnyttes i praksis. Vanskeligheden ligger i den sekventielle natur af optimeringsproblemet, som kræver baglæns induktion. Observeret analogi mellem det beskrevne beslutningsproblem og amerikansk options prissætning, det nærværende arbejde foreslår en meget effektiv algoritme baseret på Least Squares Monte Carlo metoden (LSM), som er blevet udviklet som en algoritme til prisfastsættelse amerikanske optioner. Det væsentligste bidrag i det nærværende arbejde er udviklingen af en effektiv algoritme baseret på LSM, som kaldes forbedret LSM (eLSM). Som vist i eksemplerne er effektiviteten af den foreslåede algoritme op til størrelsesordenen 100. På grund af dens effektivitet bliver det muligt at udnytte beslutningsstøttesystemer til forskellige realtid beslutningsproblemer. Selvom algoritmen er udviklet primært til anvendelse af realtid beslutningstagning for naturkatastrofer, kan algoritmen umiddelbart benyttes for andre typer af beslutningsproblemer, der deler samme problemstillinger og karakteristikker. Disse omfatter beslutningsproblemer i kvalitetskontrol og strukturel monitorering.

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Chapter 1

Introduction

1.1 Background

In the face of an emerging natural hazard event, decision makers have to decide whether or not warnings and preparation orders are communicated to the affected people and if so which order is optimal. This decision problem is typically solved on the basis of two issues: what is the forecast intensity of the natural hazard event and what are the resistances of the affected engineering structures relative to the intensity of the natural hazard event. The present thesis investigates an algorithm to find the optimal decision in such situations in real-time.

Before going into details on the procedure of how to find the optimal decision, some background information about natural hazards and their consequences is provided. A *hazard* is an extreme phenomenon that is a potential threat to human life and economy within a given time period and area (EM-DAT, 2013). When a hazard is caused by natural processes, it is called *natural hazard*. Natural processes include geophysical and hydrometeorological processes.[1] The first-mentioned processes are related to the solid earth like earthquakes, volcano eruption and dry mass movement (e.g. rockfall), whereas the latter are related to meteorological, hydrological and climatological processes like storm, flood, wet mass movement (e.g. snow avalanche), drought, extreme temperature and wild fire. In general, the intensity of a natural hazard event is high leading to large adverse consequences and the probability that it occurs is low.

[1]Natural hazards can also be caused by biological processes; e.g. disease epidemics and insect/animal plagues. Therein the same decision problems occur and the approach presented here can be applied. However, these are not considered in the present thesis.

In recent years, natural hazards have caused large numbers of casualties (injured people and fatalities) and high economic damage (e.g. cost of repair, reconstruction as well as interruption of operations and production). Hereafter, casualties and economic damage resulting from a natural hazard event are collectively referred to as consequences. Figure 1.1 illustrates the recent development of (a) the number of natural disasters[2] reported, (b) the number of reported people that were affected by natural disasters, and (c) the estimated damages resulting from natural disasters within the years 1975 - 2012. The data are provided by EM-DAT (2013).

EM-DAT (2013) defines a disaster as "a situation or event, which overwhelms local capacity, necessitating a request to national or international level for external assistance; an unforeseen and often sudden event that causes great damage, destruction and human suffering". Note that the data present only those natural hazards that led to a natural disaster. In fact, there are more situations in which natural processes are detected that are likely to evolve to a natural hazard and eventually to a disaster. Also in these situations, decision makers have to decide whether to prepare for an impact or not.

Before conclusions are made from the presented figures, a general comment to the data provided by EM-DAT (2013) is given in the following. First of all, note that the economic damage as it is provided by EM-DAT (2013) represents only the value of immediate damage when the disaster occurs; i.e. only direct damages are included such as directly reported casualties, repair costs and replacement costs and not (future) indirect damages such as cost of production interruption, unemployment, and market destabilization (EM-DAT, 2013). If indirect damages were also included, the data would show higher consequences. Secondly, the increasing trend presented in Figure 1.1 can be explained partly with several factors, which are elaborated in the following. From Figure 1.1(a) it is seen that the number of natural disasters increased until about the year 2002; thereafter it is slowly decreasing. For the increase of natural disaster occurrences, one reason, which is often cited, is the growth of population and economy, especially in risk prone locations. Furthermore, it is guessed that climatic change contributes and will contribute further to the increase of the number of occurrences and/or their intensity, see e.g. Huppert & Sparks (2006), UNISDR & WMO (2012), Rougier et al. (2013, Chapter 1), Smith (2013). However, the small decrease in occurrences may be within the range of random variability. Apparently, the growth of

[2]For a disaster to be listed in EM-DAT (2013), one or more of the following criteria must be fulfilled: (i) Ten (10) or more people reported killed; (ii) Hundred (100) or more people reported affected; (iii) Declaration of a state of emergency; or (iv) Call for international assistance.

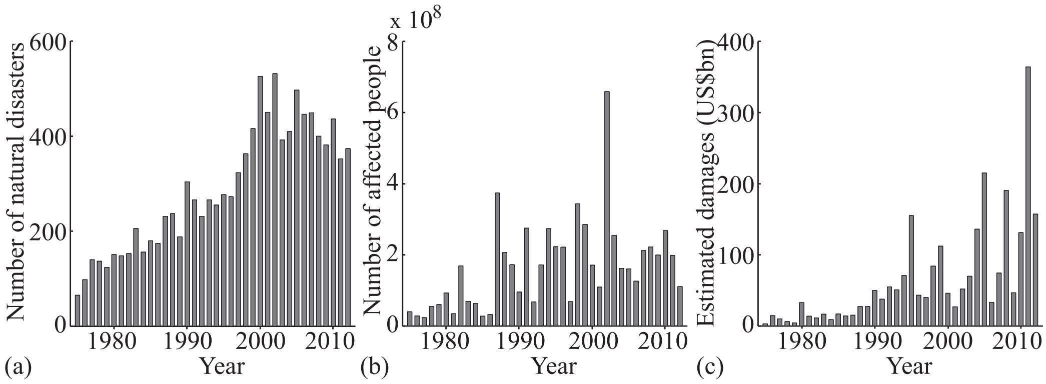

Figure 1.1: *Development of (a) the occurrence of natural disasters in the world over the last years, 1975 - 2012. The figures show (b) the number of affected people and (c) the estimated damages. The data are provided by EM-DAT (2013).*

population and economy can also be related to the increase of the total consequences (i.e. the number of affected people and estimated damage). Rougier et al. (2013, Chapter 1) state that according to the World Bank disaster impact assessment of the years 1960 - 2007 (Okuyama & Sahin, 2009), the total consequences increased but at the approximate rate the global gross domestic product increased. According to Smith (2013) if the consequences are normalized (i.e. the number of fatalities is adjusted to the number of people at risk and the economic damages to the price), then there is no evidence that economic damage increases. Another reason for the rise of total consequences may be the change in data recording methods (Smith, 2013). UNISDR & WMO (2012) identifies further factors that contribute to the consequences: "poorly planned and managed urbanization, environmental degradation, poverty and weak governance".

However, the conclusion from Figure 1.1 is that the consequences related to natural disasters worldwide are considerable and the overall aim should be to reduce these consequences. In order to achieve this aim, a significant effort has been devoted to risk assessment and risk reduction in the field of natural hazard risk management, see e.g. Zschau & Kuppers (2003), World Bank and United Nations (2010), ASCE (2011), Smith (2013), Rougier et al. (2013). Therein the focus lies on answering questions such as how to optimally mitigate risk, how to prepare for a hazard event, how to respond during the emergence of a hazard event and how to respond after the impact.

While the occurrence of natural hazard events cannot be prevented, the consequential risk can be mitigated prior to such events through, for example, adequate urban planning, risk adapted building codes and standards, or protective structures. All types of public structures and infrastructure (e.g. infrastructures for transportation, energy and communication) as well as private and industrial engineering structures (e.g. buildings, refineries and offshore platforms) are collectively referred to as engineering facilities hereafter. Conventionally, engineering facilities are designed and built for a specific service and withstand the majority of the possible load situations during their service time. However, they are not designed to resist the entire range of possible intensities of natural hazard events. This follows from economic and technical reasons and is also rational in a risk based decision framework.

The limited resistance implies that engineering facilities may fail and collapse in extreme events. Their failure often results in casualties and economic damage. In order to keep these consequences as low as possible, in case an engineering facility is likely to fail in an extreme event the option to commence risk reducing measures is considered in the overall strategy of risk management. Here, risk reducing measures include to shutdown operations as well as to evacuate people, livestock and property in the face of an emerging natural hazard. However, these measures are only relevant when the affected area can be warned in order to respond in time. Important examples in which such decisions are presently utilized are related to (i) the shutdown of refineries and fixed offshore platforms subject to tropical cyclones and storm surges, (ii) the closure of railways or roads subject to avalanches, and (iii) the evacuation of people from urban habitats and other engineering facilities subject to natural hazard events like storms, floods, landslides, tsunamis, wildfires, avalanches and volcanic eruptions.

The natural hazards of the aforementioned examples have in common that indicators can be observed prior to the impact, which facilitates the forecast of the severity of the consequences. Additionally, there is time to react once the emerging hazard is detected. If this holds, it can be expected that a significant amount of fatalities and economic damage can be prevented if the adequate risk reducing measure is commenced in time. As illustrated in Figure 1.2, the most frequently occurring natural hazards with these characteristics are flood, storm, mass movement (wet), extreme temperature, drought, wild fire, volcano eruption and tsunami. These events contribute to more than half of the fatalities and economic damage of the last 20 years. Considering the proportion of the consequences resulting from these natural hazards combined with their characteristics (forecast and reaction possibility), it is likely that enhancing the algorithm used to find the optimal decision, in such a way that it is sufficiently fast, will contribute to a reduction of the consequences. The present work focuses on developing such an efficient algorithm.

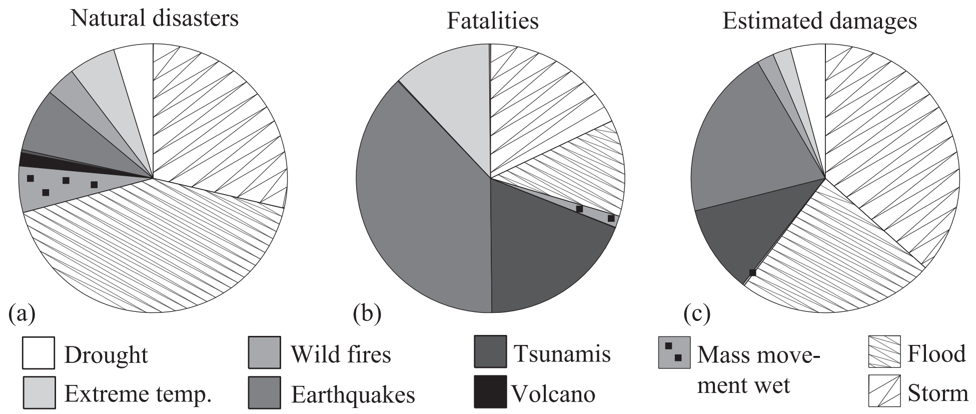

Figure 1.2: *Proportions of the main natural disasters in the world over the last 20 years, i.e. 1992 - 2012. The figures show the proportion of (a) the main natural disasters as well as the corresponding proportion of (b) fatalities and (c) estimated damage. The data are provided by EM-DAT (2013).*

As mentioned earlier, the decisions made in the face of emerging natural hazard events are not only based on the estimated resistance of the systems affected but also on the forecast of the intensity of the hazard. In order to forecast the future development of the emerging natural hazard event, models are required. For many natural hazards, statistical or physical models or a combination of both are available to describe the processes that characterize them. For instance, the track and the intensity of a tropical cyclone can basically be formulated as a function of the translation speed, the translation angle and the central pressure.

When the relevant information and models are available, the question is how should the information be utilized in order to find the optimal decision in the face of an emerging natural hazard event. The optimal decision is defined as the one that lies within the set of decision alternatives available and maximizes, using the information available, the utility function that reflects the preferences of the decision maker. Usually a small number of decision makers is responsible to make decisions on possible actions. The term *decision maker* refers to one or more decision makers; assuming that they have one common perspective and preferences. The decision makers perspective and preferences are represented through the utility function, which however does not necessarily reflect her own preferences. The optimal decision depends on the perspective of the decision maker: for example decision makers representing a societal perspective have different preferences from the ones of decision makers with an economic or private focus.

In this thesis, the utility is represented in terms of the consequences that result from the decision alternative(s) chosen and the state of nature. The set of decision alternatives comprises the decisions to commence possible risk reducing measures and the decision to wait in order to collect further information. The state of nature concerns the state of the considered facility that is possibly subject to an impact of a hazard event; for instance the state of the facility could be described as damaged, which leads to consequences, or not damaged, which leads to no consequences. Furthermore, the state of nature depends on natural phenomena that are related to the hazard development and also on characteristics of the affected area and engineering facility(s) that influence the resulting consequences. These characteristics include the location (e.g. in the mountains, at the coast line or in the ocean), accessibility and size (e.g. number of affected people).

In the case of emerging natural hazard events, local decision makers of the affected area obtain usually relevant information and support from experts and organizations like the Tropical Cyclone Warning Centers[3] or the Pacific Tsunami Warning Center[4] or other warning centers depending on the location and type of hazard. These organizations operate so-called *Early Warning Systems* (hereafter abbreviated by EWS). They have the task to detect, track and forecast emerging natural hazard events and if necessary disseminate information and warnings, see Section 2.3. EWS are widely installed and it is shown that they contribute to the reduction of natural hazard consequences when utilized correctly, see Zschau & Kuppers (2003), Rogers & Tsirkunov (2011), Golnaraghi (2012).

When precursors of an emerging hazard event are detected (e.g. through satellites or sensors) by an EWS, the information is processed in such a way that the information can be utilized for finding the optimal risk reducing measure. The information is processed with decision support systems (hereafter abbreviated by DSS). Typically, these systems return the estimated time at which the hazard impact is expected to occur, the estimated occurrence probability as well as its projected future intensity. However, it seems that none of the DSS provides information about the consequences related to the available decision alternatives. Knowing in addition what the expected consequence of a decision alternative is, facilitates finding the optimal decision as it can help to understand and compare the different consequences from the decision alternatives. The expected consequences would be interesting for public decision makers as well as for individual decision makers such as an operator of an offshore platform, because the consequences are

[3]See e.g. `http://www.nhc.noaa.gov/aboutrsmc.shtml`.

[4]See e.g. `http://ptwc.weather.gov/`.

typically largest when engineering facilities fail during operation. Whereas consequences can be still large if the operations are stopped unnecessarily, since their re-starts may require long time until they are back to normal operation; implying opportunity losses due to business interruption.

In order to be prepared to warn of an emerging natural hazard event, DSS are designed and installed for individual regions. The design may be based on the findings of a (probabilistic) risk assessment in which the relevant natural hazard events and the possible consequences are analyzed; further details are provided in Section 2.1. The difficulty to design a DSS for real-time decisions in the face of emerging natural hazard events results from the following three characteristics:

(a) *Complexity of natural hazards*
In order to compute the future consequences a model describing the development of the natural hazard is necessary. Modeling natural hazards is difficult as their development is not completely understood so far; i.e. forecasts based on physical equations do not accurately represent the real world. Moreover, they are often characterized by multiple continuous, time dependent processes with an infinite number of possible states. Therefore, probabilistic models are introduced to describe the development. Using these probabilistic models, the probability distribution of possible scenarios and their consequences can be estimated.

(b) *Limited time frame available for decision making*
As natural hazards develop continuously in time and often rapid, decisions should be made right after new information becomes available. Recent advances in information technology make it possible to access information in (near) real-time. This should be utilized in DSS to improve decision making in the face of natural hazard events. Hereafter, the decision problem described above is called *real-time decision problem.*

(c) *Sequential nature of decision problem*
In the real-time decision problem of consideration, decisions are made successively in response to new information. This leads to the typical problem of sequential optimizations, which is the exponential increase of combinations of the states of the underlying random processes and decision alternatives with increasing number of considered time steps and number of decision alternatives.

In practical applications, it is often the case that the optimization problem cannot be efficiently solved in real-time by using standard techniques such as Monte Carlo simulations, decision trees, influence diagrams or numerical methods. An optimization problem is solved efficiently by an algorithm if its computational time is "optimal" in the sense that the computational time, which is needed to obtain the optimal decision at a given level of accuracy, is minimal compared to the pace the hazard evolves. In order to meet the time constraints, common solution approaches are for example the approximation of the state space of the stochastic processes characterizing the natural hazard, the reduction of the number of time steps when a decision can be made and/or the reduction of the number of available decision alternatives. As the computational time matters, complex probabilistic forecast models cannot be used for modeling natural hazard events within a DSS as long as no efficient algorithm is available.

Approaches proposed in the literature, which solve the real-time decision problem in a sequential manner and include a probabilistic hazard model, are relatively rare. Examples are Considine et al. (2004), Regnier & Harr (2006) and Nishijima et al. (2009). The first two references do not focus on real-time decision making itself. They investigate the value of weather forecast for oil-companies and society, respectively. The third introduces the sequential decision framework as it is used in the present work; yet a crude approximation of the state space is used to estimate the expected consequences. In Anders & Nishijima (2011) it is shown that this approach is inefficient and may lead to suboptimal decisions. This shows the relevance to develop efficient algorithms that incorporate adequate probabilistic models.

1.2 Objective

Sequential decision problems have been investigated for a long time; see e.g. Bellman (1957), Raiffa & Schlaifer (1961), DeGroot (1970), Puterman (1994). Therein several approaches have been introduced to solve related optimization problems. Nevertheless, to the knowledge of the author, only the aforementioned examples investigate complex decision problems similar to the real-time decision problems introduced earlier.

In situations in which decision makers need to make decisions under time pressure, as it is in the face of emerging natural hazard events, it is important to be able to solve real-time decision problems efficiently. An efficient algorithm can contribute to the reduction of consequences by providing optimal decisions, which minimize the expected future consequences. The development of such an efficient algorithm is the objective of this thesis.

1.3 Focus

Natural hazards can be categorized into groups with characteristic attributes. Attributes that characterize natural hazards are:

- Frequency
- Types and magnitude of consequences
- Pace of development
- Size of affected area
- Availability of precursors

Natural hazard events are rare events that result in large adverse consequences. The frequency and the consequences of individual natural hazard events vary from location to location. Note that for example earthquakes happen every day all over the world especially near fault-zones, yet many of them are not considered as natural hazards since those with a small magnitude cause in general no severe damage. Furthermore, the more often a hazard occurs in an area the better are people prepared and the engineering facilities in the area are adapted to the associated risk. The adaptation reduces in general the consequences. However, in areas where a hazard occurs less frequently or has never been observed before, already medium intense hazards can cause high consequences.

Natural hazards evolve at different paces and have a different scale with respect to the sizes of the affected area. For instance, gravitational hazards like rock-falls, landslides and avalanches have a very short time period of sliding land once they are triggered, yet only affecting relatively small areas; whereas hazards like earthquakes or tsunamis occur also within a very short time or relatively short time after the triggering event, but the affected area is relatively large. Slowly emerging hazards like floods (caused by precipitation), bush fires, volcanic ash clouds and tropical storms may induce damages over considerably large areas. Hazards caused by climatic changes, like the sea level rise, may evolve over centuries and affect the entire Earth.

In regard to the availability of information on precursors of an emerging hazard event, there is a significant difference between the types of natural hazards. For example, earthquakes occur typically without precursors, while forecasts or several precursors are available before a tropical cyclone strikes. Other examples for which precursors may be identified are tsunamis. The triggering event may be an earthquake or a mass movement, which can be detected, another natural precursor may be the drawback of the water below the low-tide level.

The possibility to collect information on precursors has the advantage that in combination with models the impact can be estimated. In cases the natural hazard event evolves also relatively slowly, then there is some time to react on the precursors. This is in general not possible for natural hazards that evolve fast and have no precursors (e.g. earthquakes). For the fast evolving gravitational hazards mentioned above, there may be precursors available as long as the regions concerned are under close observation. In this case processes that cause gravitational hazards, such as precipitation, can be monitored and used to forecast the triggering event and intensity. However, once an event is triggered, there is little or no time to react; therefore precautious risk reduction measures have to be undertaken prior to the triggering of these events.

Considering the aforementioned characteristics, the focus of this thesis lies on natural hazard events for which

- The time horizon is limited to a relatively short time period (hours or days).
- The impact and thus the consequences can be quantified.
- The natural hazard develops relatively slowly allowing for reactive decision making.
- Precursors are available to update the probability of an impact and its intensity.
- A probabilistic hazard model can be formulated to forecast future states.

Further, in order to support the subjective of this thesis, i.e. to develop an efficient algorithm for solving the considered real-time decision problems, the application of this algorithm is illustrated by means of examples. However, due to the time constraints of the Ph.D. project, the focus thereby lies on examples that assume various simplifications especially in the consequence model. For instance, throughout the thesis only cost optimization is considered. This implies that all consequences are modeled in monetary terms; i.e. the factors representing life safety or casualties are not considered. As the time horizons considered in this thesis are relatively short, discounting of the consequences is not included. In this thesis only random processes are investigated that are related to the natural processes which may lead to the considered natural hazard events. Other factors such as the time necessary to complete a risk reducing measure or the resistance of the considered structure are assumed to be deterministic; both can be included in the framework as additional random processes straightforwardly. The examples and the proposed algorithm focus on sequential decision problems, although the framework introduced in Chapter 3 takes basis in the sequential/pre-posterior decisions analysis; the extension of the algorithm to the pre-posterior decision problems (i.e. including sequential Bayesian updating) is left for future research.

1.4 Approach

In order to achieve the objective, the framework introduced in Nishijima et al. (2009) is applied to formulate the real-time optimization problem. This framework takes basis in the sequential/pre-posterior decisions analysis (Raiffa & Schlaifer, 1961). Given the framework, an algorithm is developed to solve efficiently the real-time optimization problem. In order to find potential algorithms, which might be taken as a basis, the following two research hypothesis were formulated:

1. The considered real-time decision problem is similar to the American option pricing problem.
2. American option pricing algorithms can be adapted to the real-time decision problem such that real-time information can be used efficiently.

When approaching the question whether the second hypothesis can be accepted or not, several algorithms developed for American option pricing were tested. The most promising algorithm is found to be the Least Squares Monte Carlo method (herafter abbreviated by LSM method) proposed by Longstaff & Schwartz (2001). It is shown that the algorithm can be adapted to the present purpose and constitutes the basis of this thesis.

1.5 Outline

The interrelation of the chapters in the thesis is illustrated in Figure 1.2.

Chapter 2 provides the fundamentals and the state-of-the-art of the research fields relevant to the topic. It further addresses the thesis work in a relevant context. The relevant research fields are probabilistic risk assessment, decision theory and early warning systems.

Chapter 3 first characterizes the decision problem in consideration and then introduces the mathematical terms used in the subsequent chapters. It includes the mathematical terms used in sequential/pre-posterior decision analysis, the definition of the real-time decision framework and the optimization problem.

Chapter 4 introduces the mathematical background as well as the idea of the proposed algorithm. This chapter presents the main part of the thesis. It gives a short introduction to American option pricing, which is relevant in order to understand the similarities as well as the differences of the decision problems. Thereafter, the adaptations of the LSM method to the real-time decision framework as well as an enhanced version of the LSM method are presented.

Chapter 5 presents a scheme of a decision support system (DSS) which illustrates the interrelations between the model components. It facilitates the application of the framework. After the general introduction of the structure of the DSS, three sections follow. Each section describes a different model component (module) of the DSS; these are the hazard module, the consequence module and the optimization module.

Chapter 6 presents two examples that illustrate how the DSS can be applied in practice. The computational advantage of the adapted algorithm compared to two traditional methods (a Monte Carlo method and a numerical integration method) is also presented. The findings from the examples motivate the use of the adapted algorithm for complex sequential decision problems.

Chapter 7 concludes the work with the discussion and the outlook of future work.

Three annexe are included which support the context of the thesis. *Annex A* illustrates step by step the implementation of the first example introduced in Chapter 6 by means of numerical results and Matlab code. *Annex B* presents two theoretical examples introduced in DeGroot (1970, Chapter 12). These examples show that the proposed algorithm is also applicable to engineering problems like quality control. *Annex C* provides two conference papers that were written during the Ph.D. project.

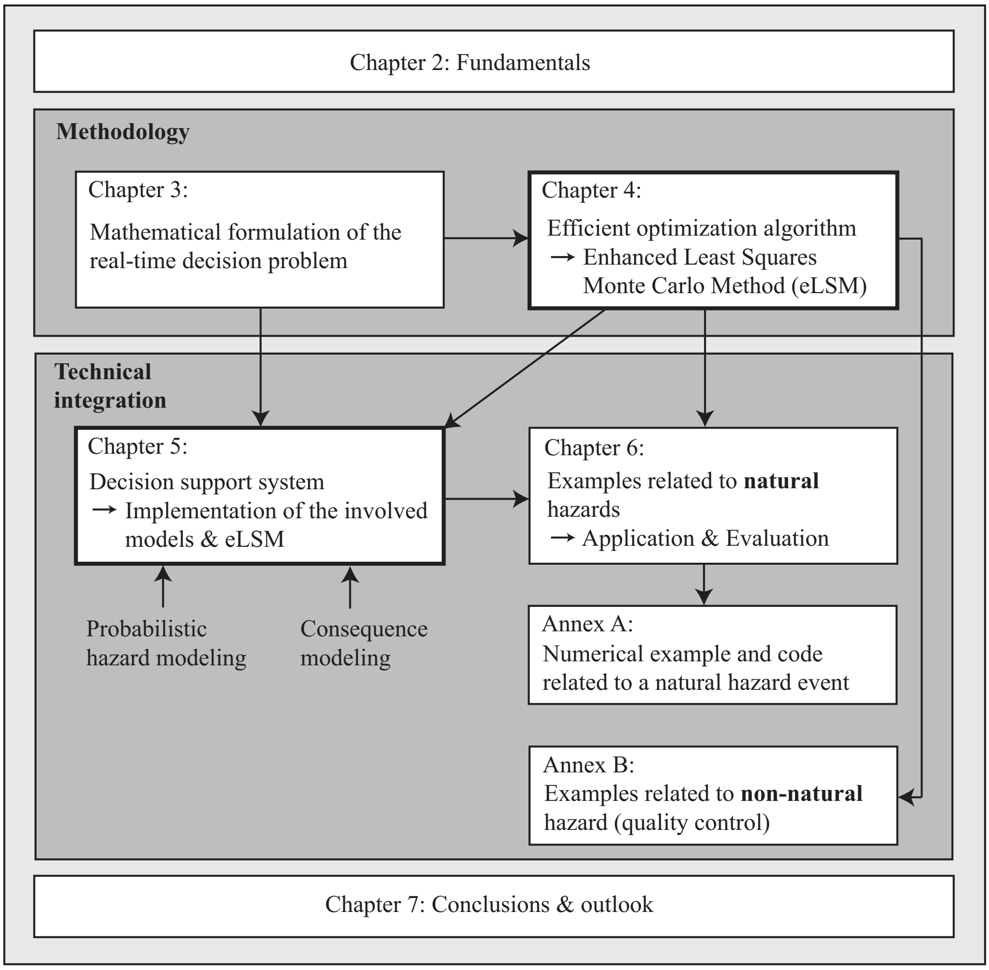

Figure 1.3: *Outline of the thesis. Thick framed boxes highlight the main contributions of the thesis.*

Chapter 2

Fundamentals for decision making within warning systems

The aim of this chapter is to provide the fundamentals and state-of-the-art of three research areas relevant to the thesis: probabilistic risk assessment (PRA); decision theory; early warning. Within the first section, the basics of probabilistic risk assessment are introduced, which is necessary to understand and model the impact of hazard events and its resulting consequences. This is important for making optimal decisions. The fundamentals to find the optimal decision is provided in the second section. The section introduces the fundamentals to formulate and solve the considered decision problem by applying the ideas of sequential/pre-posterior decision analysis. The third section outlines a potential field of application for the proposed framework; namely within early warning systems for finding an optimal decision on risk reducing actions in the face of an emerging natural hazard event.

2.1 Probabilistic risk assessment

The overall procedure to identify, analyze and evaluate potential failure scenarios and corresponding consequences is referred to as *risk assessment*. Risk assessment that includes the analysis of the uncertainties of effects leading to a failure is named *probabilistic risk assessment* (PRA).[1] Random effects that lead to a failure of engineering systems can be distinguished in internal and external hazard processes. Internal hazard processes are for example fa-

[1]In the literature, similar problems and approaches to those considered in probabilistic risk assessment are known under the terms *probabilistic risk analysis*, see e.g. Kaplan & Garrick (1981), or *quantitative risk analysis* and *probabilistic safety analysis*, see Bedford & Cooke (2001).

tigue and design errors. External hazard processes include natural hazards, explosions or collisions. A detailed introduction to PRA can be found, for example, in Stewart & Melchers (1997), Bedford & Cooke (2001) and Haimes (2004). In the following the basics are introduced.

2.1.1 Definition of risk

ISO (2009) defines risk as an *effect of uncertainty on objectives*. The cause of such effect is considered to be an event that occurs randomly and influences the performance of an engineering system. The effect can have either positive or negative consequences related to the objective.

In the present work, the main focus lies on assessing the risk related to engineering systems due to natural hazard events. A system is "an organized or connected group of objects" (OED, 2013) and examples of engineering systems are engineering facilities as well as services of entire infrastructure systems (e.g. health care, road network, public transportation system, energy provision). In general, engineering systems provide the built environment and related services for modern human society. If a random effect such as a natural hazard event occurs, the related loads can lead to the collapse of the complete or part of the engineering system or to the discontinuity of its use. The performance of the engineering system is not ensured thereafter, which in turn can lead to adverse consequences such as casualties and/or economic losses.

Since the effect E involves uncertainty, the probabilities p_i, $i = 1, 2, ...,$ of all possible states E_i of the effect need to be assessed for the quantification of the corresponding risk R.

Technical risk R is defined as the sum over the possible consequences multiplied by the corresponding probability of occurrence (Faber & Stewart, 2003); i.e. the expected consequences of the random effect E:

$$R = \sum_{i=1}^{\infty} p_i C(E_i) \tag{2.1}$$

where the function $C(E_i)$ denotes the consequence of E_i. This definition requires that all possible consequences as well as their occurrence probabilities are well defined and can be quantified.

It should be noted that dependent on the formulation of the decision problem and information available other tools representing the uncertainty of effects leading to failure may be relevant. Such tools are the conditional distribution (Haimes, 2004, Chapter 8), or the whole distribution function of the risk is relevant (Kaplan & Garrick, (1981) or Rougier et al. (2013), Chapter 2). Decision problems where e.g. the whole or part of the distribution function is of interest include the evaluation of scenarios that cause an insurance company to become insolvent. These scenarios are investigated in order to ensure that the insurance companies have enough capital to cover also extreme situations up to a certain threshold or probability as it is required by the Swiss Solvency Test or the Solvency II regulations.

2.1.2 Probabilistic risk assessment

According to Kaplan & Garrick (1981), for a PRA the following three questions need to be investigated:

1. What can happen? (i.e. What can go wrong?)
2. How likely is it that that will happen?
3. If it does happen, what are the consequences?

PRA is a widely used concept to quantify risk for decision making. Exemplary application fields are finance, insurance, medicine and engineering. In the field of natural hazards, Grossi & Kunreuther (2005), Smith (2013) or Rougier et al. (2013) provide a description of procedures and methods for PRA.

2.1.3 Uncertainties involved in probabilistic risk assessment

Even if it would be possible to predict[2] where and when a hazard event occurs (including its intensity), the answer to the three questions would not be straightforward. In order to answer these questions there are more uncertainties involved than those related to the prediction of the time and place of the hazard event. As a matter of fact in practice the answers to these questions involve significant uncertainties related to the forecast of the following variables:

1. The occurrence of the hazard process in time and space
2. The intensity of the hazard process if it occurs

[2]In this thesis, the following distinction between the terms *to predict* and *to forecast* is made: *to predict* is to state, on the basis of knowledge or reasoning, that an event will happen in the future, whereas *to forecast* is to estimate, conjecture or imaging what is likely to happen in the future.

3. The resistance of the considered system
4. The consequences

Furthermore, it is usually the case that the available information involves also uncertainties, e.g. the data obtained from tests or the information about the state of precursors is often uncertain due to measurement errors.

In order to incorporate the uncertainties, the related variables must be modeled in probabilistic manners. Therein the uncertainties should be differentiated according to their origin (see e.g. Faber (2005), Der Kiureghian & Ditlevsen (2009)). Three types are usually considered; these are the inherent natural variability, the model uncertainty and the statistical uncertainty. The former is often referred to as *aleatory uncertainty* and the latter two as *epistemic uncertainties*.

The distinction between aleatory and epistemic uncertainty is often not trivial. As Faber (2005) points out, aleatory uncertainty (or a mixture of aleatory and epistemic uncertainty) about a future state of a system transforms into pure epistemic uncertainty as soon as the future state is observed or realized. The difference is that before the state is realized, no information is available to reduce uncertainty, which changes as soon as the state is realized, since information can be collected (e.g. by testing). This implies that the type of uncertainty depends on the prevailing conditions. The difficulty of understanding this change of "perspective" is described in Spiegelhalter (2011). The author uses a simple example flipping a fair unbiased coin. Before flipping the coin, people are aware of the aleatory uncertainty and that the state of the coin is unpredictable; after flipping and in case the coin is still covered (i.e. the state of coin is realized but unknown), people hesitate about the uncertainty in the state, realizing that now the uncertainty is only related to their lack of knowledge. In Der Kiureghian & Ditlevsen (2009) it is noted that making this distinction facilitates the understanding of which uncertainty is reducible (e.g. by testing) and which is less likely to be reducible.

In practice, the type of uncertainty may be subjective and if considered in either way it makes no difference (Paté-Cornell, 2012). Where Paté-Cornell (2012) sees a practical problem is that two popular images, "perfect storms" and "black swans", representing aleatory and epistemic uncertainty are related to terms like "extreme unlikely" and "unthinkable". She points out that this relation implies severe consequences when these terms are used as ubiquitous justification (ex-post) for the failure to apply proactively risk reducing measures, although often precursors are available, but not considered, when assessing potential hazard events.

This thesis neglects, for simplification, the transition of aleatory uncertainty to epistemic uncertainty and considers only aleatory uncertainty; implying that only the reduction of the risk associated to aleatory uncertainty is investigated. Following the above reasoning, this should be understood as follows: the risk associated to aleatory uncertainty of a random variable is reduced by "waiting"; i.e. waiting until future states of the underlying random variables are realized and can be observed. It is assumed that in case the state of a random variable is realized, it can be observed without any uncertainty.

2.1.4 Hazard process assessment

Potential hazard events are typically assessed using historical data and probabilistic models; although historical events show that these tools are not sufficient to foresee all combinations of effects that have a positive probability. The accidents at the Fukushima reactors in 2011 are a recent example of a chain of effects that were not considered in the design with such intensities. Two interrelated natural hazards, an earthquake and a subsequent tsunami, resulted in the failure of several safety components which in turn led to radioactive release. As noted by Ramana (2011), the Fukushima accidents demonstrate the difficulty to understand and model common-cause or common-mode due to natural hazards. However, Paté-Cornell (2012) notes that in the case of the Fukushima reactors, the risk related to the conjunction of the two events with the resulting intensity could have been estimated based on the prevailing information and existing PRA methods. She points out that the applied evaluation techniques as well as the negligence of historical events[3] led to the severe underestimation of the risk. Although it was known that such scenarios are likely to occur in the area, it appears that the responsible decision makers of the reactors decided not to anticipate this information and retrofit the reactors, see also Acton & Hibbs (2012). Acton & Hibbs (2012) mention several causes why the owners did not follow international best practice and standards; (1) there is a focus on seismic safety in Japanese nuclear industry, leading to the exclusion of other possible hazards; (2) nuclear professionals may have failed to use the available knowledge; and (3) many believed that a severe accident was simply impossible. These points clearly show that it is not only difficult for engineers or scientists to assess

[3] According to Paté-Cornell (2012) at least two earthquake events resulting in a tsunami wave above the design criteria occurred. She lists one in the year 869 and in the year 1611. The information is taken from the National Oceanic and Atmospheric Administration (NOAA) available online at http://www.ngdc.noaa.gov/hazard.

unknown hazard processes, but also to communicate the risk associated to these low-probability events.

However, it can be assumed that as knowledge increases with appropriate research efforts and more experience, the proportion of failures resulting from unforeseen natural hazard events (i.e. unknown effects) will decrease over time (Stewart & Melchers, 1997, Chapter 2).

In order to assess the probabilistic characteristics of a hazard process related to an engineering system, two random factors have to be considered: how often does a hazard process occur and what intensity does it have, when it occurs. The probability of the occurrence and the intensity of hazard processes can be estimated using historical data and probabilistic models. The *Probabilistic Model Code* (JCSS, 2001, Part 2) provides for instance the basics on the probabilistic modeling of variable loads due to natural phenomena such as wind load and snow load. An adequate assessment of the hazard processes, which can be significant for the considered engineering system, form not only the basis for the related hazard modeling for the forecast but also for the related consequence assessment.

2.1.5 Consequence assessment

The assessment of consequences requires to understand possible failure states. In natural hazard events failures are caused by the additional (extreme) loads that exceed the resistance of the system. The load as well as the resistance include uncertainties and usually the number of possible combinations is large. For each combination, the state of the system can be assessed, in case the physical response is understood and the circumstances are known. If for instance an engineering facility such as an offshore platform collapses completely under certain conditions one knows the lost value; whereas, if it collapses only partially, the resulting consequences are in general subject to uncertainty (Kübler, 2006, Chapter 4). This uncertainty comes from the lack of knowledge about the partial collapse; often the details about the degree of damage and the affected components are unknown.

Consequences are often represented by the number of injured people and fatalities as well as economic damages, see e.g. EM-DAT (2013). However, consequences such as cultural and environmental damages, which are often not straightforward to assess, should also be considered, see Kübler (2006, Chapter 4), JCSS (2008). These consequences can be further differentiated between material and immaterial consequences. Assessing the immaterial consequences is difficult if not impossible. In risk assessment, a third distinction is made between direct and indirect consequences. Usually the focus lies on the assessment of direct consequences; whereas often indirect consequences are simply considered through risk-averse utility functions within the decision making (JCSS, 2008). This simplification may come at the cost of relevant information, especially in cases where indirect consequences are an important factor and the total consequences might be under or overestimated by this crude simplification (Faber & Maes, 2003). Apparently, the differentiation between the types of consequences is subjective and problematic. The assessment of immaterial and indirect consequences is in general extremely difficult, especially in the case of system failures with long term consequences that are hardly understood such as radioactive release.

As mentioned above, the total consequences can be distinguished, amongst others, between direct and indirect consequences. In the following this distinction is considered specifically.

Indirect consequences can result directly from the damage state of the constituent (e.g. cost of business interruption) but also from direct consequences (e.g. in case of radioactive release the death rate may increase later on due to higher cancer rate). These indirect consequences are collectively called *event imposed indirect consequences*. Other indirect consequences are caused by society and are a result of the perception of system changes; examples of such indirect consequences include the loss of reputation or credibility of responsible companies or decision makers. These *societal imposed indirect consequences* occur usually after extremely stressful situations where the decision maker is not fully informed and where a sub-optimal decision is preferred to no decision (Schubert et al., 2007).

In order to compute the risk related to the possible damage states of a system, the associated probability distribution needs to be known. This probability distribution is characterized by the conditional probabilities of the system being in the damage states given the intensity of the hazard event. The conditional probabilities can be determined by so-called fragility curves.

Shinozuka et al. (2000) provide the basic ideas on how to analyze statistically (empirically and analytically) fragility functions. For different hazard events different fragility functions apply, since the intensity of the hazard translates to a different type of load on the engineering system. Likewise for different types of constituents different fragility functions represent their resistance for the same hazard. Further approaches and examples for the assessment of fragility functions for seismic hazards can be found in Ravindra (1990), Straub & Kiureghian (2008), Bayraktarli (2009), Jaiswal et al. (2011) and a vast literature survey is provided in Rossetto et al. (2013); of wind related hazards see for example Ellingwood & Rosowsky (2004), Li & Ellingwood (2006), Smith & Caracoglia (2011), Mardfekri & Gardoni (2013) and of flood hazards see for example van de Lindt & Taggart (2009), van der Meer et al. (2009).

Recent work that consider the assessment of fragility curves as well as the modeling of consequences due to natural hazard events are summarized in the following. Bayraktarli (2009) investigates an example of seismic risk assessment for answering the question whether to retrofit a certain type of house or not. Therein the procedure of computing fragility functions as well as direct and indirect consequences are illustrated. Schubert (2009) proposes a generic risk model and considers especially the risk due to rock-fall. The thesis includes a detailed description on the modeling of direct and indirect consequences, which is illustrated by means of the risk related to rock-fall galleries. The modeling of loss of life in flood events is investigated by Jonkman (2007). Dutta et al. (2003) consider the modeling and estimation of economic loss due to floods.

Note that, an alternative to the usage of a fragility function is the use of a so-called *damage function*. The damage function relates the intensity of the hazard to the resulting damage state of the considered constituent in percentage, where 0% refers to no damage and 100% refers to the collapse of the constituent (see e.g. Grossi & Kunreuther (2005), Dutta et al. (2003)).

So far, only the damage state of the considered system is considered to be uncertain, but there are more factors that are relevant for the consequence assessment and include uncertainties. For example, in a tropical cyclone event the resulting consequences are highly related to the lead time (i.e. the time before the tropical cyclone strikes) and the time necessary in order to complete the commenced risk reducing measures (e.g. the time needed to evacuate people from the affected area) - both, the lead time as well as the completion time are uncertain. A part of the involved uncertainties arises from the unpredictable behavior of the affected people, which may lead, for example, to delays in the evacuation process. For details on the modeling of evacuations (considering the time period starting from the warning until the evacuation is completed) it is referred to Jonkman (2007). Other situations that are difficult to foresee are situations in which important back-up or rescue facilities and infrastructures may not be in place, because they were not installed before or failed in the hazard event. These potential situations make the estimation of the time necessary for completing the risk reducing measures difficult. Furthermore, the shorter the lead time, the higher are the costs for risk reducing measures because the time needed to complete the risk reducing measure may not be sufficient and additional emergency actions may be necessary. For deciding when it is best to commence an order, the following extremes need to be considered: for early orders costs may arise due to unnecessary risk reducing measures or business interruption; whereas for late orders more expansive actions may be necessary to actually perform and complete the risk reducing measures.

2.1.6 Failure probability assessment

The probability of failure of a system is a function of the resistance of the system and the intensity of the considered hazard process as a variable load. In practice, the decision problems considered in this thesis are usually so complex that there are no analytical solutions available to calculate the probability of a failure state accurately. Methods such as numerical integration and crude Monte Carlo methods are computational expensive. In order to obtain an estimate of the probability of failure in a reasonable time, many approximation methods have been introduced. These methods are included in the field of *structural reliability analysis*. Well-known methods are the first-order reliability method (FORM), the second-order reliability method (SORM) or simulation based methods (e.g. Monte Carlo methods using importance sampling and subset sampling). A broad overview of structural reliability methods is described for example in Melchers (2001), Ditlevsen & Madsen (2005) or Madsen et al. (1986). Note that in complex problems approximations can lead to large deviations from the true value and therefore to suboptimal decisions. Furthermore, the assessment of the failure probability involves similar difficulties as mentioned in the assessment of the hazard processes; namely how to assess the probability of an hazard process for which no historical data is available, instead it is only known that it did not occur so far.

2.1.7 Methods for probabilistic risk assessment

Well established methods in order to assess the probability of possible states of a system are event trees and fault trees, see e.g. Paté-Cornell (1984), Faber (2009). These methods allow to graphically represent possible scenarios. This facilitates the understanding of common-cause effects. Within these methods, the failure probabilities are estimated using the methods mentioned in Section 2.1.6. Given these probabilities the risk is estimated according to Equation (2.1). For a probabilistic risk assessment, methods such as Monte Carlo simulation (see e.g. Stroeve et al. (2009)), Bayesian modeling frameworks (see e.g. Graf et al. (2007), Bayraktarli (2009), Kelly & Smith (2009)) as well as various combinations have been introduced.

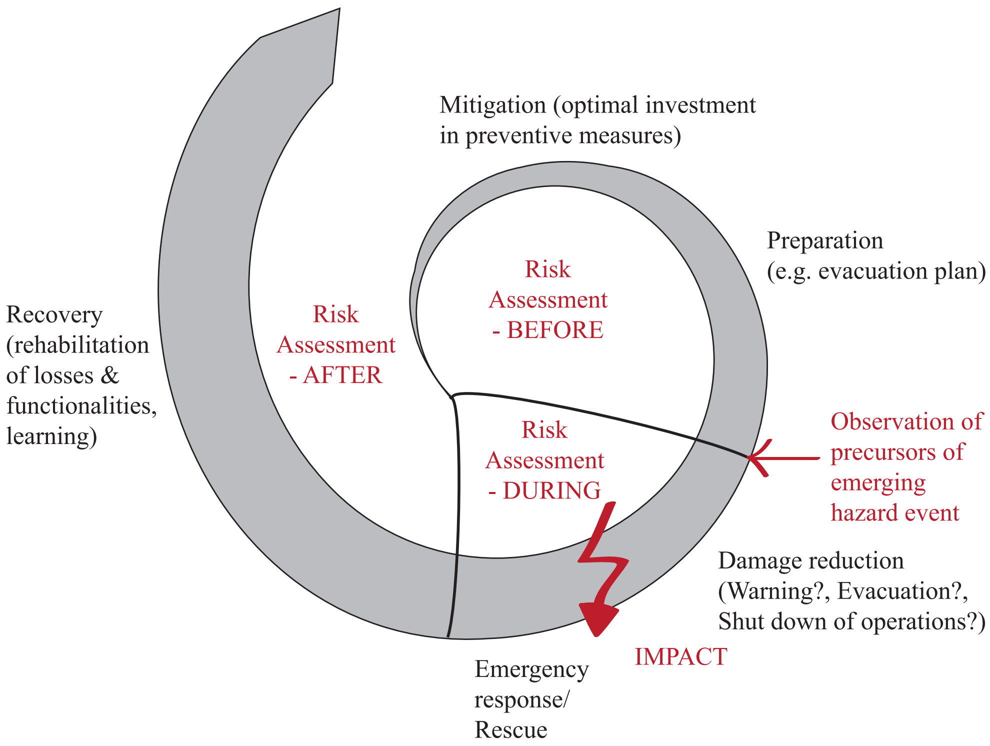

Figure 2.1: *Risk assessment spiral. PRA has different objectives in the three phases of hazard risk management.*

2.1.8 Probabilistic risk assessment in risk management

In risk management, PRA is commonly used to collect information regarding risk reducing measures. Based on this information, decisions are made on the design and the implementation of the optimal risk reducing measures. The objective of risk assessment changes relative to the occurrence of a hazard event; i.e. there is a phase before, during and after the hazard event as described in JCSS (2008). This follows from the differences in the boundary conditions and the decision alternatives available during the three phases. Figure 2.1 illustrates these phases and the related objectives.

In the first phase, before the hazard event, the objective is to obtain the optimal investment in preventive measures to mitigate the risk through adequate design (e.g. as given in the codes and standards), retrofit as well as installation of protective structures. Another objective is how to prepare for an occurrence with emergency strategies such as an evacuation plan.

The phase during the hazard event is defined as the time period from the point in time when an emerging hazard is detected until the emergency response is completed. In this time period the impact of the hazard event may occur. The focus is first (before the impact) on the limitation of consequences in the short-term by deciding whether to commence risk reducing measures such as warnings, evacuation orders or shut-down of operations. Then immediately after the impact the objective is to define and implement rescue, evacuation and first-aid strategies.

After the hazard event, the priority is set on the recovery of the affected infrastructures and/or facilities. This includes the rehabilitation of the damages and recovering functionality.

In Figure 2.1 the spiral form is chosen to illustrate the state of knowledge. It should increase through learning from the performance of the risk management related to the hazard event. The learning process is especially important, when the "spiral" comes back again to the state where decisions are made about risk mitigation and preparation for the next hazard event. This phase is somehow similar to the phase before the impact but with increased knowledge.

2.2 Decision theory

Decision theory provides the mathematical formulation of the procedure to find an optimal decision among two or more decision alternatives. In engineering, many decisions are made based on (probabilistic) risk assessment. Probabilistic concepts and decision theory provides the foundation to make optimal and consistent decisions for planning, designing, operating and managing engineering systems; see e.g. Benjamin & Cornell (1970), Ang & Tang (2007), Faber (2012). The objective is to maximize the overall life cycle benefit under constraints such as the fulfillment of legislative safety requirements and budget limits. However, in the face of an emerging natural hazard the objective is focused on minimizing the consequences.

2.2.1 Components of decision making

Three components are relevant in any decision problem:

1. The state of nature
2. The set of decision alternatives
3. The utility function

The state of nature is unknown and characterizes the decision problem. Within the context of decision problems related to engineering systems subject to natural hazard events, examples are (i) the state of an offshore platform in a tropical cyclone event describing whether the offshore platform collapses or not; (ii) the state of a city in a storm event describing whether the amount of precipitation leads to flooding or not; and (iii) the state of a habitat subject to extreme temperature whether temperature leads to fatalities or not. Typically, the development of the natural processes that characterize the natural hazard and thus the state of nature cannot be influenced by the decision maker; however the related consequences can be reduced through adequate measures. The state of nature is denoted by the random variable Θ.

The set of decision alternatives, denoted by A, represents possible actions from which the decision maker can choose. In the phase before a natural hazard event emerges, risk mitigation measures include retrofitting of existing buildings, the construction of dikes or tsunami walls. In the face of an emerging natural hazard event, immediate actions include risk reducing measures such as evacuation and shut-down of operations.

The utility function related to a decision maker represents her preferences over the consequences resulting from the state of nature and the decision alternative. A realization $U(a, C(\theta, a))$ of the utility is a function of the decision a and the consequence $C(\theta, a)$ related to the state of nature $\Theta = \theta$. The utility function should be consistent with the choice of decisions. This is guaranteed by the axioms of utility given by von Neumann & Morgenstern (1944). For instance, from the axioms it follows that if the decision maker prefers decision alternative $a^{(2)}$ to $a^{(1)}$ and $a^{(3)}$ to $a^{(2)}$, then she prefers $a^{(3)}$ to $a^{(1)}$ and the utility function must be defined such that

$$U(a^{(1)}, C(\theta, a^{(1)})) < U(a^{(2)}, C(\theta, a^{(2)})) < U(a^{(3)}, C(\theta, a^{(3)})) \tag{2.2}$$

The consistency and the rationality of the choice of a decision are provided by the axioms and the findings of Savage (1954). Under the assumption of rational decision making the optimal decision a^* is the one that maximizes the utility $U(a, C(\cdot, a))$; i.e.

$$a^* = \arg\max_{a \in A} U(a, C(\cdot, a)) \tag{2.3}$$

This *normative* assumption is taken in this thesis; although it has been shown for example by Kahneman & Tversky (1979) that individuals do not act in general according to these axioms.

2.2.2 Types of decision making conditions

Following Luce & Raiffa (1957), the decision theory can be applied to three conditions; that are conditions of certainty, "risk" and uncertainty. When solving a decision problem the condition is defined through the choice of model and the information available or used. Under the first condition each decision alternative leads to a known consequence; under "risk" condition, the decisions may lead to several possible consequences but the probability of their occurrence is given; whereas under the third condition the distributions are also unknown, i.e. the possible decisions lead to consequences for which the occurrence probabilities are not given and need to be estimated. In the present work, the second condition is of relevance; that is, the decision problem is considered to be under the condition of "risk" as it is assumed that the distribution of the underlying random variables are known. Therefore the optimization problem (2.3) is rewritten using the expectation operator $E[\cdot]$ related to the distribution of Θ:

$$a^* = \arg\max_{a \in A} E[U(a, C(\Theta, a)] \tag{2.4}$$

2.2.3 Sequential and pre-posterior decision analysis

In the considered decision problem, the state of nature evolves over time such that the question arises whether it is optimal to collect further information to reduce uncertainty or to make a decision immediately. This type of problem may be treated within the framework of sequential decision analysis, see e.g. Bellman (1957), or Bayesian decision theory, see e.g. Raiffa & Schlaifer (1961), DeGroot (1970) and Benjamin & Cornell (1970).

In sequential decision analysis observations are assumed to be obtained one after the other. After each observation a decision is made, which is either to choose an action or to take another observation. For further details see Wald (1947), Wald (1950), Berger (1980). A common approach to solve sequential decision problems is the so-called Dynamic Programming approach by Bellman (1957).

The pre-posterior decision analysis is an approach to decide sequentially whether to conduct, for example, an experiment to gain more information or not, before one makes another observation. The idea is to compare the expected utility of making an immediate decision (i.e. without further observations) and the expected (posterior) utility that is obtained including possible future observations. This makes pre-posterior decision analysis an useful technique for risk assessment in general, and in civil engineering in particular. For example, in the field of civil engineering, it has been utilized for the assessment of the performance of existing structures and to optimize maintenance plans for the deterioration of structures on the basis of risk minimization. In these engineering decision problems, prior information is available and additional information can be "bought" by using different methods such as inspection; see Straub (2004) and Kübler (2006). Presently, the Bayesian pre-posterior decision analysis provides a concept for risk assessment and management of engineered facilities, and generally for engineering decision making, see JCSS (2008) and Faber et al. (2007a).

The mathematical formulations relevant in the present thesis are given in Section 3.2.

2.2.4 Methods to solve sequential and pre-posterior decision problems

An analytical solution is generally not available for practical sequential/pre-posterior decision problems. Only for a few simple examples an analytical solution is provided, see e.g. DeGroot (1970, Chapter 12). Two simple examples, for which the solution is known analytically, are revised in Annex B.2.2. These examples help to understand the nature of the sequential/pre-posterior decision problems.

In this thesis sequential/pre-posterior decision problems must be addressed for which no analytical solution is available. The literature available on methods that solve such decision problems is extensive. Various solution approaches have been proposed. A selection of available approaches that formulate and solve sequential/pre-posterior decision problems includes: decision tree models, Monte Carlo simulation approaches, influence diagrams, Bayesian inference, neural networks, and in particular reinforced learning to solve Markov decision processes (policy or value function approach) or partial observable Markov decision processes, stochastic mesh method or the least-squares Monte Carlo method; these can be found for example in DeGroot (1970), Jensen & Nielsen (2007), Kjaerulff & Madsen (2008), Puterman (1994), Haykin (1999), Powell (2011), Littman (1996), Sutton & Barto (1998) and Glasserman (2004). These models often assume conditions restricting the application or they introduce crude approximations such as (1) simplification of a non-stationary stochastic process by a first-order Markov process or Markov chain; (2) approximation of a continuous random process by a discretized model; (3) the choice of a finite or infinite time horizon; (4) reducing the number of decision alternatives available; or (5) reducing the number of time steps at which a decision can be made.

2.3 Early warning systems as tool for risk management

Early warning systems (EWS) are not only an important tool for the risk management of natural hazard threats, but also for warnings of other hazards such as fire in buildings (Paté-Cornell, 1986), or the inspection and maintenance of e.g. airplanes (Lakats & Paté-Cornell, 2004), power plants (see e.g. Renders et al. (1995), Hashemian (2011)), or the prevention of collisions of vehicles (see e.g. Ding & Zhou (2013), (Wu & Wang, 2000)) or financial crisis (see e.g. Davis & Emanuel (1991), Ciarlone & Trebeschi (2005), Bussiere & Fratzscher (2006)). In these examples, often similar characteristics of the underlying random processes can be found as those described in Section 3.1.

This section introduces the main elements of EWS and gives a brief overview on literature describing existing EWS with the focus on the application to natural hazard events.

2.3.1 Early warning systems for natural hazards

Figure 1.1 in Chapter 1 illustrates the development of consequences in the world due to natural hazard events in the last 37 years. The increase of the consequences in the late 20th Century resulted in an increasing awareness of people for the associated risks, which in turn led to an increasing demand for protection against natural hazard events. Protection can either be accomplished through implementing risk reducing measures for risk mitigation before a natural hazard event occurs or for emergency response during the event. The latter requires well prepared early warning systems (EWS). The outcomes of the World Conference on Disaster Reduction held in Kobe, Hyogo, Japan in 2005, commit "the international community to address disaster reduction and to engage in a determined, results-oriented plan of action for the next decade" (WCDR, 2013). Its final report, the Hyogo Framework of Action 2005-2015, prioritizes five actions; among them to "Identify, assess and monitor disaster risks and enhance early warning" (UNISDR, 2005). This fostered research and further implementation of EWS, which according to Rogers & Tsirkunov (2011) has saved many lives and property in recent years.

2.3.2 Definition and elements of early warning systems

In the list of terminologies of the United Nation International Strategy for Disaster Reduction UNISDR (2009) an EWS is defined as "the set of capacities needed to generate and disseminate timely and meaningful warning information to enable individuals, communities and organizations threatened by a hazard to prepare and to act appropriately and in sufficient time to reduce the possibility of harm or loss."

Thereafter it is mentioned that a people-centered EWS integrates four main elements:

1. *Knowledge of risk*: Obtained by (probabilistic) risk assessment that provides relevant information to define priorities for mitigation as well as response strategies and to design early warning systems.
2. *Monitoring, analysis and forecasting of the hazard*: Monitoring systems combined with forecasting models provide timely risk estimates.
3. *Communication or dissemination of alerts and warnings*: Communication systems deliver warning messages to the local and regional governmental agencies responsible for the areas that are likely to be affected.
4. *Local capability to respond to the warning*: Coordination, good governance and appropriate action plans need to be available beforehand. Additionally, the affected people should be aware of the potential hazards and accordingly informed or trained what to do in such situations.

Failure of any part of the EWS will imply failure of the whole system. These elements basically correspond to those of an EWS applied to warn individuals such as owners of industrial systems (e.g. owner of an offshore platform).

2.3.3 Existing EWS and decision support in EWS

The recent advances in information technology provide new possibilities to monitor natural processes. Technologies such as sensors, satellite imagery, radar or automated weather stations provide information and indication about emerging natural hazards. The information is in principle accessible from everywhere. This possibility facilitates the idea of a global early warning system as proposed at the World Conference on Disaster Reduction in 2005 by the United Nations.

Natural hazards are a global problem affecting developing as well as developed countries. The EWS utilized in developed countries are in general well integrated and accepted by the population. This is in general not the case in developing countries. In these countries, the World Bank and the United Nations provide technical support and guidance to install EWS in risk-prone areas. The advances and the state of art of the implementation of EWS can be found for instance in Zschau & Kuppers (2003), UNISDR (2010), Rogers & Tsirkunov (2011), UNEP (2012), and Golnaraghi (2012).

The focus of these reports lies on the application of the aforementioned elements to people-centered EWS.

Common EWS are installed with priority to save lives, which is ethical from a societal point of view. However, the framework applied in this thesis uses as decision criteria the estimate of the expected consequences in terms of monetary units. Emphasizing economic loss is usually important in industrial applications where decisions are made on whether to shut down operations or not. Note that casualties can be considered additionally within the framework e.g. by constraining the decision optimization to a minimum acceptable human risk or directly by monetizing loss of life.

The focus of this thesis lies on the decision optimization related to the first and second element mentioned above; which is to assist decision makers to find an optimal decision. This part of the EWS is referred to as *Decision Support System* (DSS) hereafter. In the following, some existing DSS are provided for tropical cyclones and for snow avalanche hazards, since these are considered in the examples in Chapter 6.

DSS for tropical cyclones

Tropical cyclones (TC) are also known in the Atlantic and eastern North Pacific as hurricanes, in the western North Pacific as typhoons or in India as cyclones. These are intense rotating storm events that originate over warm tropical waters (typically 26.5°C or greater); low atmospheric pressure, high winds and heavy rainfalls are the main characteristics (Ahrens, 2009, Chapter 15). These characteristics make them to a potential natural hazard event. A well-known example is Hurricane Katrina in 2005 with over 1'300 estimated fatalities and an estimated financial loss of $100 billion (Kirlik, 2007).

The information for early warnings facing TC are in general provided by meteorological agencies such as the Japan Meteorological Agency (JMA) or the National Hurricane Center (NHC). The relevant information includes for example the expected time of landfall, the expected intensity and the potential affected zones. With this information public officials as well as private decision makers have to decide whether or not to recommend the evacuation of people in potentially affected areas, to shut-down operations, or other risk reducing measures. State of the art DSS such as the HURREVAC software of the Federal Emergency Management Agency (FEMA) provide regional recommendation for the "evacuation decision time" at each time step when new information about the present state and future forecasts are available, see FEMA (2012) for further details. The recommendation is based on the tropical storm forecast of the NHC and the Weather Service. The *evacuation decision time* is defined as the latest point in time by which an evacuation should be ordered so that it is completed successfully. It is computed by subtracting the estimated evacuation time from the estimated time when the strongest winds of the TC arrive at the considered area. For the estimation of the arrival time, the worst-case scenario is taken where the TC hits directly the area; however with the translation speed and wind of the NHC forecast.

Kirlik (2007) investigates the HURREVAC DSS by means of the evacuation in the case of hurricane Katrina in 2005. He states that HURREVAC provides information when a decision needs to be made, but not what decision should be made. Furthermore, HURREVAC provides the information on worst-case scenarios, which meant in the case of hurricane Katrina that the entire gulf coast (Florida to Texas) should have been evacuated (Kirlik, 2007). HURREVAC provides the forecast uncertainty in two ways: using an error cone[4] and numerical strike probabilities. In case of New Orleans the mean track of the cone was directly above New Orleans, yet the strike probability was estimated by 17% with the 72-hr forecast 56 hours before the landfall. Kirlik (2007) claims that the actual burden is then on the decision makers to identify a true hazard and to try to avoid false alarm. A similar study of the EWS in case of hurricane Katrina by Einstein & Sousa (2007) pointed out that no warning system nor an adequate risk management plan existed for the actual hazard scenario; meaning that the flooding due to the dike breaks were not adequately considered as a possible consequence from the prevailing storm surge in the risk assessment. This example shows also how important reasonable risk assessment prior to a hazard event is, which enables decision makers to estimate the risk related to the decision alternatives available.

Summarizing the findings from the reports mentioned above, HURREVAC has basically three drawbacks: (i) it uses only worst-case storm scenarios to compute the evacuation decision time, which is rather conservative and may lead to false alarm; (ii) it does not provide which decision is optimal, decision makers need to interpret the probabilities; (iii) it does not consider possible future realizations of a hurricane nor consequences or the corresponding evacuation decision times in the computation of the present evacuation decision time; i.e. it solves only one-time decision problems[5]. The last mentioned drawback actually fails to motivate decision makers to plan

[4]The error cone represents the average error associated with the NHC forecast track; see for details FEMA (2012).

[5]One-time decision problems are decision problems in which one decision can be made out of the set of terminal decision alternatives; the decision to postpone making a terminal decision is not available. A simple example of a near-real time decision problem using one-time decision making is given in Nishijima et al. (2008).

an evacuation or shut-down of operation in situations in which the information indicates to do nothing, however this can be important when the TC takes an unexpected transition in future times and an evacuation might be too late or much more expensive due to the reduced time. Regnier (2008) state similar conclusions and propose a dynamic model, which is similar to the presented real-time decision framework. However, her optimal decision is based on a simple statistical hurricane track model, a first-order Markov chain model, introduced in Regnier & Harr (2006). Using this model, 10'000 hurricane tracks are simulated by crude Monte Carlo simulations. Given these simulations the total costs related to the decision alternatives are computed. Regnier & Harr (2006) note that the most likely tracks the model produces are not necessarily close to the track forecast of NHC; i.e. the forecast ability of the model is not sufficient for real-time decision making. The potential costs are defined as a function of the fixed lead times; outcomes are compared and discussed. The computational time is not mentioned, however the authors note that one reason, why the real-time decision making is not implemented, is that the stochastic model to simulate hurricane tracks is not designed for forecasting.

DSS for snow avalanches

Snow avalanches are hydro-meteorological natural hazards. They are further classified as mass movement hazards like rockfall, landslides or debris flow. In Figure 1.2, it can be seen that snow avalanches do not belong to the most significant natural hazards on a global scale, but in Switzerland, for example, they affect important industries such as tourism and transportation. According to the statistics of SLF (Swiss Federal Institute for Snow and Avalanche Research) every year about 200 avalanche accidents are reported and in average 25 people die. In Switzerland the last “avalanche winter” was in 1998/99, when people actually died in buildings and on roads (SLF, 2013). During this winter 28 people in inhabited areas were subject to an avalanche, 17 of whom died, 131 snow sport tourists were subject to avalanches, 19 died, and the economic damage is estimated to be over CHF 600 million (SLF, 2000).

During the winter months, the SLF publishes every day two avalanche bulletins that include the forecast of avalanche danger levels of the following day. The danger levels are illustrated with a colored map. The bulletin is kept general and no local risk assessment is provided. To establish the report, information is obtained from observers (about 180 persons), automated measuring stations (about 100 stations), MeteoSwiss or other weather agencies and reports of actual avalanches. The statistical software tool NXD provides historical data about weather, snow conditions and avalanches; by using the nearest neighbor approach similar historical events are determined to estimate the future avalanche danger. Furthermore, it is planned that software like SNOWPACK and ALPINE3D can be used to simulate physical processes to evaluate the avalanche danger; both are still in the development phase and deterministic.

An overview of other models that estimate the avalanche danger is given in Fromm & Adams (2012). Most of the models mentioned therein, are based on the nearest neighbor approach. Another approach cited in Fromm & Adams (2012) is the numerical avalanche prediction scheme proposed by Floyer & McClung (2003). They apply an extensive variance analysis and a canonical discriminant analysis to determine the variables that contribute most to the prediction of an avalanche or non-avalanche day. The variables contributing most are: the amount of new precipitation, present temperature, snowpack depth, foot penetration and present temperature trend. These variables are used to build a set of functions that allow to predict whether a future time period can be classified as either an avalanche or a non-avalanche period.

Chapter 3

Real-time decision framework

This chapter consists of five sections. The first section introduces the characteristics of the considered decision problem that are required in order to apply the proposed framework. Additionally, the section shows the limitations of the frameworks application. The second section provides the general formulation of *sequential/pre-posterior* decision analysis, which is followed by the description of the characteristics and interrelations of the random variables underlying the decision problem. The proposed real-time decision framework is introduced in Section 3.4. The differences between the formal sequential/preposterior decision framework and the real time decision framework are pointed out as well as the proposed adaptations. Thereafter the mathematical formulation of the optimization problem is provided. Parts of this chapter are adapted from Nishijima et al. (2009) and Anders & Nishijima (2011).

3.1 Characterization of the decision problem

The considered decision situation is specified by the following characteristics, see Nishijima et al. (2009):

1. The hazard process emerges relatively slowly and allows for reactive decision making.
2. Prior to the impact of the hazard, various types of information can be obtained, which can be utilized to predict its severity.
3. Decision makers have options for risk reducing measures, which may be commenced at any time, supported by the information available.
4. The decision making is subject to uncertainties, part of which might be reduced by collecting further information.

5. The decisions must be made fast, in (near) real-time.

The decision problems that can be characterized by these five characteristics are hereafter referred to as *real-time decision problems*. The characteristics are interrelated. As these interrelations may not be obvious at a first glance, they are pointed out in the following in order to facilitate the understanding of the decision problems considered:

(i) Precursors are in general available only if the hazard process emerges relatively slowly.

(ii) It is only possible to decide on risk reducing measures if decision makers are aware of the emerging natural hazard, which is only realistic in case the hazard process evolves relatively slowly and/or precursors are available.

(iii) Further information can only be collected if the hazard process and/or its precursors emerges with such a slow pace that time is available to postpone risk reducing measures. The available time frame should be in such a way that another decision can be made before an impact occurs; otherwise one-time decision making is sufficient.

(iv) The decisions have to be made fast since risk reducing measures take time until they are completed successfully. How fast the decisions need to be made is in turn related to the pace of the development of the natural hazard event or its precursors; i.e. the faster the development, the shorter is the time period that is available for making a decision and completing a risk reducing measure.

From the possibility to postpone the decision for risk reducing measures in order to collect further information the following typical problem arises: when a decision is postponed, time passes until the next decision is made, which implies the reduction of uncertainty but also the reduction of available time for commencing and completing risk reducing measures, in case they are necessary.

The decision to postpone making a terminal decision and to collect further information can be described as “waiting”. This “waiting” is often associated with costs; these *information costs* consist of two components. The first component includes those costs that arise when further measurements are taken or further data collected in order to improve forecasting. The second component is related to the costs that arise when the time is too short to complete a risk reducing measure. These costs include costs that would not arise if the risk reducing measure was commenced and completed in time. For example, this is the case when people have to be evacuated from a flooded area, property like vehicles are left in a flood zone, ships are still in a harbor or offshore platforms are still in operation when a tropical cyclone strikes.

As mentioned in Section 2.1, two types of uncertainty should be distinguished; i.e. aleatory uncertainty and epistemic uncertainty. Note that, whereas the reduction of the risk associated to both types of uncertainty is relevant in general, only the former is considered in this thesis in order to emphasize the essential ideas in the proposed framework. In the following, it is briefly described how the risk of both types of uncertainties can be reduced in principle.

Epistemic uncertainty is reduced by collecting more information to update the probability model representing the random phenomena underlying the decision problem. As mentioned above, this may involve costs. Hence, collecting more information is worth undertaking only if the corresponding expected information costs are smaller than the expected value of the additional costs arising from a potentially suboptimal decision. In the examples presented in Chapter 6, the risk associated to aleatory uncertainty may be reduced by "waiting". Namely, as time goes by it becomes more apparent whether the impact of a natural hazard event occurs or not.

By postponing the decision the probability is reduced that the decision maker makes a suboptimal decision; but, in turn, the probability increases that risk reducing measures are undertaken too late if they are necessary. This leads to a trade-off between the risk of making a sub-optimal terminal decision and the risk of making this decision too late. This trade-off problem is similar to that of the error type I and II in hypothesis testing. Assuming that the null-hypothesis is that the hazard occurs in a certain time period, then error type I stands for rejecting the null-hypothesis when it is true.[1] In this case the initiation of risk reducing measures is too late, insufficient or not ordered, although the hazard occurs. Whereas error type II represents the case where risk reducing measures are initiated, when no hazard occurs; i.e. the case of a false alarm. Sometimes this may also include the cases where the initiation is a lot too early or excessively. The similarity to hypothesis testing is mentioned here, as the sequential (decision) analysis introduced in Section 3.2 can be seen as a sequential hypothesis testing where the sample size or the time horizon is not fixed in advance.

[1] For an introduction to hypothesis testing see e.g. Savage (1954), Benjamin & Cornell (1970), Ang & Tang (2007).

3.2 Mathematical formulation of the sequential decision procedure

In order to solve the decision problem that is defined by the characteristics given in Section 3.1, the sequential/pre-posterior decision analysis provides the mathematical basis. In this section the general formulation of the sequential and pre-posterior decision analysis is presented in accordance with DeGroot (1970) and Berger (1980). Therein the idea of sequential and pre-posterior decision analysis is introduced mainly for hypothesis testing and experimental design. For the purpose of illustration the exemplary decision problem involved in the quality control of a product series is considered. For further reading it is referred to Raiffa & Schlaifer (1961) and Benjamin & Cornell (1970).

As mentioned in Section 2.2, there are three key components in a decision problem: the state of nature Θ, the set of decision alternatives A and the utility function $U(\cdot,\cdot)$. For instance, Θ may represent the quality of a product series that specifies the decision problem whether to buy the product series or not.

Let $\{Y_t\}_{t=1,2,\ldots}$ denote a sequence of random variables that can be observed for the estimation of the state of nature. In case of quality control of a product series, Y_t represents the quality of the product that is observed at time t, assuming that at each time step $t = 1, 2, \ldots$ the quality of a product is observed. The range of samples of Y_t is denoted by $\mathcal{Y}_t$. Let $\underline{\mathbf{Y}}_t$ be the vector of random variables that can be observed at times $s = 1, 2, \ldots, t$; i.e. $\underline{\mathbf{Y}}_t = (Y_1, Y_2, \ldots, Y_t)$. Assume that $\underline{\mathbf{Y}}_t$ has the conditional probability density function (cpdf) $f_t(\underline{\mathbf{y}}_t|\theta)$, conditional on the realization θ of the state of nature, and the conditional cumulative distribution function $F_t(\underline{\mathbf{y}}_t|\theta)$ on the range of samples $\mathcal{Y}_1 \times \mathcal{Y}_2 \times \cdots \times \mathcal{Y}_t$. If the random variables $Y_1, Y_2, \ldots$ are independent and identically distributed from a common cpdf $f(y|\theta)$, then the cpdf of $\underline{\mathbf{Y}}_t$ is

$$f_t(\underline{\mathbf{y}}_t|\theta) = \prod_{s=1}^{t} f(y_s|\theta) \qquad (3.1)$$

The sequence $\{Y_t\}_{t=1,2,\ldots}$ represents a *sequential sample* from the cpdf $f(y_t|\theta)$ where the observations are taken sequentially. The information available if no observation is taken is represented by the variables $\underline{Y}_0$ and $\underline{\mathbf{Y}}_0$.

The decision problem can be formulated at each time step: is it optimal to make decision $a \in A$ or is it better to wait and take another observation. In case a decision is made, no further observation can be taken; i.e. the sampling is terminated. Hereafter, these decisions are called *terminal decisions.*

Taking another observation may reduce the uncertainty about the quality of the product series, but additional costs arise. The total costs depend on the number n of observations taken and the decision $a \in A$ made thereafter. The number n represents the time step at which a terminal decision taken and represents the time horizon of the index of the random variable Y_t; i.e. $t = 1, 2, \ldots, n$. In this section n is not assumed to be fixed in advance.

Denote the *loss function* by

$$L(a, \theta, n) \tag{3.2}$$

Assuming that the utility function is linear, the loss function $L(a, \theta, n)$ is the sum of the loss $L(a, \theta) = -G(a, \theta)$ due to decision a and the observation cost $C(n)$ after n observations are taken. $G(a, \theta)$ denotes the gain as a function of the realization (a, θ). If the utility function is non-linear, then

$$L(a, \theta, n) = -U(a, G(a, \theta) - C(n)) \tag{3.3}$$

Here, only those decision problems are considered for which it can be assumed that a terminal decision is made after a finite number of observations. Denote $\pi(\theta)$ the prior density function of the unknown state of nature. The prior density function $\pi(\theta)$ can be updated when information becomes available. For instance, if random variables Y_s, $s = 1, 2, \ldots, t$ are distributed according to the joint probability density function $f_t(\underline{\mathbf{y}}_t|\theta)$ and $\underline{\mathbf{y}}_t$ is observed after t samples, then the posterior density function is obtained by applying Bayes rule:

$$\pi(\theta|\underline{\mathbf{y}}_t) = \frac{f_t(\underline{\mathbf{y}}_t|\theta)\pi(\theta)}{\int_\Theta f_t(\underline{\mathbf{y}}_t|\theta)\pi(\theta)d\theta} \tag{3.4}$$

The calculation of the posterior density function is straightforward if the random variables Y_s, $s = 1, 2, \ldots, t$ are independent and identically distributed. In this case the joint probability function $f_t(\underline{\mathbf{y}}_t|\theta)$ is given by Equation (3.1). In case the update is requested after each observation, Bayes rule can be applied as follows: assume the prior is now given by $\pi(\theta|\underline{\mathbf{y}}_{t-1})$ and the observation y_t is made at time t, then the corresponding posterior probability density function is

$$\pi(\theta|y_t, \underline{\mathbf{y}}_{t-1}) = \frac{f(y_t|\underline{\mathbf{y}}_{t-1}, \theta)\pi(\theta|\underline{\mathbf{y}}_{t-1})}{\int_\Theta f(y_t|\underline{\mathbf{y}}_{t-1}, \theta)\pi(\theta|\underline{\mathbf{y}}_{t-1})d\theta} \tag{3.5}$$

Before the optimization problem in the sequential/pre-posterior decision problem is introduced, note that the set of decision alternatives A changes over time, which is denoted by the index t; i.e. A_t. Namely, after a terminal decision is made at time t, no further decision is available at time $t+1$; i.e. the decision set A_{t+1} is empty. It is convenient to divide the decision set into two mutually exclusive subsets:

$$A_t = A_t^{(c)} \cup A_t^{(s)}, \quad A_t^{(c)} \cap A_t^{(s)} = \emptyset$$

where $A_t^{(c)}$ consists of one decision alternative $a_t^{(0)}$ to "wait" and collect further information (i.e. $A_t^{(c)} = \{a_t^{(0)}\}$) and $A_t^{(s)}$ is the set consisting of the terminal decisions. Note that, in regard to the subsequent application, this formulation of time dependent decision sets is different to the formulation introduced in DeGroot (1970) and Berger (1980), yet it represents the same set of decision alternatives.[2]

In general, the aim is to maximize the expected utility over the set of decision alternatives. Since here, the loss or cost is investigated, the optimization problem is defined using the minimum-operator.[3]

In case no terminal decision is made up to time t, the optimal decision a_t^* at time t is identified as the one that minimizes the expected loss at time t conditional on the collection of information up to time t (Nishijima et al., 2009):

$$\begin{aligned} &E_{\theta|\underline{\mathbf{y}}_t}[L(a_t^*(\underline{\mathbf{y}}_t),\theta,t)|\underline{\mathbf{y}}_t] \\ &= \begin{cases} \min\limits_{a_t \in A_t} E_{\theta|\underline{\mathbf{y}}_t}[L(a_t(\underline{\mathbf{y}}_t),\theta,t)|\underline{\mathbf{y}}_t], & \text{for } t = 0,1,...,n-1 \\ \min\limits_{a_t \in A_t^{(s)}} E_{\theta|\underline{\mathbf{y}}_t}[L(a_t(\underline{\mathbf{y}}_t),\theta,t)|\underline{\mathbf{y}}_t], & \text{for } t = n \end{cases} \end{aligned} \tag{3.6}$$

where the expectation is computed with respect to the updated distribution. Furthermore, for $t = 0, 1, \ldots, n-1$ the conditional expected value of the loss associated to the decision $a_t^{(0)}$ is defined through the equation

$$\begin{aligned} &E_{\theta|\underline{\mathbf{y}}_t}[L(a_t^{(0)}(\underline{\mathbf{y}}_t),\theta,t)|\underline{\mathbf{y}}_t] \\ &= \int E_{\theta|\underline{\mathbf{y}}_t}[L(a_{t+1}((\underline{\mathbf{y}}_t, y_{t+1})),\theta,t+1)|\underline{\mathbf{y}}_t] f(y_{t+1}|\underline{\mathbf{y}}_t) dy_{t+1} \end{aligned} \tag{3.7}$$

[2] DeGroot (1970) introduces a sampling plan in which at least one observation is taken. The sampling plan is characterized by a stopping set $B_n \in \mathcal{Y}_1 \times \mathcal{Y}_2 \times \cdots \times \mathcal{Y}_n$, $n = 1, 2, \ldots$ with the following property: sampling is terminated after observing $(y_1, y_2, \ldots, y_n)$, if $(y_1, y_2, \ldots, y_n) \in B_n$. Further he introduces a decision rule $\delta = \{\delta_0, \delta_1(\underline{\mathbf{y}}_1), \delta_2(\underline{\mathbf{y}}_2), \ldots\}$, where $\delta_t(\underline{\mathbf{y}}_t)$ represents the action to be taken in case sampling is terminated after $\underline{\mathbf{y}}_t$ is observed. Whereas Berger (1980) introduces a sequential decision procedure $\mathbf{d} = (\tau, \delta)$ consisting of the stopping rule τ and the decision rule δ. $\tau = \{\tau_0, \tau_1(\underline{\mathbf{y}}_1), \tau_2(\underline{\mathbf{y}}_2), \ldots\}$, where $\tau_t(\underline{\mathbf{y}}_t)$ represents the probability that sampling is terminated and a decision is made after $\underline{\mathbf{y}}_t$ is observed. The decision rule is defined like the one introduced by DeGroot (1970).

[3] DeGroot (1970) and Berger (1980) use the infimum-operator instead of the minimum-operator. In fact the infimum is called minimum, if the infimum is an element of the considered set.

From Equation (3.7) it can be seen that for the decision $a_t^{(0)}$ at time t, the optimization requires to know all optimal decisions at future times, $t+1, t+2, \ldots, n$; hence, backward induction is required.

3.3 Characteristics of random processes underlying the decision problem

In the previous section the general idea of sequential and pre-posterior decision analysis is provided; this section introduces the characteristics and interrelations of the random processes underlying the decision problem of consideration. The content is presented in accordance with Nishijima et al. (2009) and Anders & Nishijima (2011).

Let Z denote the random variable of relevance to the consequences in a decision problem; e.g. the state whether the (maximum) wind speed exceeds a certain threshold $\tilde{z}$ during a storm event or not. The exceedance of the threshold reflects the impact; i.e. if the wind load exceeds the expected wind resistance of the considered engineering system implies that the storm leads to consequences. In accordance with Section 3.2 the random variable Z is similar to the unknown state of nature Θ. However, Z is not fixed at the beginning; but it is realized latest at the time horizon n. n is the number of points in time at which observations can be made. For example, the maximum wind speed during a storm event is defined as a sequence of random variables $\{Z_t\}_{t=0,1,\ldots,n}$, where Z_t denotes the maximum wind speed in the time period $[0,t]$. Let Z be equal to one, in case the emerging storm evolves to a natural hazard event leading to an impact during the time period $[0,n]$, and Z equal to zero, otherwise; i.e.

$$Z = \begin{cases} 1, & \text{if } Z_\tau \geq \tilde{z} \text{ for some } \tau \in [0,n] \\ 0, & \text{otherwise} \end{cases} \tag{3.8}$$

where both cases have a certain probability at each time step as long as $Z_t < \tilde{z}$ and $t < n$. As soon as there is a first time step $\tau \leq n$ for which Z_t exceeds the threshold (i.e. $Z_\tau \geq \tilde{z}$), the state of Z is known. However, latest at time $t = n$, the state of Z is either equal to one or to zero, which is typically observable; this is different compared to other sequential/pre-posterior decision problems such as the ones in quality control or inspection planning. The sequence $\{Z_t\}_{t=0,1,\ldots,n}$ is called the *hazard index* and is characterized by a sequence of random variables $(Y_0, Y_1, \ldots, Y_n)$, which are required for calculating the probabilistic characteristics of Z.

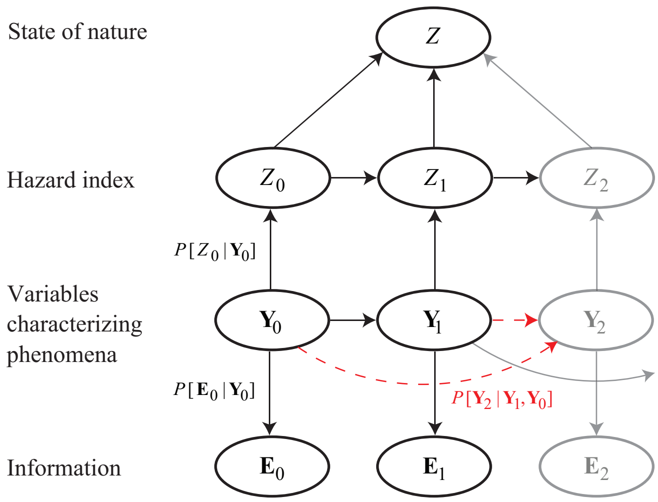

Figure 3.1: *Structure and interrelations of the relevant random processes. The variables characterizing the random phenomena are characterized by a second-order Markov model. This scheme is adapted from Nishijima et al. (2009).*

Prior to or at the last time step n, a terminal decision must be made. Denote by $\underline{\mathbf{E}}_n = (E_0, E_1, ..., E_n)$ a sequence of random variables representing the observed information at the respective time steps. The observed information can be utilized to reduce the uncertainty associated with the future states of Y_t and in turn with Z. Note that the variables Z, Z_t, Y_t and E_t, $t = 0, 1, ..., n$, can be scalar or vector. Hereafter, if necessary a vector variable will be denoted by a bold letter to avoid ambiguity.

The illustrative relationship between the variables is shown in Figure 3.1 for the case that $\mathbf{Y}_t$ is represented by a second-order Markov model. Each node represents a variable and each directed edge link represents the probabilistic dependency between the connected variables. For instance, the edge link directed from the node $\mathbf{Y}_0$ to the node $\mathbf{E}_0$ represents that the random variable $\mathbf{E}_0$ is characterized by the conditional probability $P[\mathbf{E}_0|\mathbf{Y}_0]$. When more than two edge links are directed to a node, it signifies that the random variable represented by the node is characterized by the conditional probability on the variables represented by the nodes from which the edge links are directed. For instance, in Figure 3.1 the random variable $\mathbf{Y}_2$ depends on the random variables $\mathbf{Y}_0$ and $\mathbf{Y}_1$ (illustrated by the dashed red arrows); the node $\mathbf{Y}_2$ is characterized by the conditional probability $P[\mathbf{Y}_2|\mathbf{Y}_1, \mathbf{Y}_0]$. When all the conditional probabilities are given that correspond to the directed edge links and the (unconditional) probabilities for the nodes to which no edge link is directed, conditional probabilities of any variable in the graph can be calculated. Hence, the probabilistic characteristics of the random phenomena underlying the decision problem can be completely defined.

In the present decision framework (introduced in Section 3.4) it is important to compute the conditional probability of the state of Z conditional on the information $\underline{\mathbf{E}}_t$ and the conditional probabilities of $\mathbf{E}_{t+1}$ given $\underline{\mathbf{E}}_t$, $t = 0, 1, \ldots, n$. In this thesis, it is assumed that the probabilistic models and algorithms are available to calculate the conditional probabilities relevant in the considered decision problems. For the examples in Chapter 6 the conditional probabilities are represented through regression models. However, in case the conditional probabilities have to be computed, Bayesian Probabilistic Networks for instance provide the basis to represent the interrelations between the relevant variables. Given the structure of the decision problem, several generic algorithms are available to compute the conditional probabilities; some are presented e.g. in Jensen & Nielsen (2007).

3.4 Framework for real-time decision support

The framework introduced in this chapter is called *real-time decision framework*. In the following, the formulation of the framework is introduced in accordance with Nishijima & Anders (2012).

The real-time decision framework is based on the sequential and preposterior decision procedure presented in Section 3.2. Whereas the principal ideas of the decision procedure are the same, the following differences between the decision problems are found:

- The finite time horizon is defined in advance.
- The process $\mathbf{Y}_t$, $t = 0, 1, \ldots$, characterizing the phenomena are not necessarily independent and identically distributed; typical examples are non-stationary first- or higher-order Markov processes.
- The decision procedure is not only terminated through making a terminal decision, but also in case the state of nature becomes realized; i.e. also in case the impact of the natural hazard is occurred.
- The observed information $\mathbf{e}_t$ does not necessarily equal the state $\mathbf{y}_t$ or a deterministic function of $\mathbf{y}_t$. This is the case when observation or measurement errors are considered; if so, $\mathbf{e}_t$ may be probabilistically dependent on $\mathbf{y}_t$ and is characterized by a conditional distribution function given $\mathbf{y}_t$.

The idea of the real-time decision framework is illustrated in Figure 3.2. The state of the natural hazard changes over time, which is represented

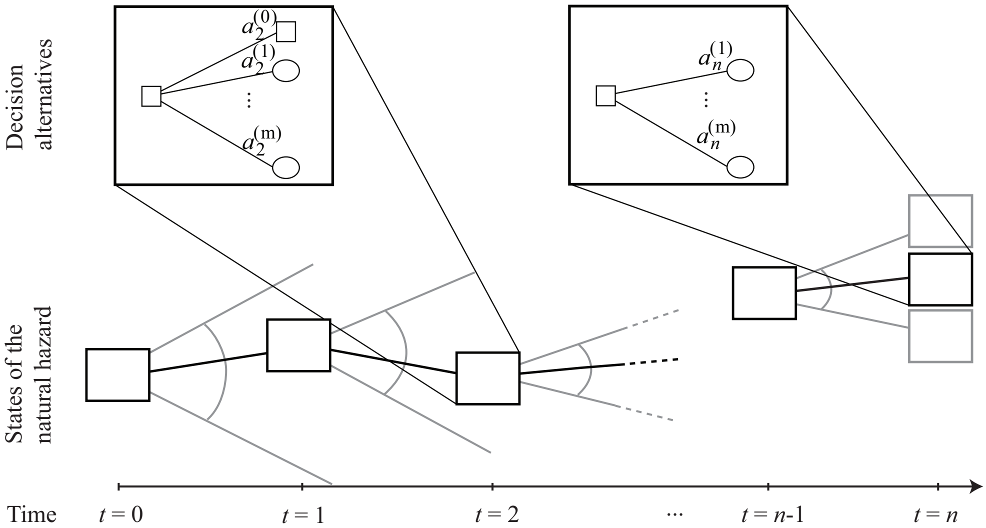

Figure 3.2: *Decision tree representing the real-time decision problem.*

through the hazard index $\{\mathbf{Z}_t\}_{t=0}^n$. The hazard index is characterized by the underlying random sequence $\{\mathbf{Y}_t\}_{t=0}^n$, as illustrated in Figure 3.1. At each time step $t = 0, 1, \ldots, n$ new information becomes available. The new information can be utilized to evaluate the present state of nature and to update the probabilistic characteristics of the underlying random processes. In case the impact of the natural hazard is realized, no further decision can be made. A decision is made at time t only if no impact occurred before. The points in time when a decision can be made are illustrated by the decision nodes (represented by the squares in the lower part of Figure 3.2). In order to find the optimal decision, the updated probabilistic characteristics are utilized to estimate the future states of the underlying random processes as well as the future states of the hazard index. As mentioned before the set of decision alternatives A_t consists of the decision to "wait" $a_t^{(0)}$ and the set of terminal decisions $A_t^{(s)} = \{a_t^{(1)}, a_t^{(2)}, \ldots, a_t^{(m)}\}$. The decision alternatives are illustrated in the upper part of the figure. An oval after a terminal decision represents the fact that no further decision can be made and the set of decision alternatives is thereafter empty; i.e. only the future state of nature will influence the outcome of the consequences. Whereas, in case the decision $a_t^{(0)}$ is made, the square thereafter represents the fact that future decisions are made that influence the corresponding consequences, in addition to the future state of nature. However, at the time horizon n the decision to wait is not available anymore, which implies a terminal decision has to be made.

Let $\underline{a}_t$ denote the vector of decision alternatives taken prior and at time t; i.e. $\underline{a}_t = (a_0, a_1, \ldots, a_t)$. Making no terminal decision up to time t implies that $\underline{a}_t = (a_0^{(0)}, a_1^{(0)}, \ldots, a_t^{(0)})$.

Assuming a decision is made at time t, then the optimal decision a_t^* is identified as the one that maximizes the expected utility at time t conditional on the collection of the information up to time t. This is analog to Equation (3.6):

$$E[U_t(a_t^*, \mathbf{Z})|\underline{a}_{t-1}, \underline{\mathbf{e}}_t] \\ = \begin{cases} \max\limits_{a_t \in A_t} E[U_t(a_t, \mathbf{Z})|\underline{a}_{t-1}, \underline{\mathbf{e}}_t], & \text{for } t = 0, 1, ..., n-1 \\ \max\limits_{a_t \in A_t^{(s)}} E[U_t(a_t, \mathbf{Z})|\underline{a}_{t-1}, \underline{\mathbf{e}}_t], & \text{for } t = n \end{cases} \tag{3.9}$$

where for $t = 0$ the decision $\underline{a}_{-1}$ should be regarded as zero and for the time steps $t = 0, 1, ..., n-1$ and $a_t^{(0)}$,

$$E[U_t({a_t}^{(0)}, \mathbf{Z})|\underline{a}_{t-1}, \underline{\mathbf{e}}_t] \\ = \int E[U_{t+1}(a_{t+1}^*, \mathbf{Z})|\underline{a}_{t-1}, a_t^{(0)}, \underline{\mathbf{e}}_{t+1}] f(\mathbf{e}_{t+1}|, \underline{\mathbf{e}}_t) d\mathbf{e}_{t+1} \tag{3.10}$$

Here, $U_t(a_t, \mathbf{z})$ represents the utility function. It is a function of the decision alternative a_t and the realization $\mathbf{z}$ of the hazard index $\mathbf{Z}$ that is relevant for the decision problem. The vector $\underline{\mathbf{e}}_t = (\mathbf{e}_0, \mathbf{e}_1, ..., \mathbf{e}_t)$ is the collection of the information available up to time t.

Comments to the real-time decision framework

In the examples presented in Chapter 6, it is assumed that

$$\mathbf{y}_t = \mathbf{e}_t, \ (t = 0, 1, ..., n) \tag{3.11}$$

namely, the states of the underlying random processes relevant to the decision problem are known to the decision maker without uncertainty. Thus, the symbols $\mathbf{y}_t$ and $\mathbf{e}_t$ are utilized interchangeably. $f_t(.|\mathbf{e}_t)$ is the conditional probability density/mass function of information $\mathbf{E}_{t+1}$ given $\mathbf{E}_t = \mathbf{e}_t$. This simplification is introduced for illustrative reasons. The uncertainty of $\mathbf{E}_t$ can be included straightforward in the framework if relevant; however, the computational cost will increase and the algorithm may convergence at a slower rate.

The framework can be extended for decision problems where it is relevant that not only one "terminal" decision can be made. In this case the "terminal" decisions are not actually terminating the decision procedure but further decisions can be made at subsequent time steps. The possibility to make several terminal decisions subsequently is similar to the decision to wait. Likewise, for the computation of the expected utility the future optimal decisions have to be known. This can be achieved for example by applying again least squares methods or implementing the eLSM as a sub-algorithm.

Assuming that a terminal decision needs s time steps taking δ time (one time interval is defined by $[t, t+\delta]$) until it is effective, then the latest point in time τ at which this decision should be included is $\tau = n - s\delta$. After time τ considering this decision does not make sense as long as no benefit is introduced for commencing the associated risk reducing measure although it is not fully completed before a hazard impact occurs or before the time horizon. In practice, it may be relevant to introduce a partial reduction of consequences as soon as a risk reducing measure is commenced.

Note that, the decision "to do nothing" is introduced for formal reasons. It results from the fact that at the time horizon the decision to wait (i.e. $a_n^{(0)}$) is not available. Typically, it is then optimal to do nothing instead of commencing a risk reducing measure.

3.5 Reformulation of optimization problem

The decision alternatives in $A_t^{(c)}$ and $A_t^{(s)}$ have different characteristics for computing the respective expected utilities. The computation of the expected utilities for the terminal decisions $a_t \in A_t^{(s)}$ is straightforward and can be estimated for example relatively easy with Monte Carlo simulation; whereas the computation of the expected utility for the decision $a_t^{(0)} \in A_t^{(c)}$ is not straightforward. As it is noted in Section 3.2, for the decision $a_t^{(0)}$ the optimization requires to know all optimal decisions in the future time steps. In order to obtain these optimal decisions, it is required to evaluate all possible future realizations of the underlying random phenomena combined with the decisions available. Usually in practice there is no analytical solution for the expected utility given in Equation (3.10). The large number of states that need to be evaluated makes the numerical computation expansive. The most common approach to solve the optimization problem is to use backward induction as introduced by Bellman (1957) under the name *dynamic programming*. The idea of dynamic programming is to reduce the optimization problem introduced in Equation (3.10) to smaller optimization problems that can be easily solved. Starting at the time horizon n, where the decision problem is straightforward to solve, the estimate of the expected utility respective to the optimal decision (i.e. the maximized expected utility) is obtained; moving one time step backward (to time step $t = n-1$) the maximal expected utilities are estimated using the solutions at time $t = n$. This procedure is repeated backward in time until the initial time step. Note that, the more time steps, underlying random phenomena and decision alternatives are considered, the more states of possible combinations need to be evaluated in order to estimate the expected utility corresponding to the decision to wait.

In order to emphasize the dynamic programming structure and the differences in the computation of the expected utility for the terminal decisions and the decision to wait, the optimization problem given in Equation (3.9) is reformulated in accordance with Anders & Nishijima (2012):

$$q_t(\underline{a}_{t-1}, \underline{\mathbf{e}}_t) = \begin{cases} \max\left\{h_t(\underline{a}_{t-1}, \underline{\mathbf{e}}_t), c_t(\underline{a}_{t-1}, \underline{\mathbf{e}}_t)\right\}, & \text{for } t = 0, 1, \ldots, n-1 \\ h_t(\underline{a}_{t-1}, \underline{\mathbf{e}}_t), & \text{for } t = n. \end{cases} \tag{3.12}$$

Here,

$$q_t(\underline{a}_{t-1}, \underline{\mathbf{e}}_t) = E[U_t(a_t^*, \mathbf{Z}) | \underline{a}_{t-1}, \underline{\mathbf{e}}_t] \tag{3.13}$$

$$h_t(\underline{a}_{t-1}, \underline{\mathbf{e}}_t) = \max_{a_t \in A_t^{(s)}} l_t(\underline{a}_{t-1}, a_t, \underline{\mathbf{e}}_t) \tag{3.14}$$

$$l_t(\underline{a}_{t-1}, a_t, \underline{\mathbf{e}}_t) = E[U_t(a_t, \mathbf{Z}) | \underline{a}_{t-1}, \underline{\mathbf{e}}_t], \quad a_t \in A_t^{(s)} \tag{3.15}$$

$$c_t(\underline{a}_{t-1}, \underline{\mathbf{e}}_t) = E[q_{t+1}(\underline{\mathbf{e}}_t, \mathbf{E}_{t+1}) | \underline{a}_{t-1}, \underline{\mathbf{e}}_t] \tag{3.16}$$

The function $q_t(\underline{a}_{t-1}, \underline{\mathbf{e}}_t)$, $t = 0, 1, ..., n$, is the maximized expected utility, hereafter abbreviated as MEU. The functions $h_t(\underline{a}_{t-1}, \underline{\mathbf{e}}_t)$ and $c_t(\underline{a}_{t-1}, \underline{\mathbf{e}}_t)$ are named stopping value function (SVF) and continuing value function (CVF), respectively.

From the Equations (3.12)-(3.16), it becomes apparent that the evaluation of the SVF does not require backward induction, whereas the evaluation of the CVF requires backward induction. However, no matter how complex the structure of the decision optimization problem may seem, $c_t(\underline{a}_{t-1}, \underline{\mathbf{e}}_t)$ is only a function of $\underline{\mathbf{e}}_t$. Furthermore, if the underlying random process $\{\mathbf{Y}_t\}_{t=0}^n$ follows s^{th}-order Markov process, $c_t(\underline{a}_{t-1}, \underline{\mathbf{e}}_t)$ is a function effectively of the last s information, $\mathbf{e}_{t-s+1}, \mathbf{e}_{t-s+2}, ..., \mathbf{e}_t$.

In this thesis, only cases where one terminal decision can be made are considered. This implies that if a decision can be made at time t, it is required that the vector of previous decisions consists only of decision to wait; i.e. $\underline{a}_{t-1} = (a_0^{(0)}, a_1^{(0)}, \ldots, a_{t-1}^{(0)})$. Thus the vector $\underline{a}_{t-1}$ is neglected in the functions (3.12)-(3.16).

Chapter 4

Proposed optimization algorithm

The optimization algorithm proposed in this chapter is the core of the thesis. The chapter consists of five sections. At first, to understand the basic idea of the proposed algorithm, its original application to American option pricing is introduced in the first three sections. The first two sections provide the technical details and the third a literature review on the application, convergence rate and previously introduced modifications. The fourth section highlights similarities as well as differences between the American option pricing problem and the real-time decision problem. This is followed by the proposal of the enhanced algorithm that is applicable to the real-time decision problems considered in this thesis. The main ideas of this chapter are published in Anders & Nishijima (2011) and Anders & Nishijima (2012).

4.1 Introduction to American option pricing

From a mathematical point of view, the decision problem formulated in the last chapter is similar to the problems of the American option pricing in the field of finance, see Anders & Nishijima (2011).

An *option* is a financial instrument that gives the owner the right to execute (sell or buy) a predefined number of shares of, for example, a stock at a predefined execution price and before or at the predefined time horizon n. An option that can only be executed at the time horizon n is called *European option*; whereas an option that can be exercised at any time before or at the time horizon of the option is called *American option*.[1] The appropriate price

[1]For early references see e.g. Karatzas (1988) or Cox et al. (1979); for a broad overview on option pricing in general see e.g. Hull (2012).

of an American option is identified by comparing the expected (discounted) benefits gained by executing the option and by not executing (i.e. postponing the execution); the maximum value of these two benefits is regarded as the price of the option. In order to assess the expected benefit gained by not executing the option, the prices of the option (i.e. its expected benefits) at future times must be known; thus, a backward induction is required.

Let $\mathbf{Y}_s$ denote the stochastic process with first-order Markov property that represents the price of an underlying asset at time s, on which an American option is defined. In the case where $\mathbf{Y}_s$ is a continuous process in time, as before, it is approximated by discretization $\mathbf{Y}_t$, $t = 0, 1, \ldots, n$. The probabilistic characteristics of the first-order Markov process $\mathbf{Y}_t$ is then characterized by the transition probability density function $f_t(\mathbf{y}_{t+1}|\mathbf{y}_t)$ from $\mathbf{Y}_t = \mathbf{y}_t$ to $\mathbf{Y}_{t+1} = \mathbf{y}_{t+1}$, $t = 0, 1, \ldots, n-1$. The initial state $\mathbf{Y}_0 = \mathbf{y}_0$ is known.

Define the *continuing value function* (CVF) $c_t(\mathbf{y}_t)$ and the *stopping value function* (SVF) $h_t(\mathbf{y}_t)$ respectively for the American option. The CVF $c_t(\mathbf{y}_t)$ represents the expected benefit gained by not executing the option at time t, given the state of the process $\mathbf{Y}_t = \mathbf{y}_t$. The SVF $h_t(\mathbf{y}_t)$ represents the benefit gained by executing the option at time t, given $\mathbf{Y}_t = \mathbf{y}_t$. Typically, the value of $h_t(\mathbf{y}_t)$ can be assumed to be analytically known for all times t given $\mathbf{y}_t$; it is calculated by comparing the execution price and the price $\mathbf{y}_t$ of the underlying asset at time t. The benefit $q_t(\mathbf{y}_t)$ of the American option with any given state $\mathbf{Y}_t = \mathbf{y}_t$ is then written as:

$$q_t(\mathbf{y}_t) = \begin{cases} \max\left\{h_t(\mathbf{y}_t),\ c_t(\mathbf{y}_t)\right\}, & t = 0, 1, \ldots, n-1 \\ h_t(\mathbf{y}_t), & t = n \end{cases} \tag{4.1}$$

and the CVF can be expressed by:

$$c_t(\mathbf{y}_t) = E[q_{t+1}(\mathbf{Y}_{t+1})|\mathbf{Y}_t = \mathbf{y}_t] \tag{4.2}$$

With this setting, the pricing of an American option is formulated as: to identify the maximized expected benefit $q_0(\mathbf{y}_0)$ under the random process $\mathbf{Y}_t$ with the known initial state $\mathbf{Y}_0 = \mathbf{y}_0$. This is similar in the decision problem formulated in Equations (3.9) and (3.10).

For an overview of algorithms available to solve American option pricing it is referred to Glasserman (2004) and Brandimarte (2006).

4.2 Description of LSM method

The main technical problem of the option pricing formulated in the previous section is the evaluation of the conditional expectation in Equation (4.2). In principle, for any given state $\mathbf{Y}_t = \mathbf{y}_t$ at time t ($\leq n-1$) the conditional expected value $E[q_{t+1}(\mathbf{Y}_{t+1})|\mathbf{Y}_t = \mathbf{y}_t]$ can be estimated by *Monte Carlo Simulations* (hereafter abbreviated by MCS) of the underlying process. However, in order to estimate the conditional expected value above, the conditional expected values $E[q_{t+1}(\mathbf{Y}_{t+2})|\mathbf{Y}_{t+1} = \mathbf{y}_{t+1}]$ for the individual realizations $\mathbf{y}_{t+1}$ at time $t+1$, which are simulated by MCS starting from the state $\mathbf{Y}_t = \mathbf{y}_t$, must be evaluated. This requires another set of simulations corresponding to each of the realizations $\mathbf{y}_{t+1}$. Consequently, the total number of the required simulations increases exponentially as a function of the number n, which usually is not computationally feasible. The Least Squares Monte Carlo method (LSM method) by Longstaff & Schwartz (2001) circumvents this, by employing a least squares method. In the LSM method, the CVF $c(t, \mathbf{y}_t)$ are approximated with certain functions for all time steps, and these functions are estimated by regression utilizing a single set of realizations of $\mathbf{Y}_t$ simulated by Monte Carlo method.

Before introducing how this is performed, it is noted that the CVF at time t is a function only of $\mathbf{y}_t$, because the probabilistic characteristics of the underlying first-order Markov process at future times is fully characterized by the state $\mathbf{Y}_t = \mathbf{y}_t$ at time t. In the context of American option pricing, this means that if the price of a stock follows a first-order Markov sequence, the price of its American option is a function only of the current stock price. Therefore, under regular conditions (among others that the CVF is square integrable see Longstaff & Schwartz (2001) and Stentoft (2004b) for detail), the CVF can be represented by an appropriate set of basis functions $L_k(\mathbf{y}_t)$, $k = 0, 1, \ldots$, with respect to the state $\mathbf{Y}_t = \mathbf{y}_t$, i.e.

$$c_t(\mathbf{y}_t) = \sum_{k=1}^{\infty} r_{t,k} L_k(\mathbf{y}_t) \tag{4.3}$$

with the coefficients $r_{t,k}$, $k = 0, 1, \ldots$ for $t = 0, 1, \ldots, n-1$. In the regressions, this is approximated as:

$$c_t(\mathbf{y}_t) \approx \sum_{k=1}^{K} r_{t,k} L_k(\mathbf{y}_t) \tag{4.4}$$

with a finite number K of basis functions.

A list of basis functions can be found in Abramowitz & Stegun (1972, Chapter 22). The coefficients $r_{t,k}$ are estimated using least-squares method; i.e. by minimizing the sum of the squared distances between the observed realizations of the dependent variable $q_{t+1}(\underline{\mathbf{y}}_{t+1})$ in the data set and their fitted values. In matrix form the coefficients are obtained by solving:

$$\mathbf{r}_t = \arg\min_{\mathbf{r}} \|\mathbf{q}_{t+1} - \mathbf{L}_t \mathbf{r}\|_2^2 \tag{4.5}$$

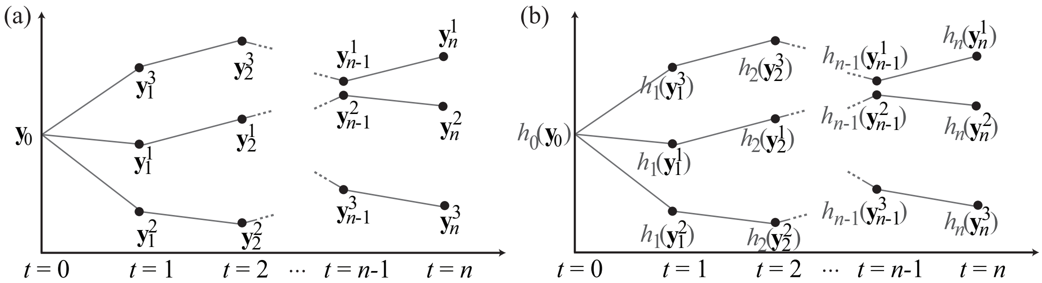

Figure 4.1: *Illustration of (a) the realizations* $\mathbf{y}_t^i$*, and (b) the stopping value functions given the realizations* $\mathbf{y}_t^i$ *of exemplary paths* $i = 1, 2, 3$, $t = 0, 1, \ldots, n$.

where $\mathbf{r}_t = (r_{t,1}, r_{t,2}, \ldots, r_{t,K})$, $\|\cdot\|_2$ denotes the Euclidean norm, $\mathbf{L}_t$ is a $b \times K$ matrix consisting of values of basis functions $\{L_{t,k}(\cdot)\}_{k=1}^K$, which are functions of realizations of $\mathbf{y}_t$, and $\mathbf{q}_{t+1}$ the $b \times 1$ vector of realized MEU in time $t+1$; i.e. $\mathbf{q}_{t+1} = (q_{t+1}(\mathbf{y}_{t+1}^1), q_{t+1}(\mathbf{y}_{t+1}^2), \ldots, q_{t+1}(\mathbf{y}_{t+1}^b))$.

In the following the steps of the algorithm of the LSM method are described:

The **first step** is, by MCS, to generate a set of b independent paths of the random process $\mathbf{Y}_t$ according to the transition density function $f_t(\mathbf{y}_{t+1}|\mathbf{y}_t)$, $t = 0, 1, \ldots, n-1$, with the initial condition $\mathbf{Y}_0 = \mathbf{y}_0$. These realizations of the paths are denoted by $\mathbf{y}^i = (\mathbf{y}_0^i, \mathbf{y}_1^i, \ldots, \mathbf{y}_n^i)$, $i = 1, 2, \ldots, b$. Note that, $\mathbf{y}_0^i = \mathbf{y}_0$ for all paths. Three exemplary paths are illustrated in Figure 4.1(a).

The **second step** is to compute the values of the SVF $h_t(\mathbf{y}_t^i)$ for each realization $\mathbf{y}_t^i$, $t = 0, 1, \ldots, n$ and $i = 1, 2, \ldots, b$; see Figure 4.1(b). In the case of American option pricing, these values are usually directly given through a deterministic function.

The next steps are performed backwards in time, since the MEU is defined backward recursively. The **third step** is to determine the MEU $q_n(\mathbf{y}_n^i)$ at time $t = n$. At time n, the option expires; hence, the realization of the benefit $q_n(\mathbf{y}_n)$ for each individual path is determined by $q_n(\mathbf{y}_n^i) = h_n(\mathbf{y}_n^i)$ according to Equation (4.1).

Having obtained the MEU at time step n, the **fourth step** is to move one time step backward to $t = n-1$. Each realization $q_n(\mathbf{y}_n^i)$ is related to the observation $\mathbf{y}_{n-1}^i$ so that the data set $(\mathbf{y}_{n-1}^i, q_n(\mathbf{y}_n^i))$, $i = 1, 2, \ldots, b$, is obtained. This data set is utilized to approximate

$$c_{n-1}(\mathbf{y}_{n-1}) = E[q_n(\mathbf{Y}_n)|\mathbf{Y}_{n-1} = \mathbf{y}_{n-1}] \tag{4.6}$$

with Equation (4.4) by least squares method, see Figure 4.2. The estimated CVF is denoted by $\hat{c}_{n-1}(\mathbf{y}_{n-1})$. Here, it should be emphasized that the

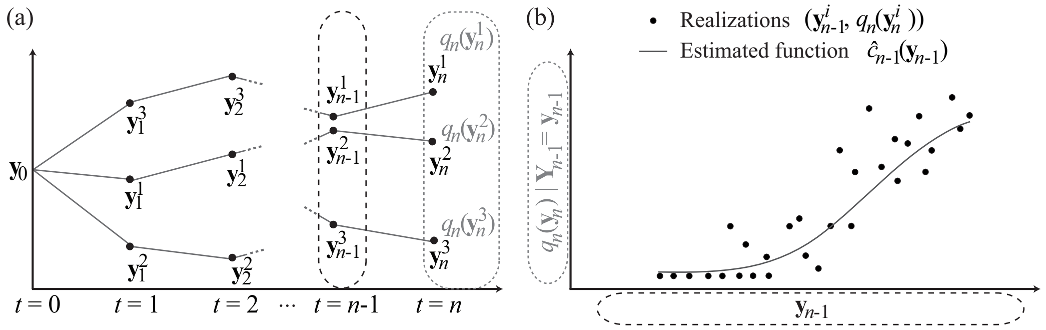

Figure 4.2: *Illustration of the estimation of the continuing value function $c_{n-1}(\mathbf{y}_{n-1})$ given the observations $\mathbf{y}^i_{n-1}$ of the paths $i = 1, 2, \ldots, b$.*

individual realizations $q_n(\mathbf{y}^i_n)$ associated to the realizations $\mathbf{y}^i_{n-1}$ are "shared" to estimate $E[q_n(\mathbf{Y}_n)|\mathbf{Y}_{n-1} = \mathbf{y}^j_{n-1}]$ $(j \neq i)$, but also to interpolate and extrapolate the estimates over the support of $\mathbf{Y}_{n-1}$ where no realizations are available in the simulation.

In the **fifth step** the approximation of the CVF $\hat{c}_{n-1}(\mathbf{y}_{n-1})$ for time step $t = n - 1$ is utilized to determine the realizations of $q_{n-1}(\mathbf{y}_{n-1})$, i.e. $q_{n-1}(\mathbf{y}^i_{n-1})$, according to Equation (4.1):

$$q_{n-1}(\mathbf{y}^i_{n-1}) = \begin{cases} h_{n-1}(\mathbf{y}^i_{n-1}), & \text{if } h_{n-1}(\mathbf{y}^i_{n-1}) > \hat{c}_{n-1}(\mathbf{y}^i_{n-1}) \\ q_n(\mathbf{y}^i_n), & \text{otherwise} \end{cases} \tag{4.7}$$

Moving to time step $n - 2$, each realization $q_{n-1}(\mathbf{y}^i_{n-1})$ is related to a state $\mathbf{y}^i_{n-2}$, such that the data set $(\mathbf{y}^i_{n-2}, q_{n-1}(\mathbf{y}^i_{n-1}))$, $i = 1, 2, \ldots, b$, is obtained. Then, in the same way as at time step $t = n - 1$, the CVF is estimated (denoted by $\hat{c}_{n-2}(\mathbf{y}_{n-2})$) and again according to Equation (4.1) the values of the MEU at time $n - 2$ are determined

$$q_{n-2}(\mathbf{y}^i_{n-2}) = \begin{cases} h_{n-2}(\mathbf{y}^i_{n-2}), & \text{if } h_{n-2}(\mathbf{y}^i_{n-2}) > \hat{c}_{n-2}(\mathbf{y}^i_{n-2}) \\ q_{n-1}(\mathbf{y}^i_{n-1}), & \text{otherwise.} \end{cases} \tag{4.8}$$

This procedure is repeated until time $t = 1$, such that for all paths the maximum expected benefit $q_1(\mathbf{y}^i_1)$ is obtained.

In the final **sixth step** $\hat{c}_0(\mathbf{y}_0)$ is obtained by the average of the realizations $q_1(\mathbf{y}^i_1)$, $i = 1, 2, \ldots, b$, since the initial value of all paths equals $\mathbf{y}_0$:

$$\hat{c}_0(\mathbf{y}_0) = \frac{1}{b} \sum_{i=1}^{b} q_1(\mathbf{y}^i_1) \tag{4.9}$$

Finally the price of the American option $q_0(\mathbf{y}_0)$ is obtained as $\hat{c}_0(\mathbf{y}_0)$, which represents the price of the considered American option.

4.3 Comments to the application of the LSM method

Approaches using Monte Carlo simulation combined with regression methods are proposed by Keane & Wolpin (1994) to solve dynamic programming problems, and Carriere (1996) as well as Tsitsiklis & Van Roy (1997) for American option pricing (Longstaff & Schwartz, 2001). In the literature, it is often mentioned that these methods are similar, yet there are significant differences in the performance as the investigations in Stentoft (2012, 2013) illustrate. The author concludes from his investigations that the LSM method leads to less biased estimators[2], especially in cases where multiple early exercises were considered. Furthermore, the LSM method has been proven to be flexible in the application to other fields where multiple early exercises and/or path dependent stochastic processes are relevant. Examples in which the LSM method worked well include

- The valuation of mortgage-backed securities when the borrowers have to refinance their credits at a certain rate, see Longstaff (2005).
- The pricing of real options[3], see e.g. Gamba (2008) and Alesii (2008).
- For pricing life and pension insurances, see e.g. Andreatta & Corradin (2003), Bacinello (2008), Bacinello et al. (2010), Bauer et al. (2010).
- The measuring and managing of longevity risk, see Boyer & Stentoft (2013).

Besides Longstaff & Schwartz (2001), the convergence properties of the LSM method are investigated in Clément et al. (2002), Moreno & Navas (2003), Glasserman & Yu (2004) and Stentoft (2004a, 2004b). Their findings are reviewed in the following.

Longstaff & Schwartz (2001) and Clément et al. (2002) show that the approximation using the least squares method converges to the true price as the number K of basis functions (regressors) increases and that for a fixed number K of basis functions the approximation of the CVF using MCS converges to the CVF approximated by the basis functions. In Moreno & Navas (2003) as well as Stentoft (2004a) the convergence is analyzed with

[2] An estimator of a parameter is called *biased* if its mean value is different from the true value of the estimated parameter, see e.g. Faber (2012, Chapter 5.7.2), and this difference is called *bias*.

[3] A real option is similarly defined as an option in Finance; it gives the right to the owner to decide on exercising the option in the future. However, a real option is often related to a business decision like an investment. Further details can be found e.g. Schwartz & Trigeorgis (2001).

respect to various types of basis functions, increasing degree of the basis functions and increasing numbers b of paths. Both find that the convergence of the LSM method may not be guaranteed in general.

Glasserman & Yu (2004) find a relationship between the number b of paths and the number K of regressors that has to be fulfilled in order to converge. The relationship is that the number b of paths has to grow exponentially in the number K, considering the worst case convergence for Brownian motions and geometric Brownian motions. This finding is diminished in Stentoft (2004b). It is shown that the increase of b should be polynomial in K if the state space of the underlying random processes is bounded; i.e. for the regression extreme values are neglected. Limiting the state space by neglecting extreme values is the main difference compared to the approach of Glasserman & Yu (2004). Gerhold (2011) points out that this is the reason for the different results of convergence rate, which means that the tails of the distributions of the underlying random processes cause the results found in Glasserman & Yu (2004). The assumption of the bounded state space is a simplification that may have no influence on the result as the lower and upper bound can be chosen arbitrarily small and large respectively. This includes also the smallest or larges value of the sampled realizations of the underlying random processes. Whether this assumption is valid in practice is, according to Gerhold (2011), not clear and further investigations should be conducted to answer the question how strong the truncation effects the convergence rate. However, following the arguments of Stentoft (2004b), he shows that under mild conditions the result of the LSM method converges also in general multi-period settings; these conditions include:

- The state space is bounded.
- The CVF is sufficiently smooth.
- The probability is zero that the value of the CVF and the value of the SVF are equal.

Stentoft (2004b) concludes that in terms of the mean squared error, it is worth to increase the number b of paths, rather than the number K of basis functions.

The investigations of Moreno & Navas (2003) and Stentoft (2004a) show that the result of the LSM method is robust with respect to the choice of the type of basis function and its degree when applied to relatively simple American options. Stentoft (2004a) proposes to use simple polynomials with a relatively low degree (2 or 3), which is efficient considering the computational time needed for the regression procedure. Similar investigations are given in the examples in Chapter 6 of this thesis. The results support the findings that simple polynomials are sufficient.

The results of the applications in Chapter 6 indicate a bias. The bias is also observed in the case of American option pricing and is investigated, for example, in Stentoft (2004a, 2013). In Stentoft (2004a) two types of sources are identified:

- The approximations of the CVF result in a low bias (i.e. $E[\hat{q}_0] \leq q_0$); this bias should vanish as the number of regressors increases.
- Using the same paths to estimate the CVF and to calculate the SVF results in a high bias (i.e. $E[\hat{q}_0] \geq q_0$); this should vanish as the number of paths increases.

The resulting bias is in general not known and depends on the number b of paths and the number K of regressors (Stentoft, 2004a). In order to reduce the bias, Glasserman & Yu (2004) propose to use an independent set of MCS to estimate the CVF at each decision step, which leads, however, to a higher computational demand.

As mentioned above the method is relatively robust and flexible, which allows for modifications while preserving the convergence properties. Among others the following modifications (compared to the original algorithm proposed by Longstaff & Schwartz (2001)) are proposed in the literature:

- Use of all paths for the estimation of the CVF.
- Application of variance reduction methods for the simulation of the independent paths.
- Generalization of the underlying random processes to GARCH and to Lévy processes.
- Application of different types of regression methods to estimate the CVF.

These modifications are introduced briefly in the following. In the original LSM method, as proposed by Longstaff & Schwartz (2001), only "in-the-money" paths are used in the regression to approximate the CVF at each time step. The "in-the-money" paths denote those paths that have a strictly positive value at the considered time step. Longstaff & Schwartz (2001) use only these values for the estimation of the CVF since the decision whether to exercise the option at time t or not is only relevant if the option has a positive value. This is equivalent to introducing a lower bound to the region over which the CVF must be estimated and reduces computational time. In this thesis all paths are used, which is in accordance with the investigations of Clément et al. (2002), Bacinello (2008), Stentoft (2013).

In order to improve the LSM method with regard to the rate of convergence the application of variance reduction methods is investigated among others by Lemieux & La (2005) and Areal et al. (2008) for American option pricing. Lemieux & La (2005) apply important sampling, control variates as well as randomized quasi-Monte Carlo methods. The authors use one example for which it turns out that, whereas randomized quasi-Monte Carlo methods consistently reduce the variance, for importance sampling and a control variate the reduction is sensitive to the parameters used in the example. Further investigations are provided in Areal et al. (2008) by means of several examples as well as various types of basis functions. Implementing several quasi-Monte Carlo methods, Areal et al. (2008) find similar results as Lemieux & La (2005); i.e. the low-discrepancy sequences (Halton (1960), Niederreiter (1992) and Sobol (1967) sequences) provide better results. They also applied the low discrepancy sequences introduced in Faure (1982), but without obtaining an improvement in the convergence rate. Besides that, they consider the method of moments, control variates, Brownian bridge and combinations of the considered methods. In general, it turns out that a relatively specific combination of these methods is preferable for simple options. In the examples considered in Chapter 6 only the method of antithetic variates[4] is applied as suggested in Longstaff & Schwartz (2001) and is straightforward to implement. Nevertheless, it is assumed that implementing, for example, quasi-Monte Carlo methods or randomized quasi-Monte Carlo methods may lead to an improvement if well chosen.

An extension to the LSM with regard to the underlying random processes has been introduced first in Stentoft (2005) to processes with a time-varying volatility in form of GARCH processes[5], in Stentoft (2008) to processes with a time-varying volatility with conditional skewness and leptokurtosis (GARCH models with normal inverse Gaussian distribution) and in Gerhold (2011) to Lévy processes[6]. The idea of the present thesis to apply the LSM method to time-dependent higher-order Markov processes was built upon the state

[4]The definition of antithetic random variables can be found for example in Kroese et al. (2011): a pair (X, X^*) is antithetic if X and X^* have the same distribution and are negatively correlated; antithetic estimator $1/N \sum_{i=1}^{N/2}(X_i + X_i^*)$ is unbiased with variance $Var(X)(1 + \rho_{X,X^*})/N$ where ρ_{X,X^*} is the correlation of X and X^*.

[5]Generalized Autoregressive Conditional Heteroskedastic processes introduced by Bollerslev (1986).

[6]Lévy processes "are essentially stochastic processes with stationary and independent increments" Applebaum (2009, Chapter 1); examples include Brownian motion, Poisson, Gamma and Normal inverse Gaussian processes.

of knowledge of the first two articles; the latter confirms that also other stochastic processes like Poisson processes and Gamma processes lead to the convergence of the algorithm.

Kohler & Krzyzak (2012) and to some extent Areal et al. (2008) consider various different regression methods when the LSM method is applied to price American options with simple Markov processes. For instance, Kohler & Krzyzak (2012) use non-parametric regression estimates like the least squares estimate with complexity penalties including spline, neural networks, smoothing splines and orthogonal series estimates. They mention that the bias may decrease by using these methods, but the corresponding computational time is increased significantly (it is in the range of hours) compared to the computational time of the original LSM algorithm, which is in the range of minutes. Considering the significant increase of computational cost, these methods are not considered in this thesis.

4.4 Similarities and differences to the real-time decision problem

By comparing the characteristics of the procedure for pricing an American option described in Section 4.1 in practice with those for making real-time decisions described in Section 3.1, the following similarities are found:

- Decisions have to be made fast in accordance with the accessibility of information.
- The decisions are made any time in a predefined finite time frame.
- The decisions are affected by underlying Markov processes.
- At each decision phase the procedure to determine the price of an American option (cf. Equation (4.1)) is equivalent to the procedure to determine the MEU (cf. Equation (3.12)); i.e. the price of an American option when it is exercised and the price when it is not are compared.

In the last mentioned item the choice to exercise the American option corresponds to the choice of making a terminal decision and the associated price function corresponds to the SVF. The choice not to exercise the American option corresponds to taking decision $a^{(0)}$ (to postpone) and the associated price function corresponds to the CVF as formulated in Equation (3.10) where all the future prices have to be known.

Besides these similarities, there are also differences between the settings of American option pricing and real-time decision problems:

- In American option pricing the underlying random phenomena are in general characterized by stationary, first-order Markov processes, which is not necessarily the case for real-time decision problems.
- In American option pricing the realizations of the SVF are known, which is in general not the case for real-time decision problems in engineering applications.
- In American option pricing the set of terminal decisions comprises only two alternatives: to execute or not to execute the option; whereas in real-time decision problems more decision alternatives can be relevant.

The second difference is especially important, since this implies that further uncertainties are involved in the solution and further computations are required to estimate the SVF.

4.5 Proposal of eLSM

This section presents the extensions to the LSM method for the purpose of applying the LSM method to the real-time decision framework. It can be used to find the optimal decision in the face of an emerging natural hazard event in (near) real time.

The proposed algorithm is developed over two subsequent steps. The first step is to introduce the necessary extensions to overcome the differences. These extensions result in a first adapted version of the LSM method (Anders & Nishijima, 2011). The first version is presented in the first part of this section. However, as it is found that the computational performance of the first version is not sufficient, a further enhancement is applied. This enhanced version of the adapted LSM method is also the final version that was introduced in Anders & Nishijima (2012) and is presented in the second part.

Part 1: Extensions of LSM for real time decision problems

As mentioned in Section 4.4, real-time decision problems in the face of emerging natural hazard events and the pricing of American options differ mainly in the following two settings: (1) the underlying random processes may not be characterized by first-order Markov processes and (2) the SVF, i.e., the expected utilities corresponding to terminal decisions, are not analytically known and the evaluation of the stopping values often requires MCS.

Concerning the first difference, the LSM method can be straightforwardly extended: by approximating the CVF defined by Equation (4.2) with a set of basis functions whose arguments include all the states of the underlying processes to the extent that these states can fully characterize the probabilistic characteristics of the processes in the future.

The second difference, concerns the fact that in the real-time decision problems of consideration the SVF cannot be evaluated analytically, unlike in the case of American option pricing. Therefore, in the first version of the extended LSM method MCS are introduced to estimate the SVF, see Anders & Nishijima (2011). These additional MCS are computationally expensive if a large number of MCS is required in order to converge. The additional computational effort increases proportional to n. However, in the examples presented in Chapter 6 of this thesis, it is found that one additional simulation may be sufficient in order to estimate the SVF, but to the expense of a greater bias. The reason why one additional information may be sufficient is that the information of the simulated and estimated values of the SVF of different paths is "shared" in two ways: (1) the simulated values of the SVF are used in the least squares regression for estimating the CVF and (2) at the initial time step, the average of all b MEU (i.e. implicitly also the simulated values of the SVF) at time $t = 1$ are taken to estimate $c_0(\mathbf{y}_0)$. Hence, precise estimates of the individual SVF are not necessary.

The implementation of the extensions to the LSM algorithm modify the steps as introduced in Section 4.2 as follows:

- Similar to the **first step**, the set of b independent realizations (paths) of the random sequence $\mathbf{Y}_t$ is generated by MCS. Note that, the simulation is performed according to the transition density function $f_t(\mathbf{y}_{t+1}|\underline{\mathbf{e}}_t)$, $t = 0, 1, \ldots, n-1$, which is conditional on the information available. The information at time t includes all relevant historical information; i.e. if $\mathbf{Y}_t$ is a Markov process of order s, the relevant information includes $\underline{\mathbf{e}}_t = (\mathbf{e}_{t-s+1}, \mathbf{e}_{t-s+2}, \ldots, \mathbf{e}_t)$. The initial conditions are assumed to be known $\underline{\mathbf{Y}}_0 = \underline{\mathbf{y}}_0$.

- In the **second step**, the SVF is not analytically known, here the SVF is estimated using additional MCS for each of the realizations $\{\mathbf{y}_t^i\}_{t=0}^n$, $i = 1, 2, \ldots, b$, see Figure 4.3.

- The **third, fourth and fifth step** are analog to the steps introduced in Section 4.2. except that the SVF estimates that are obtained by the Monte Carlo method are used.

- The **sixth step** is to determine the value $c_0(\mathbf{y}_0)$, which is equal to $q_0(\mathbf{y}_0)$. In the real-time decision framework the aim is not only to determine the value $c_0(\mathbf{y}_0)$ but also to identify which decision is optimal.

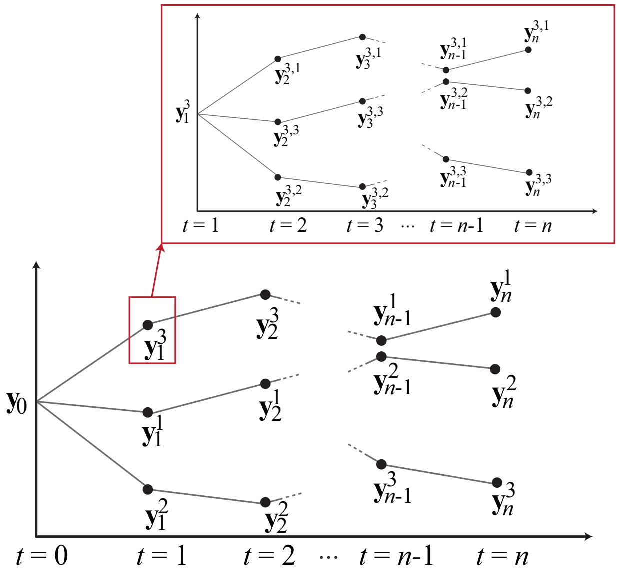

Figure 4.3: *Three paths of the underlying random sequence* $\mathbf{Y}_t$ *with an exemplary set of additional MCS to estimate the value of* $h_1(\mathbf{y}_1^3)$.

The optimal decision is identified by comparing $h_0(\mathbf{y}_0)$ with $c_0(\mathbf{y}_0)$. The SVF for the extended LSM is defined according to Equation (3.14), i.e.

$$\hat{h}_{0,\mathrm{MCM}}(\underline{\mathbf{y}}_0) = \max_{a \in A_0^{(s)}} l_{0,\mathrm{MCM}}(a, \underline{\mathbf{y}}_0) \tag{4.10}$$

In case $\hat{h}_{0,\mathrm{MCM}}(\underline{\mathbf{y}}_0) > \hat{c}_{0,\mathrm{MCM}}(\underline{\mathbf{y}}_0)$, it is optimal to stop the decision procedure and take the best terminal decision; i.e. the one that returns $\hat{h}_{0,\mathrm{MCM}}(\underline{\mathbf{y}}_0)$. In case $\hat{h}_{0,\mathrm{MCM}}(\underline{\mathbf{y}}_0) \leq \hat{c}_{0,\mathrm{MCM}}(\underline{\mathbf{y}}_0)$, it is optimal to continue with the decision process and wait for further information. The abbreviation "MCM" in the index of the functions denotes that the functions are estimated using the extended LSM method described above.

Part 2: Enhanced LSM - an enhanced version of the extended LSM

As mentioned before, in Step 2 of the extended LSM additional MCS are required to estimate the SVF. These additional MCS are, nevertheless, computationally expensive and the computational effort increases proportional to the numbers n and b. The enhanced LSM method (hereafter abbreviated by eLSM method) circumvents this by applying the least squares method also for the estimation of the SVF. The general idea is explained in the following.

As before the SVF $h_{t,\mathrm{eLSM}}(\underline{\mathbf{y}}_t)$ of the eLSM is defined as maximum of the conditional expected utilities $l_{t,\mathrm{eLSM}}(a_t^{(j)}, \underline{\mathbf{y}}_t)$ with respect to the terminal decisions $a_t^{(j)} \in A_t^{(s)}$, cf. Equations (3.14) and (3.15). Unlike the approach described in Part 1, now the functions $l_{t,\text{eLSM}}(a_t^{(j)}, \underline{\mathbf{y}}_t)$ are estimated using the least squares method, similar to the estimation of the CVF described in Section 4.2; i.e. given the realizations $\{\underline{\mathbf{y}}_t^i\}_{i=1}^b$ the functions $l_{t,\text{eLSM}}(a_t^{(j)}, \underline{\mathbf{y}}_t)$ are approximated by linear combination of basis functions $\{L_{t,k}(\cdot)\}_{k=1}^K$ and unknown coefficients $r_{t,k}^{(j)}$, $j = 1, 2, \ldots, m$:

$$l_{t,\text{eLSM}}(a_t^{(j)}, \underline{\mathbf{y}}_t) \cong \sum\nolimits_{k=1}^{K} L_{t,k}(\underline{\mathbf{y}}_t) r_{t,k}^{(j)}. \tag{4.11}$$

The coefficients $\mathbf{r}_t^{(j)} = (r_{t,1}^{(j)}, r_{t,2}^{(j)}, \ldots, r_{t,K}^{(j)})^T$ are estimated using least squares method, which is given by the following equation in the matrix form:

$$\mathbf{r}_t^{(j)} = \arg\min_{\mathbf{r}} \|\mathbf{u}_t^{(j)} - \mathbf{L}_t \mathbf{r}\|_2^2 \tag{4.12}$$

where $\mathbf{L}_t$ is the same $b \times K$ matrix consisting of values of basis functions $\{L_{t,k}(\cdot)\}_{k=1}^K$ as used for the estimation of $c_t(\underline{\mathbf{y}}_t)$ in Equation (4.4). However, instead of the MEU, here the dependent variable is $\mathbf{u}_t^{(j)}$, which denotes the $b \times 1$ vector of observed future utilities $u_t(a_t^{(j)}, \mathbf{z}^i)$, $i = 1, 2, \ldots, b$, given the realization $\mathbf{z}^i$ of the state of nature (as a function of the hazard index) related to the path i and given the decision $a_t^{(j)}$ is made at time t. Note that $u_t(a_t^{(j)}, \mathbf{z}^i)$ is a realization of $l_{t,\text{eLSM}}(a_t^{(j)}, \underline{\mathbf{y}}_t^i)$. The realization $\mathbf{z}^1$ of the state of nature related to the path $i = 1$ is illustrated in Figure 4.4(a). Figure 4.4(b) illustrates the functional relation between the observations $\underline{\mathbf{y}}_t^1$ in path $i = 1$ and the observed future utilities $u_t(a_t^{(j)}, \mathbf{z}^1)$ for $t = 0, 1, \ldots, n$; where $\mathbf{z}^1$ is a function of $\{\underline{\mathbf{y}}_t^1\}_{t=0}^n$.

Let $\mathbf{Z}$ be defined by Equation (3.8); i.e. $\mathbf{Z}$ is scalar and equal to one if the hazard index Z_t exceeds the threshold $\tilde{z}$ for $t \in [0, n]$, and Z is equal to zero otherwise. In case $Z_s < \tilde{z}$ for all $s \in [0, t]$, it follows that Z is still unknown at time t. Let $u_t(a_t^{(j)}, z^i)$ represent the utility that is realized when decision $a_t^{(j)}$ is made at time t and the future realizations of path i are given by $\{\mathbf{Y}_s^i\}_{s=t+1}^n = \{\mathbf{y}_s^i\}_{s=t+1}^n$. As described in Section 3.3, given the realizations $\{z_s^i\}_{s=0}^t$ the realization of the random variable Z in path i is a function of the future realizations $\{z_s^i\}_{s=t+1}^n$ of the hazard index Z_t. The hazard index is characterized, in turn, by the future realizations $\{\mathbf{y}_t^i\}$ of the underlying random phenomena Y_t. Based on the set of realizations $\{\mathbf{y}_t^i\}_{t=0}^n$ illustrated in Figure 4.4(a), Figure 4.4(c) represents the realizations involved in the estimation of the SVF for $t = 1$ and Figure 4.4(d) illustrates the least squares method in order to approximate the function $l_{1,\text{eLSM}}(a_1^{(j)}, \underline{\mathbf{y}}_1)$.

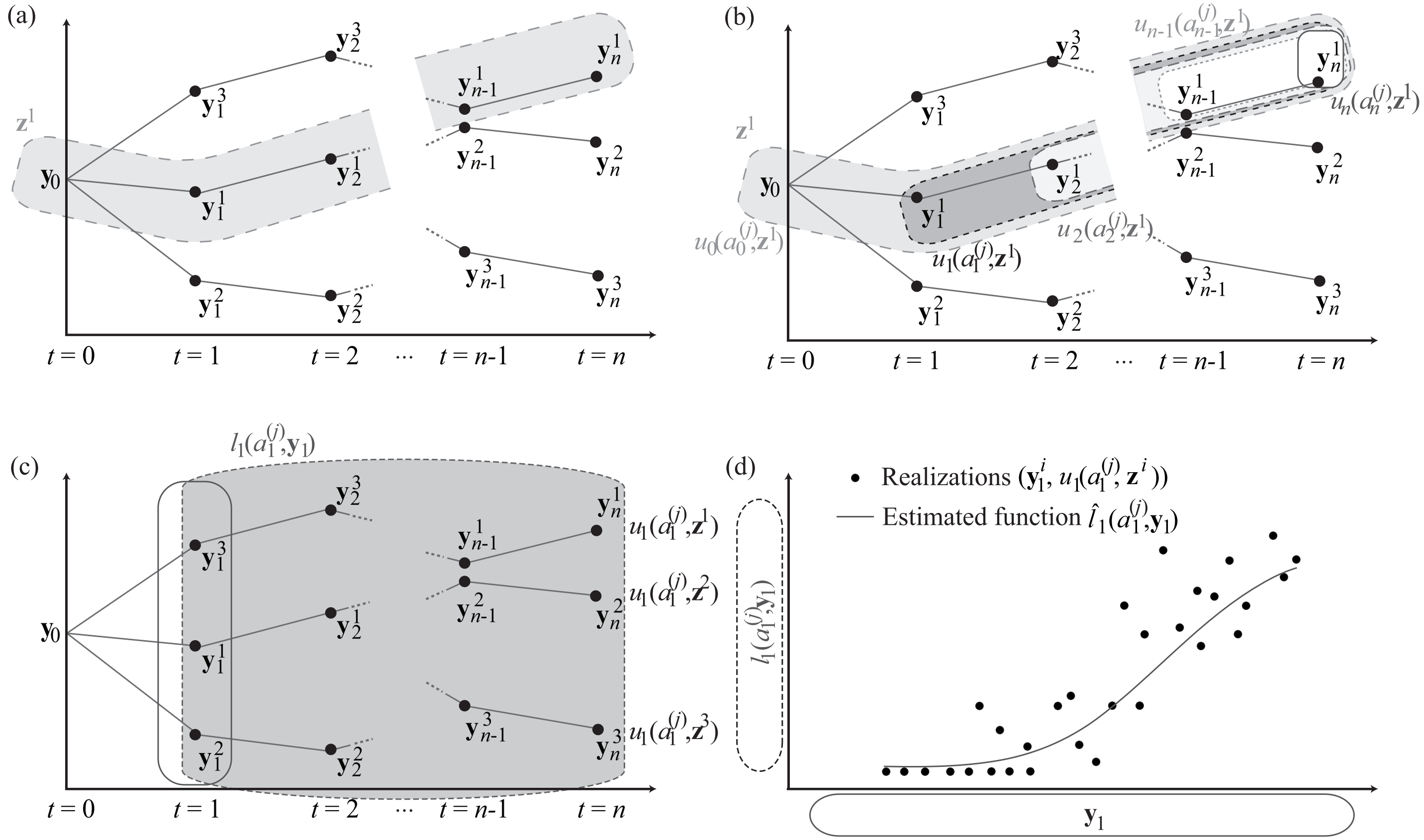

Figure 4.4: *Illustration of (a) a realization $\mathbf{z}^i$ of the state of nature for the path $i = 1$; (b) functional relation between the observations $\underline{\mathbf{y}}_t^i$ for path $i = 1$ together with the realizations of future utilities $u_t(\mathbf{z}^1, a_t^{(j)})$, $t = 0, 1, \ldots, n$; (c) realizations involved in the estimation of the SVF for $t = 1$; and (d) least squares method for the approximation of the function $l_{1,eLSM}(a_1^{(j)}, \underline{\mathbf{y}}_1)$.*

In order to avoid to introduce a bias by using the least squares estimates within the determination of the MEU, Equation (4.8) is changed to:

$$q_{t,\text{eLSM}}(\underline{\mathbf{y}}_t^i) = \begin{cases} u_t^*(a_t^*, \mathbf{z}^i), & \text{if } \hat{h}_{t,\text{eLSM}}(\underline{\mathbf{y}}_t^i) > \hat{c}_{t,\text{eLSM}}(\underline{\mathbf{y}}_t^i) \\ q_{t+1,\text{eLSM}}(\underline{\mathbf{y}}_{t+1}^i), & \text{otherwise} \end{cases} \tag{4.13}$$

where $u_t^*(a_t^*, \mathbf{z}^i)$ is the observed future utility of path i for the optimal terminal decision a_t^*.

Chapter 5

Implementation of the decision support system

In the previous chapters the real-time decision framework with its mathematical formulation as well as an efficient optimization algorithm are introduced. This chapter proposes a scheme for a decision support system (DSS) and represents the second main contribution. The scheme provides the interrelations between the different components evolved in the decision process. Each component is described in detail in the corresponding section. The scheme is hereafter used to structure the implementation of the various components required in the application examples.

5.1 Structure of the decision support system

The structure of the proposed DSS is illustrated by the flowchart shown in Figure 5.1. The flowchart presents the main components (i.e. the modules representing the units for the related algorithms and the decision alternatives available) as well as their interfaces. In the present context, the scheme of the DSS is designed for one hazard event. This can be extended to multi-hazard events as described in Schmidt et al. (2011) or as it is embedded in the HAZUS software provided by the Federal Emergency Management Agency (FEMA) of the United States of America (FEMA, 2004). Note that both software tools are designed for decisions for risk mitigation and emergency or recovery planning; i.e. for the risk assessment far before a hazard impact or after the impact and not for real-time decision support in the face of an emerging hazard event.

The modules of the DSS are the hazard, the consequence and the optimizations module; these are described briefly in the following.

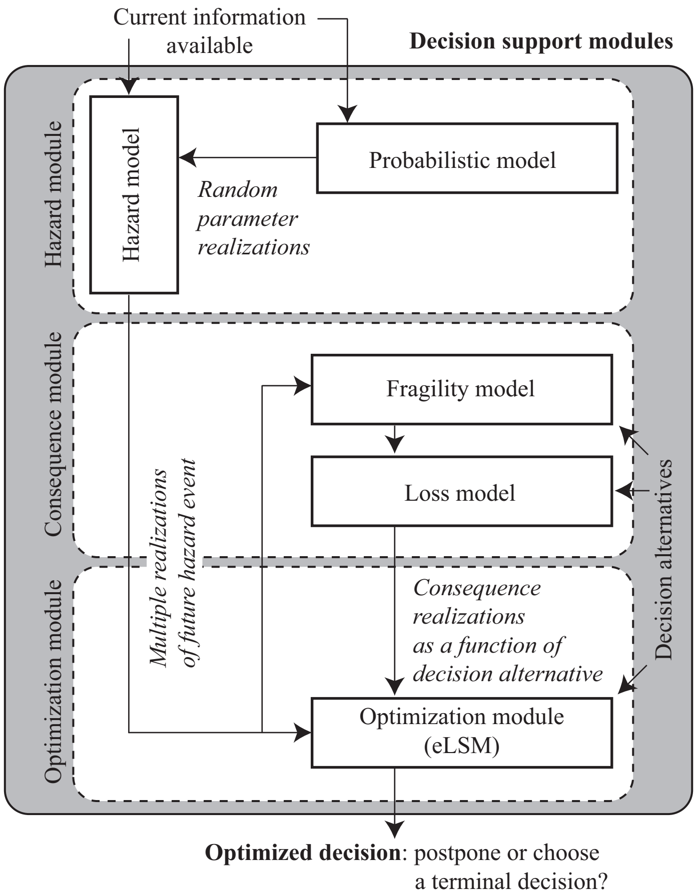

Figure 5.1: *Representation of the decision support system with the natural hazard module, consequence module and the optimization module.*

The *hazard module* for a specific hazard event consists of the following elements:

- Input parameters: current information on the state of the natural hazard event; location of the considered engineering system; area that is likely to be affected; considered time horizon n; number b of simulations; number of considered time steps; and if applicable information about parameters like the roughness length, sea surface temperature and other hazard specific parameters
- Hazard models: physical and/or statistical functions describing the future development of the hazard event
- Probabilistic models: functions characterizing aleatory and epistemic uncertainties involved in the hazard models
- Output parameters: multiple realizations of the future hazard event (i.e. the realizations $\{\mathbf{y}_t^i\}_{t=0,1,\ldots,n}^{i=1,2,\ldots,b}$ of the underlying random variables and related realizations $\{\mathbf{z}_t^i\}_{t=0,1,\ldots,n}^{i=1,2,\ldots,b}$ of the hazard index representing the intensity of the hazard event)

The *consequence module* consists of the following elements:

- Input parameters: multiple realizations of the future hazard event; the decision alternatives available; and the relevant specific information about the performance of the considered engineering system
- Fragility model: fragility functions describing the probability that a certain degree of damage (or failure state) is exceeded as a function of the hazard intensity
- Loss model: consequences as a function of the degree of damage caused in the hazard event and the choice of decision alternative
- Probabilistic models: functions characterizing the uncertainties involved in the fragility and loss model
- Output parameters: realizations of the future state of nature (occurrence of an impact) and the consequences/losses related to the future intensities of the hazard event

The *optimization module* consists of the following elements:

- Input parameters: multiple realizations of the future hazard event; decision alternatives available; realizations of the consequences (as a function of the decision alternatives and the hazard realizations); and relevant specific information for the optimization method (e.g. the choice of regression method, type and degree of basis functions, relevant variables for the estimation of the SVF and CVF)

- Optimization method: e.g. the proposed eLSM
- Output parameter: optimal decision

In Figure 5.1 the fact that in this thesis only the hazard module includes uncertainties is shown by leaving out the probabilistic models in the fragility and the loss model. The probabilistic models can be included straightforward in the other modules, which may be relevant for practical applications.

Figure 5.2 illustrates a scheme of the proposed eLSM algorithm when implemented within the DSS. The figure facilitates the understanding of the structure of the algorithm including the hazard and the consequence model. The algorithm processes the realizations provided by these models for the estimation of the SVF and the CVF. By backward induction the values of the estimated functions are compared to obtain the MEU for each path and considered time step. At the initial time step t the optimal decision is obtained based on the current information and the available models.

5.2 Hazard modeling

The hazard module includes all models that describe the random processes underlying the decision problem and may have a physical impact on the considered engineering system. The natural hazard processes considered in the thesis have been introduced before; examples are storm, avalanche, flood, wild fire and ash clouds. In a more general engineering context, hazard processes such as the progress of corrosion within a structure, the quality of a product or the uncertain cost of a construction project could be considered; however, these processes need to have the characteristics listed in Section 3.1.

In principle, there are two different approaches for probabilistic modeling; the so-called pure statistical approach and the engineering approach. Note that this differentiation is not strict, i.e. the engineering approach may involve in addition to physical models also statistical analysis. Whereas the pure statistical approach relies extensively on the statistical analysis modeling the natural hazard event in probabilistic manners. The pure statistical approach assumes a direct relationship between the observed data and the model prediction. By regression analysis the distribution of the process to be modeled is estimated using possible distribution functions. This approach is mainly applied in cases where the detailed physical mechanics are not well understood or too complex to treat in a practical model. Engineering approaches try to overcome the obvious drawbacks of the pure statistical approach such as (i) direct observations of extreme events are rare; (ii) observations may not be available at the desired location; and (iii) scientific

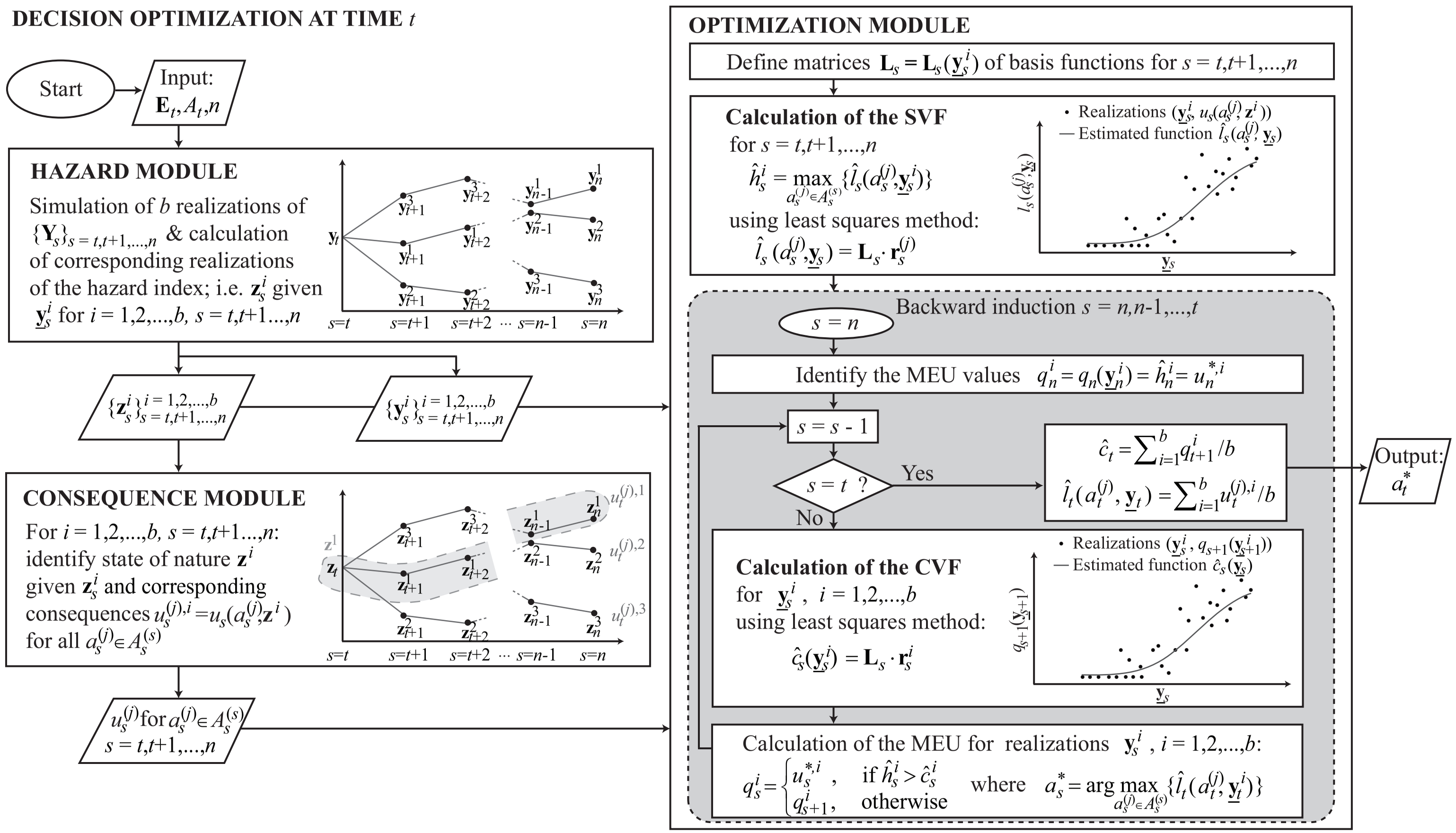

Figure 5.2: *Detailed flowchart of the algorithm for implementing the natural hazard module, consequence module and the optimization module. The following notation is used:* $u_t^{(j),i} = u_t(a_t^{(j)}, \mathbf{z}^i)$, $u_t^{*,i} = u_t(a_t^*, \mathbf{z}^i)$, $f_t^i = f_t(\underline{\mathbf{y}}_t^i)$ *for any function* $f_t(\cdot)$ *and* a_t^* *denotes the optimal decision at time* t.

knowledge and/or engineering experience cannot be included directly. For this shift an important work is provided by Cornell (1968) in the field of seismic hazard assessment. An example of a typhoon model based on an engineering approach for the northwest Pacific region has been introduced by Graf et al. (2009), which can be used for the estimation of statistics of insured portfolio losses in the insurance industry. Further examples of combined models for tropical cyclones are run by NHC, e.g. the Statistical Hurricane Intensity Prediction Scheme (SHIPS), cf. Schumacher et al. (2013). Another example for an engineered approach is provided by Straub & Schubert (2008), they provide a framework for the risk assessment of rock-fall hazards. Other software tools which facilitate decision making in regard e.g. design of structures and hazard mitigation planning are available in public domains. In the United States of America FEMA provides a suite of software tools called HAZUS (FEMA, 2004), which facilitates to estimate losses due to earthquake, hurricane and flood events in the United States of America see http://www.fema.gov/plan/prevent/hazus. Another such software is RiskScape that provides a framework for the risk assessment of several types of natural hazards in New Zealand, see https://riskscape.niwa.co.nz. A number of engineering approaches and corresponding software tools for different natural hazards have been developed if not fully quantitative; see e.g. Gruber (1998) for avalanches and see e.g. Crosta & Agliardi (2003) for rock-falls.

Whichever approach is chosen, the components involved in the modeling of possible future states of the hazard event can be subdivided with regard to two objectives; these are (1) deterministic forecast models that provide the future mean development of the hazard event, or its median, and (2) probabilistic models that represent the uncertainties of the corresponding deterministic forecast model, as illustrated in Figure 5.1. The uncertainties include aleatory and epistemic uncertainties. Probabilistic models are also introduced when the deterministic forecast models consist of physical functions in order to incorporate the forecast uncertainties, as the available physical functions do not capture the true development of the hazard event. The realizations generated with the hazard models describe possible future intensities of the hazard event to which the corresponding loads acting on the considered engineering system can be assigned. Under these additional loads the engineering system may experience damage or even collapse.

5.3 Consequence modeling

On the basis of the outputs from the hazard module (i.e. the realizations of the hazard index and the associated loads) the consequence module provides

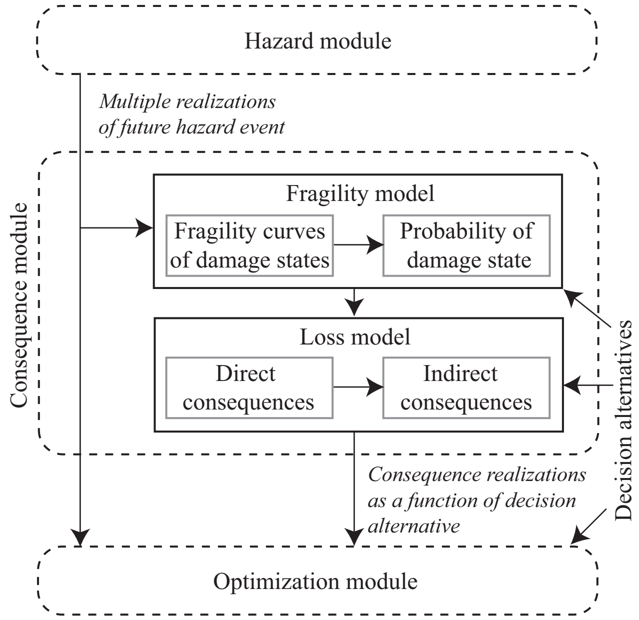

Figure 5.3: *Representation of the fragility and the loss model within the consequence module.*

the estimates of the expected total consequences related to each realization and decision alternative. The resulting consequences depend on the state of the hazard index, the choice of action and the resistance of the considered engineering system.

The components of the consequence module are represented in Figure 5.3. It includes the fragility and the loss model. The fragility model describes the resistance of the considered engineering system when exposed to a hazard event. The loss model links the damage state (or failure state) of the engineering system to the resulting consequences (casualties and/or economic loss).

As mentioned in Section 2.1.5 the resulting consequences can be distinguished, amongst others, between direct and indirect consequences. This distinction is resumed in this section. In the following, the principal description on how to calculate the expected direct and indirect consequences is presented in accordance with Faber et al. (2007b). The following description is explained by means of one hazard event with intensity Z_t; although hazard events occur often combined and should be considered so, in general.

Let the engineering system consist of n_{CON} constituents such as physical components or human activities. Each constituent is associated to a damage state C_i, which in turn is related to direct consequences $c_{\mathrm{D}}(C_i)$. The state space of the damage state is defined by a set of n_{C_i} discrete, mutually exclusive states $c_{i,j}$, $i = 1, 2, \ldots, n_{\text{CON}}$, $j = 1, 2, \ldots, n_{C_i}$, which can be extended to the continuous case.

The probability of the direct consequences $c_{\text{D}}(\mathbf{C}_l)$ that are related to the l^{th} damage state $\mathbf{C}_l$ of all n_{CSTA} combinations of damage states of all constituents of the engineering system, conditional on the hazard intensity Z_t, is characterized by the conditional probability $P[\mathbf{C}_l|Z_t]$. The risk associated to the direct consequences that result from the l^{th} damage state is equal to $c_D(\mathbf{C}_l)P[\mathbf{C}_l|Z_t]$. The risk resulting from all possible direct consequences defines the vulnerability of an engineering system. It is obtained by the following equation

$$R_D = E_Z\left[\sum_{l=1}^{n_{\text{CSTA}}} c_D(\mathbf{C}_l)P[\mathbf{C}_l|Z_t]\right] \tag{5.1}$$

where $E_Z[\cdot]$ denotes the expectation with respect to the probability distribution of the random intensity Z_t of the hazard event. The conditional probability $P[\mathbf{C}_l|Z_t]$ can be computed by using so-called *fragility functions*. A fragility function of a constituent represents the probability that the constituent is at least in a certain damage state given the hazard intensity. It is basically a cumulative distribution function representing the conditional probability of the damage state (Stewart & Melchers, 1997, Chapter 4). Three exemplary fragility functions are illustrated in Figure 5.4. The associated conditional probabilities $P[\mathbf{C}_l|Z_t]$ are illustrated in Figure 5.4(b) for one constituent that is characterized by four damage states, where the damage state $c_{i,0}$ denotes the state when no damage occurs and $c_{i,f}$ the state when the constituent collapses.

Note that the performance of the considered engineering system is not deterministic; implying that the threshold when a facility is in a certain damage state or collapses is uncertain. The uncertainty involved in the estimation of the fragility function is illustrated in Ravindra (1990). A typical choice to describe fragility functions is to use a log-normal distribution function.

Indirect consequences $c_{\text{ID}}(S_m, c_{\text{D}}(\mathbf{C}_l))$ are a function of the system state S_m associated to the indirect consequences and the direct consequences resulting from the damage state $\mathbf{C}_l$ of the engineering system. Let n_{SSTA} denote the number of possible system states. The probability $P[S_m|\mathbf{C}_l, Z_t]$ characterizes the indirect consequences given the damage state $\mathbf{C}_l$ and the hazard intensity Z_t. The associated conditional risk is $c_{\text{ID}}(S_m, c_{\text{D}}(\mathbf{C}_l))P[S_m|\mathbf{C}_l, Z_t]$. The conditional risk is used to obtain the risk due to indirect consequences, which is given by the following equation

$$R_{ID} = E_Z\left[\sum_{l=1}^{n_{\text{CSTA}}}\sum_{m=1}^{n_{\text{SSTA}}} c_{\text{ID}}(S_m, c_{\text{D}}(\mathbf{C}_l))P[S_m|\mathbf{C}_l, Z_t]P[\mathbf{C}_l|Z_t]\right] \tag{5.2}$$

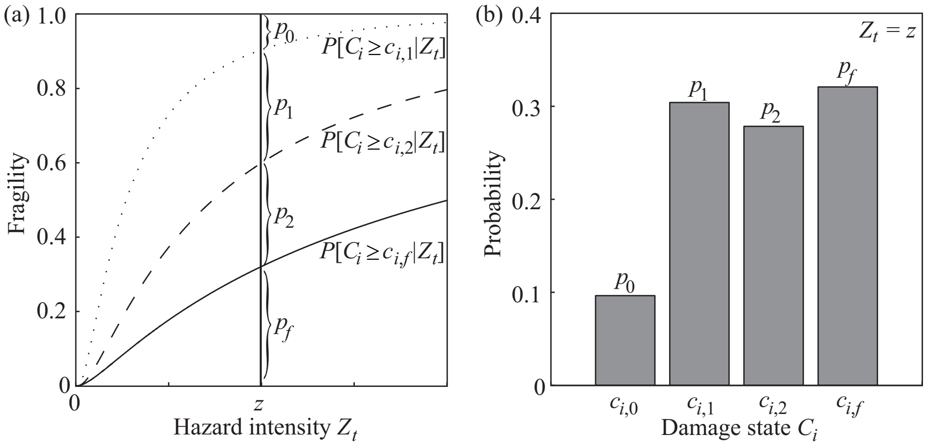

Figure 5.4: *Illustration of (a) three fragility functions for three damage states $c_{i,j}$, $j \in \{1, 2, f\}$ and (b) the probabilities p_j that the engineering system is in damage state $c_{i,j}$, $j \in \{0, 1, 2, f\}$, conditional on $Z_t = z$. The damage state $c_{i,0}$ denotes the case where the engineering system has no damage and $c_{i,f}$ the case where it fails.*

In the following a general approach for the estimation of consequences is presented when the fragility function is available. Let $\eta = (\eta_1, \eta_2, \ldots, \eta_{n_{CON}})^T$ denote the vector of inventory associated to the considered engineering system. It is assumed that to each constituent one element of inventory can be related. Further, for each element η_i the corresponding value of loss $V_{i,j} = V(\eta_i, c_{i,j})$ can be determined given the constituent is in damage state $c_{i,j}$. Then the direct consequences $c_D(\mathbf{C}_l)$ can be estimated by

$$c_D(\mathbf{C}_l) = \sum_{i=1}^{n_{CON}} V_{i,\pi_i(l)} \tag{5.3}$$

where $\pi_i(l)$ denotes the index of the damage state of constituent i corresponding to the l^{th} damage state $\mathbf{C}_l$ of all n_{CSTA} combinations of damage states of all constituents of the engineering system. This representation requires that the consequences (i.e. the values $V_{i,j}$, $i = 1, 2, \ldots, n_{\text{CON}}$, $j = 1, 2, \ldots, n_{C_i}$) have the same units, which is usually monetary units. In general, it is recommended to monetize the value of the consequences, which is straightforward for goods that are traded but not for others such as human life or the environment. For methods to determine the monetary value of goods that are not traded it is referred to Schubert (2009).

The examples in Chapter 6 do not consider explicitly a certain type of fragility function nor uncertainties within the consequence model. Instead fairly crude models are postulated to emphasize the application of the optimization algorithm. Within the models, it is assumed that only two damage states exist, these are either "no damage" or "collapse". The two states are distinguished through a boundary level $\tilde{z}$ of the intensity of the hazard; i.e. either the hazard index stays below $\tilde{z}$ then there is no damage or it exceeds $\tilde{z}$ then the system collapses. Given the damage state the value of realized consequences depends further on the choice of decisions made before. Examples of similar assumptions for the consequence model can be found for example in Katz & Murphy (1997, Chapter 6). Therein sequential decision making based on two weather and climate states (adverse or not adverse) is considered. Further, in these examples a lower and an upper boundary of the adverse consequences are assumed. Such an assumption can be made for many engineering applications which ensures the square integrability of the CVF. In cases where such boundaries cannot be defined, often the consequence model can be formulated with functions that are square integrable. The square integrability of the CVF is a requirement in order to apply the LSM approach as it is presented in this thesis.

5.4 Optimization method

The optimization module is basically the core of the decision support system. It provides the optimization algorithm returning the optimized decision and various interfaces to the hazard module, the consequence module and the set of decision alternatives, see Figure 5.2. In this thesis the implemented optimization algorithm is the proposed eLSM. However, depending on the assumptions on the conditions, defining the decision problem, other algorithms can be applied to the framework. Possible examples that are also applied in the context of American option pricing include the *Stochastic Mesh Method* by Broadie & Glasserman (2004) or the regression method proposed by Tsitsiklis & Van Roy (2001). However, it seems to be difficult to find a similar flexible algorithm with the formulation and underlying assumptions so that it can handle non-stationary higher order Markov-processes.

Chapter 6

Applications

This chapter illustrates the application of the real-time decision framework to practical examples. The procedure how to implement the hazard, the consequence and the optimization module is demonstrated by means of two examples. The efficiency of the proposed algorithm is also demonstrated with the application examples. The results support the idea that the proposed framework is useful in practice.

6.1 Real-time evacuation decisions in the face of increasing avalanche risk

The aim of this section is twofold: (1) to illustrate the application of the extended LSM as well as the eLSM and (2) to evaluate the performance of the eLSM compared to the performance of the extended LSM. For this purpose, a decision situation is considered in which a decision maker has to decide whether to order the evacuation of people in the face of an avalanche hazard, see Anders & Nishijima (2012).

6.1.1 Problem setting

Consider a village located nearby a mountain slope having a critical angle for snow avalanches. Given the prevailing winter conditions and critical snow heights, a decision has to be made whether to evacuate people from the village or not. Assume that the occurrence of a severe avalanche, causing significant damages to the village, depends only on the additional snow height S_t; i.e. S_t is the hazard index. Further, if S_t exceeds the threshold $\tilde{s}$ ($= 800$ [mm]) a severe avalanche occurs.

The weather forecast by a meteorological agency predicts that snowfall is likely within the next hours, which increases the likelihood of the occurrence of an avalanche. However, the duration and the intensity of the snowfall are uncertain. New information becomes available every 8 hours from the meteorological agency; i.e. the time interval between the subsequent decision phases is set to 8 hours ($dt = 8$). At each decision phase a decision is made according to the information available. Three decision alternatives are assumed: to evacuate the people $a^{(1)}$, not to evacuate $a^{(2)}$, and to wait $a^{(0)}$. It is assumed that the evacuation takes 16 hours to complete.

6.1.2 Hazard model

Based on the idea in Floyer & McClung (2003) the model to forecast whether a day is an avalanche or a non-avalanche day is formulated as a function of the amount of new precipitation, see Section 2.3.3. The model for precipitation is adapted from a rainfall model developed by Hyndman & Grunwald (2000). From this rainfall model, the hypothetical probabilistic snowfall model is obtained by adjusting the amount of rainfall with the ratio of the water density to the snow density.

Let X_t denote the random sequence representing the amount of snowfall in the time period $(t - dt, t]$. Hereafter, this time period is denoted by $(t-1, t]$ (i.e. the time unit is $dt = 8$) and thus $\{X_t\}_{t=0}^{n}$ for simplicity. The distribution of X_t is a mixture comprising a discrete component concentrated at $x_t = 0$ and a continuous component for $x_t > 0$. The discrete component of X_t represents the non-occurrence of snowfall and is characterized by the Bernoulli sequence J_t, whose conditional probability function is:

$$\begin{aligned} \pi_t(\mathbf{y}_{t-1}, \mathbf{y}_{t-2}) &= P[J_t = 1 | \mathbf{Y}_{t-1} = \mathbf{y}_{t-1}, \mathbf{Y}_{t-1} = \mathbf{y}_{t-2}] \\ &= l(\mu_t(\mathbf{y}_{t-1}, \mathbf{y}_{t-2})) \end{aligned} \tag{6.1}$$

where $\mathbf{Y}_t = (J_t, X_t)$ and $l(\cdot)$ denotes the logit function, which is defined as $l(\mu) = \exp(\mu)/(1 + \exp(\mu))$ if $\mu > 0$ and $l(\mu) = 0$ otherwise, and

$$\begin{aligned} \mu_t(\mathbf{y}_{t-1}, \mathbf{y}_{t-2}) = &\ \alpha_0 + \alpha_1 j_{t-1} + \alpha_2 j_{t-2} + \alpha_3 \log(x_{t-1} + c_1) \\ &+ \alpha_4 \log(x_{t-2} + c_2) + \alpha_5 t^2 \end{aligned} \tag{6.2}$$

The continuous component of X_t is strictly positive and characterizes the intensity of the snowfall. If $J_t = 1$, X_t is described by the continuous conditional density $g_t(x|\mathbf{y}_{t-1})$, $x > 0$. $g_t(\cdot|\cdot)$ follows the Gamma distribution with shape parameter κ and mean $\nu_t(\mathbf{y}_{t-1})$, where

$$\log(\nu_t(\mathbf{y}_{t-1})) = \beta_0 + \beta_1 j_{t-1} + \beta_2 \log(x_{t-1} + c_3) + \beta_3 t^2 \tag{6.3}$$

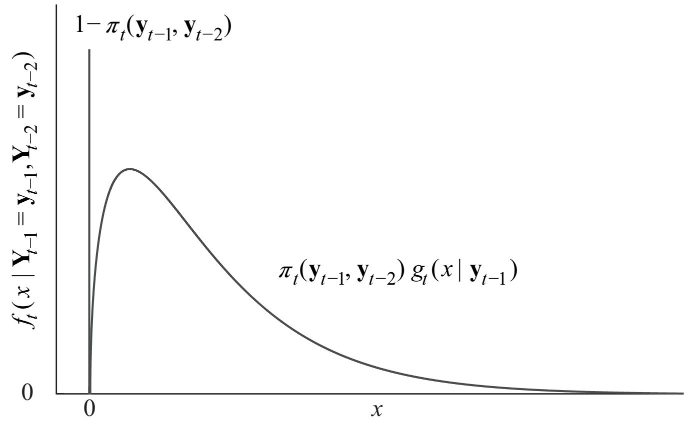

Figure 6.1: *Illustration of the probability density function $f_t(x|\mathbf{y}_{t-1}, \mathbf{y}_{t-2})$.*

Then the transition probability density function of X_t is defined as (see Figure 6.1):

$$f_t(x_t|\mathbf{y}_{t-1}, \mathbf{y}_{t-2}) = (1 - \pi_t(\mathbf{y}_{t-1}, \mathbf{y}_{t-2}))\delta_0(x_t) + \pi_t(\mathbf{y}_{t-1}, \mathbf{y}_{t-2})g_t(x_t|\mathbf{y}_{t-1}) \tag{6.4}$$

where δ_0 is the Dirac delta function.

The additional snow height is obtained by multiplying the snow intensity by the factor F_s, which accounts for the density of the snow; i.e.

$$S_t = S_t(\underline{\mathbf{y}}_t) = \sum_{s=0}^{t} F_s x_t \mathbf{1}_{\{j_s=1\}} = S_{t-1} + F_t x_t \mathbf{1}_{\{j_t=1\}} \tag{6.5}$$

where $\mathbf{1}_{\{j=1\}}$ denotes the indicator function or characteristic function which is equal to 1 in the case $j = 1$ and equal to 0 otherwise. Hence, S_t (the hazard index) at time t is characterized by the index S_{t-1} at time $t-1$ and a stochastic process composed of a second- and a first-order Markov process (X_t and J_t, respectively, in the second term in the rightmost equation of Equation 6.5). The values of the parameters of the model are summarized in Table 6.1. The time frame is set to three days; i.e. $n = 9$.

Table 6.1: *Parameters of the probabilistic snowfall model.*

Parameter	Value
(j_{-1}, j_0, S_0)	(0, 0, 0)
$\mathbf{c} = (c_1, c_2, c_3)$	$(0.15, 0.30, 0.5)$
$\alpha = (\alpha_0, \alpha_1, ..., \alpha_5)$	$(4.50, 0.26, 0.10, 0.50, 0.05, -0.20)$
κ	1.50
$\beta = (\beta_0, \beta_1, \beta_2, \beta_3)$	$(1.95, -0.20, 0.25, -0.04)$
F_s	10

6.1.3 Consequence model

The consequences are postulated as follows, see also Table 6.2. The consequence is equal to the evacuation cost $C_{Ev} = 1$ in two cases: (1) when the evacuation has been initiated but the avalanche does not occur, and (2) when the evacuation is completed before the avalanche occurs. The consequence $C_D = 10$ representing the total cost of damage is incurred if the avalanche occurs and the people are not evacuated or the evacuation was initiated too late and thus is not completed. No consequences arise only in the case when no evacuation is initiated and no avalanche occurs. The postulated consequence model ensures that the CVF is finite and thus square integrable, as the minimum consequences are zero and the maximum consequences are $C_D = 10$.

Table 6.2: *Conditions and associated consequences postulated in the consequence model.*

	Additional snow height S_t in the time period $[0, t]$	
People	$S_t > \tilde{s} = 800[mm]$	$S_t \leqslant \tilde{s} = 800[mm]$
Not evacuated	$C_D = 10$	0
Evacuated	$C_{Ev} = 1$	$C_{Ev} = 1$

6.1.4 Decision optimization

Here, the MEU in Equation (3.12) is defined by the expected consequences; i.e. the minimum operator is used and the inequality signs in Equation (4.7) and (4.13) are turned. The steps of the optimization algorithm are executed with the extended LSM and the eLSM (described in Section 4.5 Part 1 and Part 2, respectively) to obtain the optimal decision.

Step 1: By MCS, generate b independent realizations of $\{\mathbf{Y}_t\}_{t=1}^{n}$ and $\mathbf{S}^i = (S_0^i, S_1^i, ..., S_n^i)$, $i = 1, 2, ..., b$, where $S_t^i = S_t^i(\underline{\mathbf{y}}_t^i)$ and $\mathbf{y}_t^i = (j_t^i, x_t^i)$. As noted in Section 4.3 the realizations are obtained using the method of antithetic variates. The realizations $\mathbf{y}_1^i, \mathbf{y}_2^i, ..., \mathbf{y}_n^i$ are simulated according to the probability density functions in Equations (6.1) and (6.4); the paths are denoted by $\mathbf{y}^i = (\mathbf{y}_{-1}^i, \mathbf{y}_0^i, ..., \mathbf{y}_n^i)$ with the following initial values: $\mathbf{y}_{-1}^i = \mathbf{y}_{-1} = (0,0)$, $\mathbf{y}_0^i = \mathbf{y}_0 = (0,0)$ (see Table 6.1) for $i = 1, 2, \ldots, b$.

Step 2: For each $\mathbf{y}_t^i$ the value $h_t^i = h_t(\mathbf{y}_t^i, \mathbf{y}_{t-1}^i)$ of the SVF is estimated. At time $n = 9$ the consequence related to each realization and decision is assumed to be known; i.e. either s_n^i exceeds the threshold $\tilde{s}$ or not, thus $h_{n,\text{MCM}}^i = h_{n,\text{eLSM}}^i$ for all i. Further, for $t = 1, 2, ..., n-1$

(a) *with the extended LSM method*: Simulation of additional M paths $\underline{\mathbf{y}}_t^{i,m} = (\mathbf{y}_{-1}^i, ..., \mathbf{y}_t^i, \mathbf{y}_{t+1}^{i,m}, ..., \mathbf{y}_n^{i,m})$, $m = 1, 2, ..., M$, for which the observed consequences $u_t(\mathbf{s}^{i,m}, a_t^{(j)})$, $j = 1, 2$, are determined. Here $\mathbf{s}^{i,m}$ is the realization of the additional snow height related to the path realization $\underline{\mathbf{y}}_t^{i,m}$. Define

$$\hat{l}_{t,\text{MCM}}(a_t^{(j)}, \mathbf{y}_t^i, \mathbf{y}_{t-1}^i) = \frac{1}{M}\sum_{m=1}^{M} u_t(\mathbf{s}^{i,m}, a_t^{(j)}) \tag{6.6}$$

then

$$\hat{h}_{t,\text{MCM}}^i = \min\{\hat{l}_{t,\text{MCM}}(a_t^{(1)}, \mathbf{y}_t^i, \mathbf{y}_{t-1}^i), \hat{l}_{t,\text{MCM}}(a_t^{(2)}, \mathbf{y}_t^i, \mathbf{y}_{t-1}^i)\} \tag{6.7}$$

(b) *with the eLSM method*: Define

$$\hat{h}_{t,\text{eLSM}}^i = \min\{\hat{l}_{t,\text{eLSM}}(a_t^{(1)}, \mathbf{y}_t^i, \mathbf{y}_{t-1}^i), \hat{l}_{t,\text{eLSM}}(a_t^{(2)}, \mathbf{y}_t^i, \mathbf{y}_{t-1}^i)\} \tag{6.8}$$

where $\hat{l}_{t,\text{eLSM}}(a_t^{(j)}, \mathbf{y}_t^i, \mathbf{y}_{t-1}^i) = \mathbf{L}_t^i \cdot \mathbf{r}_t^{(j)}, j = 1, 2$. The vector $\mathbf{r}_t^{(j)}$ of the coefficients related to $a_t^{(j)}$ is computed by Equation (4.12). $\mathbf{L}_t^i$ denotes the i^{th} row of matrix $\mathbf{L}_t$; $\mathbf{L}_t$ consists of values of basis functions with arguments $\mathbf{y}_t$, $\mathbf{y}_{t-1}$ and S_t; e.g. for 1st order linear basis functions

$$\mathbf{L}_t = \begin{bmatrix} 1 & x_t^1 & x_{t-1}^1 & s_t^1 \\ 1 & x_t^2 & x_{t-1}^2 & s_t^2 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_t^b & x_{t-1}^b & s_t^b \end{bmatrix}. \tag{6.9}$$

For $t = 0$ set $\hat{l}_0^{(j)} = \hat{l}_{0,\text{MCM}}(a_0^{(j)}, \mathbf{y}_0, \mathbf{y}_{-1}) = \hat{l}_{0,\text{eLSM}}(a_0^{(j)}, \mathbf{y}_0, \mathbf{y}_{-1}) = \sum_{i=1}^{b} u_0(\mathbf{s}^i, a_0^{(j)})/b$, $j = 1, 2$.

Step 3: Starting at time n, for both LSM approaches, the values of $q_{n,\text{MCM}}^i = q_{n,\text{MCM}}(\mathbf{y}_n^i, \mathbf{y}_{n-1}^i)$ and $q_{n,\text{eLSM}}^i$ are set equal to $h_{n,\text{MCM}}^i$ and $h_{n,\text{eLSM}}^i$ respectively, for all i.

Step 4: Moving to time $n-1$ the values of $c_{n-1}(\mathbf{y}_{n-1}, \mathbf{y}_{n-2})$ are similarly estimated for both approaches using the least squares method as described in Section 4.2.

Step 5: Then, for each path i the values of $q_{n-1}(\mathbf{y}_{n-1}, \mathbf{y}_{n-2})$ are determined:

(a) *with the extended LSM method*:

$$q_{n-1,\text{MCM}}^i = \begin{cases} \hat{h}_{n-1,\text{MCM}}^i, & \text{if } \hat{h}_{n-1,\text{MCM}}^i < \hat{c}_{n-1,\text{MCM}}^i \\ q_{n,\text{MCM}}^i, & \text{otherwise} \end{cases} \tag{6.10}$$

(b) *with the eLSM method*:

$$q_{n-1,\text{eLSM}}^i = \begin{cases} u_{n-1}^{*,i}, & \text{if } \hat{h}_{n-1,\text{eLSM}}^i < \hat{c}_{n-1,\text{eLSM}}^i \\ q_{n,\text{eLSM}}^i, & \text{otherwise} \end{cases} \tag{6.11}$$

where $u_{n-1}^{*,i}$ denotes the observed future consequence in path i for the optimal terminal decision a_{n-1}^*.

Moving another time step backward the same procedure is repeated. This is continued until time $t = 1$ and for each path $q_{1,\text{MCM}}^i$ and $q_{1,\text{eLSM}}^i$ are determined.

Step 6: Execute Step 6 as it is described in Section 4.5 to determine the optimal decision at time $t = 0$: first compute

$$\hat{c}_{0,\cdot} = \frac{1}{b}\sum_{i=1}^{b} q_{1,\cdot}^i \tag{6.12}$$

$$\hat{h}_{0,\cdot} = \min\{\hat{l}_{0,\cdot}^{(1)}, \hat{l}_{0,\cdot}^{(2)}\} \tag{6.13}$$

then compare $\hat{h}_{0,\cdot}$ and $\hat{c}_{0,\cdot}$; in case $\hat{h}_{0,\cdot} < \hat{c}_{0,\cdot}$ it is optimal to make a terminal decision, otherwise it is optimal to continue and wait until the next time step when new information becomes available.

6.1.5 Results

To evaluate the performance of the eLSM method compared to the extended LSM method, both approaches are applied to solve the decision problem of the example. The optimal decision at the initial time is obtained by estimating the expected consequences for the three decision alternatives. Various types and degrees of basis functions are implemented; e.g. linear, Legendre and Chebyshev polynomials. Applying these basis functions, it is found that the results do not significantly differ. Thus, only the results obtained with linear basis functions are presented. Figures 6.2 and 6.3 illustrates the findings for different parameter settings of the LSM methods. Therein, Figure 6.2 shows for increasing number b of paths, $b = \{10^2, 3 \cdot 10^2, 10^3, 3 \cdot 10^3, 10^4, 3 \cdot 10^4, 10^5\}$, the convergence of the consequence estimates for the three decisions. For each b the estimates are calculated by the average of 100 computations of the indicated method. To be able to compare the results 100 different yet fixed sets of random numbers are used to generate the paths in Step 1. Hence, the estimates for the terminal decisions ($a_0^{(1)}$ and $a_0^{(2)}$) are identical for all methods; they are presented by solid lines with circles respectively squares. The following results are obtained for $b = 10^5$: $\hat{l}_0^{(1)} = 1.0192$, $\hat{l}_0^{(2)} = 0.8969$ and $\hat{c}_{0,\mathrm{eLSM}} = 0.8055$ with the eLSM method. The optimal decision is $a_0^{(0)}$, which is independent of the choice of LSM method; see Figure 6.2 . Further, the figure shows that the estimate $\hat{c}_0$ obtained by the extended LSM method with $M = 10$ is biased. Therefore it is not considered in Figure 6.3 which illustrates the convergence rate in terms of the coefficient of variation (COV) of the estimates $\hat{c}_0$ as a function of the computational time [sec]. The figure shows a significant improvement with the eLSM method in terms of computational time; a reduction up to the factor of 100. The improvement of the computational time of the eLSM method compared to the extended LSM method with only one additional MCS comes from the fact that, for each realization $\mathbf{y}_t^i$, $t = 1, 2, \ldots, n$ and $i = 1, 2, \ldots, b$, an additional path $\mathbf{y}_s^{i,m}$, $s = t, t+1, \ldots, n$ is generated; resulting in $bn(n-1)/2$ additional MCS.

6.1.6 Illustration of application

An application of the proposed approach in practice is presented in Figures 6.4 and 6.5. Figure 6.4 illustrates a hypothetical time series of the additional snow height $\{S_t\}_{t=0}^{6}$ where the threshold $\tilde{s}$ is exceeded within the time interval $(3, 4]$. The eLSM method is applied subsequently for each time step in order to compute the expected consequences for each decision alternative. The development of the three expected consequences is illustrated in Figure 6.5.

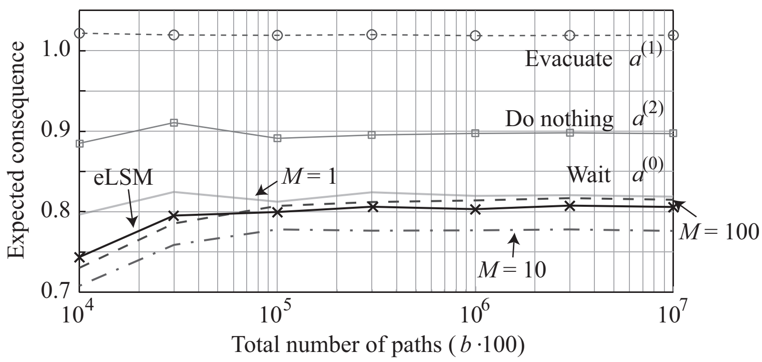

Figure 6.2: *Comparison of the results of the extended LSM method (with various numbers M of additional MCS) and eLSM method. Convergence of the average expected consequences with increasing total number of paths; the average of 100 independent realizations is presented.*

In this figure, it can be seen that the optimal decision at time $t = 0$ is $a_0^{(0)}$, whereas at time $t = 1$ it is $a_1^{(1)}$ given that the snow height increases up to time $t = 1$ as shown in Figure 6.4.

6.1.7 Discussion

In this example, the *enhancement of the extended LSM* (abbreviated by eLSM) method is applied in the context of real-time decision problems for evacuation in the face of an avalanche event. It is found that the eLSM method significantly improves the computational efficiency compared to the extended LSM method; by the factor up to 100.

6.2 Real-time operational decisions for an off-shore platform in the face of an emerging typhoon

The example investigated in Section 6.1 applies a simple statistical hazard model in order to illustrate the essence of the idea of the eLSM method. The following example studies the application of the eLSM method, in case a more complex hazard model is applied; that is a combination of physical functions and non-stationary higher-order Markov models.

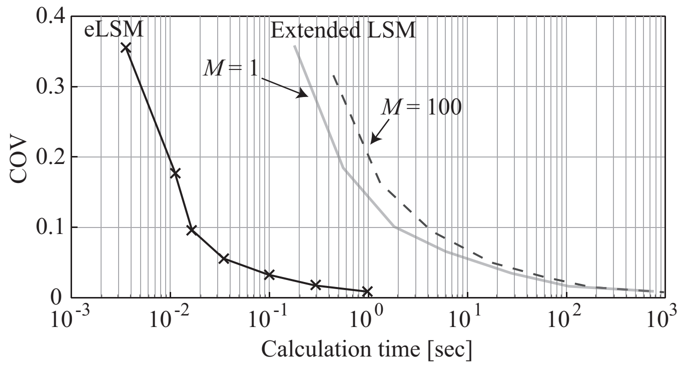

Figure 6.3: *Comparison of the results of the extended LSM method (with various numbers M of additional MCS) and eLSM method for the decision to "wait" and the terminal decisions. The expected consequence related to the terminal decisions is for all algorithms identical, as the same set of Monte Carlo realizations in Step 1 is used. Illustration of the decreasing COV of $\hat{c}_0$ related to the increasing calculation time for one LSM computation as the number b of paths increases.*

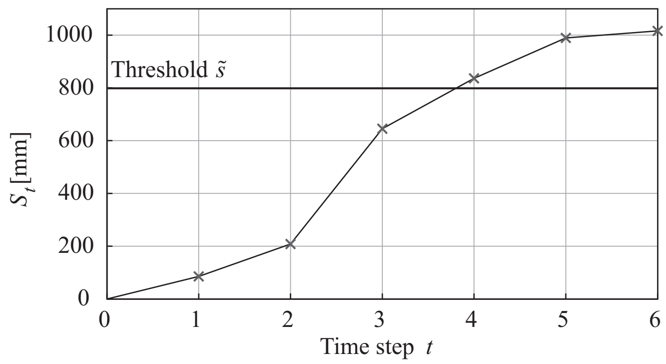

Figure 6.4: *A hypothetical time series of S_t for which the expected consequence of the three decision alternatives are estimated.*

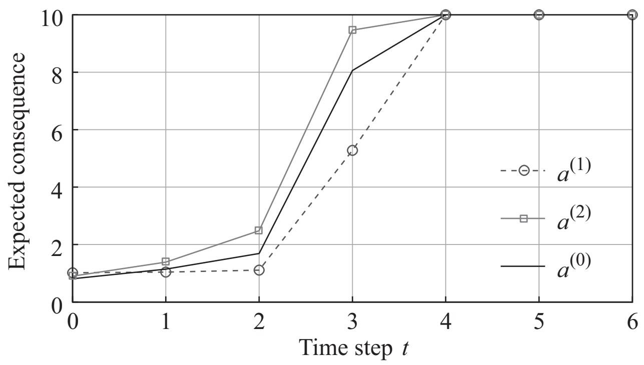

Figure 6.5: *The time series of the estimated expected consequences of the three decision alternatives corresponding to the hypothetical time series of* S_t*, cf. Figure 6.4. The estimates are calculated with the eLSM method and* $b = 10^5$.

6.2.1 Problem setting

A decision maker is faced to decide whether or not the operation of an offshore platform should be shut down in the emergence of a typhoon event. The possible decision alternatives are the terminal decisions of shut-down $a^{(1)}$, no shut-down $a^{(2)}$ and postponing the terminal decision $a^{(0)}$. When the decision maker chooses $a^{(0)}$, she can obtain further information on the state of the typhoon such as position, central pressure, translation speed and direction of the typhoon. The information is assumed to be provided by a meteorological agency once every six hours at no cost. It is assumed that the shut-down of the operation of the platform takes twelve hours after the terminal decision $a^{(1)}$ is made. In the decision problem considered here, it is assumed that the decision is terminated within 30 hours. The time frame is discretized into five time intervals of six hours; i.e. there are six time steps where information becomes available and the decisions are made. This assumption seems reasonable, since the typhoon is very likely to pass through the area relevant for the platform until the 6^{th} time step ($t = 5$), see Nishijima et al. (2009). The initial conditions, assumed in the example, are summarized in Table 6.3. Figure 6.6 illustrates the initial conditions of the emerging typhoon. It further shows the decision process as time goes by with an exemplary typhoon track.

Since the time frame of this decision problem is relatively short, discounting is not considered. In what follows, the models employed and further assumptions are explained.

Table 6.3: *Assumed initial conditions.*

Parameters	Values
Central pressures at $t = -2, -1, 0$	$930, 930, 930$ [hPa]
Translation speeds at $t = 0$	20 [km/h]
Translation angles at $t = -1, 0$	0 [°], 0 [°] (Northwards)
Position at $t = 0$	(129°E, 28°N)
SST at the location of the typhoon at $t = 0$	27.9 [°C]
Radius of max. wind speed, R_M	100 [km]
Location of the platform	(130.3°E, 31.25°N)

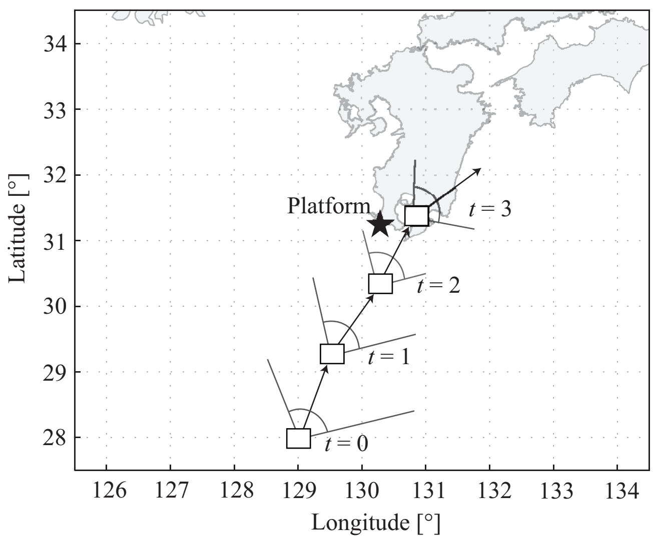

Figure 6.6: *Illustration of the transition of the typhoon and the location of the platform (after Nishijima et al. (2009)).*

6.2.2 Hazard model

The typhoon model developed by the group of Risk and Safety at ETH Zurich is employed for the modeling of the wind speed at the platform induced by the typhoon; see Graf et al. (2009). The typhoon model is composed of five components; occurrence model, transition model, wind field model, surface friction model and vulnerability model. For this example, only the transition model, the wind field model and the surface model are of relevance. In the following, a short summary of the transition model employed in the example is provided in order to show the probabilistic characteristics of the random processes underlying the decision problem. The transition model describes the transition of the state of a typhoon probabilistically. It is assumed that the state of a typhoon is characterized by three parameters: V_t representing the translation speed [km/h]; Γ_t the translation angle [°] and $P_{C,t}$ the central pressure [hPa]. These parameters are modeled by the following components of a vector Markov process:

$$\ln(V_{t+1}) = a_1 + (1 + a_2)\ln(V_t) + a_3\Gamma_t + \varepsilon_{V,t+1} \tag{6.14}$$

$$\Gamma_{t+1} = b_1 + (1 + b_3)\Gamma_t + b_2 V_t + b_4\Gamma_{t-1} + \varepsilon_{\Gamma,t+1} \tag{6.15}$$

$$P_{C,t+1} = c_1 + c_2 P_{C,t} + c_3 P_{C,t-1} + c_4 P_{C,t-2} + c_5 T_t + c_6(T_{t+1} - T_t) + \varepsilon_{P_C,t+1} \tag{6.16}$$

where T_t is the sea surface temperature (SST) at the location of the typhoon at time t. The coefficient vectors $\mathbf{a} = (a_1, a_2, a_3)^T$, $\mathbf{b} = (b_1, b_2, b_3, b_4)^T$ and $\mathbf{c} = (c_1, c_2, \ldots, c_6)^T$, as well as the distribution of the random terms $\varepsilon_{V,t+1}$, $\varepsilon_{\Gamma,t+1}$ (both are modeled by normal distributions with mean zero and standard deviations $\sigma_{V,t+1}$ and $\sigma_{\Gamma,t+1}$ respectively) and $\varepsilon_{P_C,t+1}$ (modeled by an empirical distribution) are estimated using historical data. Therein, in order to incorporate the spatial and temporal inhomogeneity of the probabilistic characteristics of the typhoon transition, these coefficients and the distributions are estimated for each individual grid area (5° latitude-by-5° longitude grids in the northwest Pacific) and for each month. The hazard index, which is in this example the 10-minute sustained wind speed u [m/s] at the platform, is calculated by the wind field model and the surface friction model. Given the state of the typhoon together with the radius of maximum wind speed R_M and the roughness length z_0 of the location of the platform, the wind speed u is calculated deterministically. Note here that the random process underlying the decision problem is expressed by $\mathbf{Y}_t = (V_t, \Gamma_t, P_{C,t})^T$ at times t, $t = 0, 1, ..., n$, and it is assumed that the precise state of the typhoon is known at each time, hence, the information $\mathbf{E}_t$ about the state of the typhoon is equal to $\mathbf{Y}_t$.

Table 6.4: *Conditions and associated costs postulated in the consequence model.*

Platform	Wind speed $u > u_c = 38[m/s]$	$u \leqslant u_c = 38[m/s]$
In operation	$C_D = 10$	0
Not in operation	$C_{PI} = 1$	$C_{PI} = 1$

6.2.3 Consequence model

The platform is assumed to be damaged if the 10-minute sustained wind speed u at the surface of the location of the platform exceeds the threshold u_c $(= 38[m/s])$, while the platform is in operation. The expected damage cost C_D is equal to 10. The platform is assumed not to be damaged if the wind speed does not exceed the threshold, or if the operation of the platform is successfully shut down, i.e. not in operation when the wind speed exceeds the threshold. However, in the latter case the cost C_{PI} arises for production interruption. Here C_{PI} is set equal to 1. The summary of the assumed consequence model is shown in Table 6.4.

Three cases are possible in which the expected damage cost C_D is incurred; the first case is the case where the decision $a^{(2)}$ is made and the wind speed exceeds the threshold u_c, the second case is where the decision $a^{(1)}$ is made but the wind speed exceeds u_c before the shut-down is completed, and the last case is the case where the decision $a^{(0)}$ is made and the wind speed exceeds u_c before the next time a decision is made. No consequences arise if and only if the decision $a^{(2)}$ is made and the wind speed does not exceed the threshold. Remember that until time n, either action $a^{(1)}$ or $a^{(2)}$ has to be chosen. The expected cost C_{PI} for production interruption is incurred if the decision $a^{(1)}$ is made and the shut-down is completed before the wind speed u exceeds the threshold (if it does), or if the decision $a^{(1)}$ is made but the wind speed does not exceed the threshold. In the example, it is assumed that R_M is constant and the current as well as the relevant previous states of the typhoon are known precisely.

Like in the example presented in Section 6.1, Equation (3.12) is reformulated to

$$q_t(\underline{a}_{t-1}, \underline{\mathbf{e}}_t) = \begin{cases} \min\left\{h_t(\underline{a}_{t-1}, \underline{\mathbf{e}}_t), c_t(\underline{a}_{t-1}, \underline{\mathbf{e}}_t)\right\}, & \text{for } t = 0, 1, \ldots, n-1 \\ h_t(\underline{a}_{t-1}, \underline{\mathbf{e}}_t), & \text{for } t = n. \end{cases} \tag{6.17}$$

with the SVF defined as

$$h_t(\underline{a}_{t-1}, \underline{\mathbf{e}}_t) = \min_{a_t \in A_t^{(s)}} l_t(\underline{a}_{t-1}, a_t, \underline{\mathbf{e}}_t) \tag{6.18}$$

6.2.4 Decision optimization

Before the steps of the eLSM method are introduced, it is noted that the underlying random process (i.e. the transition of the typhoon) is a third-order Markov process (second-order with respect to the movement and third-order with respect to the central pressure). Thus, the CVF and SVF are functions of the typhoon states of the last three time steps and written as $c_t(\mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{y}_{t-2})$ and $h_t(\mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{y}_{t-2})$, respectively. Note that $\underline{a}_{t-1}$ is herafter neglected as it is assumed that $\underline{a}_{t-1} = (a_0^{(0)}, a_1^{(0)}, \ldots, a_{t-1}^{(0)})$. The CVF is approximated with a set of basis functions with respect to $\mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{y}_{t-2}$ as in Equation (4.4). However, it is anticipated that the function may be better represented with parameters, which themselves are functions of $\mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{y}_{t-2}$ and physically more meaningful. With trial-and-errors, in this example the realizations of the translation speed v_t, the distance d_t between the location of the typhoon and the platform, and the central pressures $p_{C,t}, p_{C,t-1}, p_{C,t-2}$ are adopted. Consequently, the CVF and the SVF are assumed to be represented by $c_t^\circ(\mathbf{x}_t)$ and $h_t^\circ(\mathbf{x}_t)$, respectively, where $\mathbf{x}_t = (v_t, d_t, p_{C,t}, p_{C,t-1}, p_{C,t-2})$. To obtain the optimal decision in Situation A the eLSM is applied. As described in Section 4.5 the algorithm is characterized by the following steps.

Step 1: By MCS, generate b independent paths $\mathbf{y}^i = (\mathbf{y}^i_{-2}, \mathbf{y}^i_{-1}, \mathbf{y}^i_0, \cdots, \mathbf{y}^i_5)$, $i = 1, 2, \ldots, b$ with known initial values $\mathbf{y}_0 = (v_0, \gamma_0, p_{C,0})^T$, $\mathbf{y}_{-1} = (\gamma_{-1}, p_{C,-1})^T$ and $\mathbf{y}_{-2} = p_{C,-2}$. As in the previous example, the method of antithetic variates is applied in order to reduce the variance. Note that an individual path, in regard to the typhoon movement, is first generated as the collection of the realizations of the translation speed V_t and angle Γ_t; then, using these realizations together with the initial location of the typhoon, the locations of the typhoon at times $t = 1, 2, \ldots, 5$ are identified. Thus, the variable $\mathbf{Y}_t$ representing the state of the typhoon at time t and its realizations $\mathbf{y}_t^i$ can be (re-) composed, e.g. by the location (longitude and latitude) and the central pressure, instead of the translation speed, angle and the central pressure. Having simulated the paths, the 10-minute sustained wind speeds at 10-minute intervals are calculated by interpolating the states of the realized typhoons. However, note that in the following it is assumed that the estimated functions depend only on the realizations summarized in $\mathbf{x}_t^i$ instead of the realizations $\mathbf{y}_t^i$.

Step 2: Using the realizations of $\mathbf{x}_t$, the values of the SVF are defined by $h_t^\circ(\mathbf{x}_t) = \min\{l_t^\circ(a_t^{(1)}, \mathbf{x}_t), l_t^\circ(a_t^{(2)}, \mathbf{x}_t)\}$. The functions $l_t^\circ(\cdot, \cdot)$ are estimated using Equation (4.7) for $t = 1, 2, \ldots, n$. The estimates are denoted by $\hat{h}_t^\circ(\mathbf{x}_t)$ and $\hat{l}_t^\circ(a_t^{(j)}, \mathbf{x}_t)$ respectively, $j = 1, 2$.

Step 3: Starting at the time horizon n, the MEU values $q_5(\mathbf{x}_5^i) = \hat{h}_5^\circ(\mathbf{x}_5^i)$, $i = 1, 2, \ldots, b$ are determined (see Step 3 in Section 4.2).

Step 4: Moving to $t = 4$ for each individual path i the MEU realization $q_5(\mathbf{x}_5^i)$ is related to the realizations of $\mathbf{x}_4 = (v_4, d_4, p_{C,4}, p_{C,3}, p_{C,2})$ such that the data set $(\mathbf{x}_4^i, q_5(\mathbf{x}_5^i))$, $i = 1, 2, \ldots, b$ is obtained. This set is then used to estimate $c_4^\circ(\mathbf{x}_4)$ as in Equation (4.4) by least squares method. The estimate is denoted by $\hat{c}_4^\circ(\mathbf{x}_4)$.

Step 5: Using the estimated CVF $\hat{c}_4^\circ(\mathbf{x}_4)$ and the estimated SVF $\hat{h}_4^\circ(\mathbf{x}_4)$, the realizations of the MEU are obtained by:

$$q_4(\mathbf{x}_4^i) = \begin{cases} u_4^*(\mathbf{x}_4^i), \text{ if } \hat{h}_4^\circ(\mathbf{x}_4^i) < \hat{c}_4^\circ(\mathbf{x}_4^i) \\ q_5(\mathbf{x}_4^i), \text{ otherwise} \end{cases} \tag{6.19}$$

where $u_4^*(\mathbf{x}_4^i)$ denotes the observed future consequence in path i for the optimal terminal decision a_4^* in this path. Moving another time step backward to time $t = 3$, the same procedure is repeated; i.e. applying the least squares method to estimate the CVF $\hat{c}_3^\circ(\mathbf{x}_3)$ with the realizations $\mathbf{x}_3^i$ and then obtain the MEU $q_3(\mathbf{x}_3^i)$, $i = 1, 2, \ldots, b$. This procedure is repeated until $t = 1$, hence for each path $q_1(\mathbf{x}_1^i)$ is determined.

Step 6: This step is analogous to Step 6 in Section 6.1.4.

Method using numerical integration

The numerical integration (NI) method as it is applied in this example is described in detail in Nishijima et al. (2009). The underlying stochastic processes are defined through the three regression models given in the Equations (6.14)-(6.16); each of the processes has a random error term. As described in Section 6.2.2 the random terms have different distributions. The state space of each random term is discretized in P partitions with equal probability $1/P$. Then the boundaries of the partitions and the corresponding midpoint (the center of gravity) is obtained by numerical integration. These midpoints are used as realizations of the random terms to compute the next realization of the underlying random processes. Since in the present example three random processes are relevant, P^3 states are considered at each time step. Each state has the probability $1/P^3$. This means starting from the initial time step P^3 realizations are computed and for each of them another P^3 realizations are computed and so forth, until the time horizon. With these realizations the decision problem is solved backward in time as it is common in a decision tree. Here, the decision tree has $(P^3)^n$ branches. Note that the total number of considered branches is less, since in case the wind speed threshold u_c is exceeded, the decision process in this branch will stop and all future states have the same value.

Crude Monte Carlo method

The crude Monte Carlo method (cMCM) that is applied here, can be described similar to the NI method introduced above. However, in the cMCM at each time step C realizations of the underlying random processes are independently generated according to their probability distribution function.

Extended LSM method with additional MCS

The first version of the extended LSM method, as introduced in Section 4.5 Part 1, is also applied to the considered real-time decision problem.

6.2.5 Results

Following the algorithms described above, the optimal decision at the initial time step $t = 0$ is identified. The convergence of the result is illustrated in Figure 6.7 using the eLSM method. The average $\bar{c}_{0,eLSM}$ of 100 eLSM realizations with $b = 10^5$ (i.e. a total number of 10^7 paths) of the estimated expected cost for $a_0^{(0)}$ converges to the value 2.0021 for eLSM with a coefficient of variation (COV) $6.6 \cdot 10^{-3}$. The dashed lines above and below the average represent the boundaries of the interval of the standard deviation. The expected costs of the two terminal decisions $a_0^{(1)}$ and $a_0^{(2)}$ are estimated as 1.8437 and 2.0147 respectively. Thus, the optimal decision is identified as $a_0^{(1)}$; i.e. to shut down.

In Figure 6.8 and 6.9 the performance of the eLSM method is compared to that of the LSM method with additional MCS. Figure 6.8 illustrates the computational superiority of the eLSM method in terms of the calculation time needed to obtain a certain level of COV. Therein the computational time of 100 realizations of the eLSM method or respectively that of the LSM method with different numbers of additional MCS is illustrated. The decrease in the COV values is related to the increase in numbers b of simulated

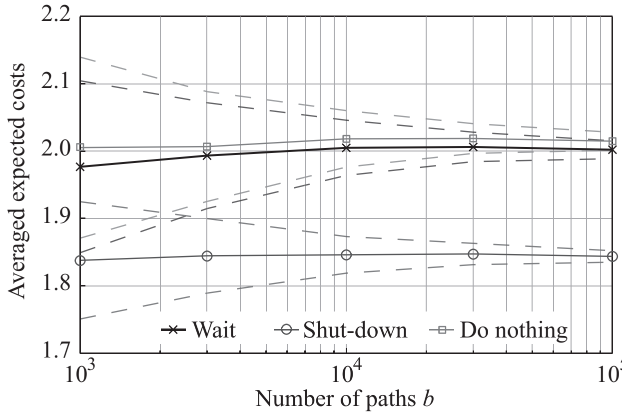

Figure 6.7: *Convergence of the estimated expected cost for $a_0^{(0)}$, $a_0^{(1)}$ and $a_0^{(2)}$ at time $t = 0$, as a function of the number b of paths. Therein for each decision alternative the average of 100 independent LSM realizations is presented by a solid line and the corresponding interval of one standard deviation by dashed lines.*

paths used in the algorithm; the values are $b = 10^3, 3 \cdot 10^3, 10^4, 3 \cdot 10^4, 10^5$. As in the previous example the calculation time of the LSM method with $M = 1$ is larger than that of the eLSM method for a given number b. This results from the higher number of total MCS, where the corresponding computational time is larger than the computational time required for the additional computations due to the application of the least squares method. Further, the computational time of the NI method with $P = 3$ is shown as a dashed vertical line. Note, there is no COV value attached with it, since it is not a stochastic method. The figure shows also the computational time for the cMCM (vertical black line). The corresponding COV value (0.1908) is not shown, since it is about three times larger than those of the other methods.

Figure 6.9 illustrates how the LSM method with additional MCS convergences as a function of the number M for decreasing COV (i.e. increasing b); whereas the eLSM method returns about the same values of the LSM method with additional $M = 1000$ MCS. It can be seen that the estimated values of the LSM method are biased-low and converge as M increases. This may be explained by the different accuracies in the estimation of the expected consequences for the terminal decision and that of waiting combined with the minimum-operator in the equation system to solve.

In the optimization, various sets of basis functions (Linear, Chebyshev, Power, weighted Laguerre and Hermite polynomials) with different truncation order K are examined. It is found that in the present example the numerical result shown above is insensitive to the choice of the basis func-

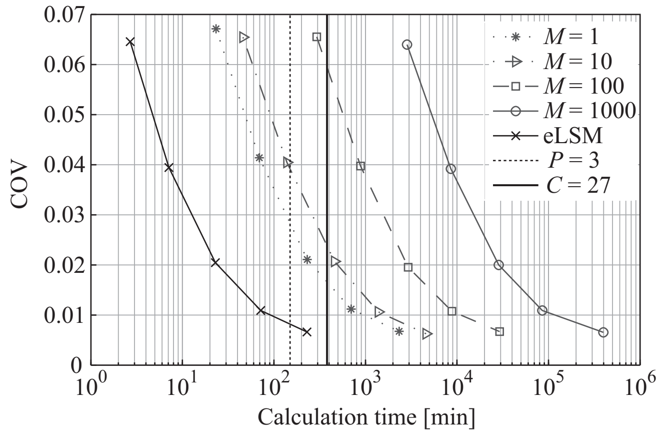

Figure 6.8: *Coefficient of variation (COV) of the estimated expected consequence for the decision to wait in the initial time step. Comparison of the eLSM (solid line with crosses) and the LSM with additional branches (other lines denoted by the different numbers M of additional MCS). The results of 100 independent eLSM/LSM realizations are used. Illustration of the superiority of the eLSM compared to the other methods.*

tions and the order K; see Figures 6.10 and 6.11 for the results obtained with $K = 1, 2$. The results presented in these figures deviate from each other less then 1%. Further it seems that the results converge parallel, which is attributed to the characteristics of the basis functions applied in the least-squares method. Deviations in the results when applying different basis function are also documented by Moreno & Navas (2003) for pricing American options. In the following the detailed results are shown for the case of first-order linear polynomials.

In order to evaluate the application of the eLSM method for this type of decision situation two additional Situations B and C with different initial conditions of the typhoon are considered. The initial conditions are presented in the Tables 6.5 and 6.6. Since in both situations the typhoon is assumed to be closer to the considered platform compared to Situation A the time horizon is set to $n = 4$. Equivalent to situation A, the NI method and cMCM are performed to obtain benchmark values.

In Table 6.7 the results of NI, cMCM as well as the LSM method with $M = 1$ additional MCS and the eLSM method (b = 100'000) are summarized. The estimates corresponding to the three decision alternatives are shown. In the case of the eLSM method and the LSM method with $M = 1$ additional MCS, the estimates of the expected cost corresponding to the terminal decisions are calculated using the average of the b paths. These paths

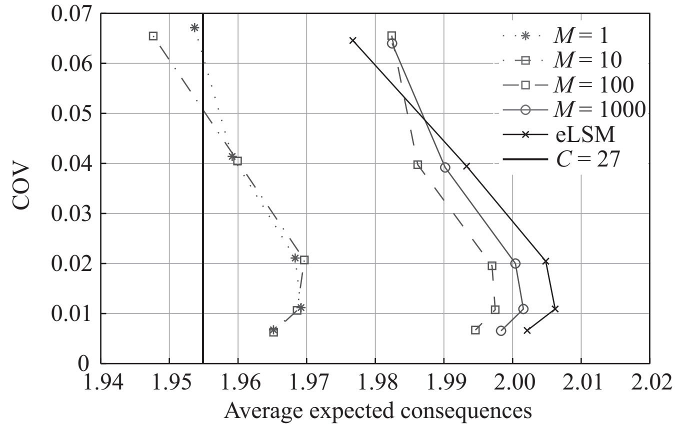

Figure 6.9: *Coefficient of variation (COV) of the estimated expected consequence for the decision to wait in the initial time step. Comparison of the eLSM (solid line with crosses) and the LSM with additional branches (lines denoted by the different numbers M of additional MCS). The results of 100 independent eLSM/LSM realizations are used. Illustration of the convergence of the methods as the number b increases, which is represented by decreasing COV.*

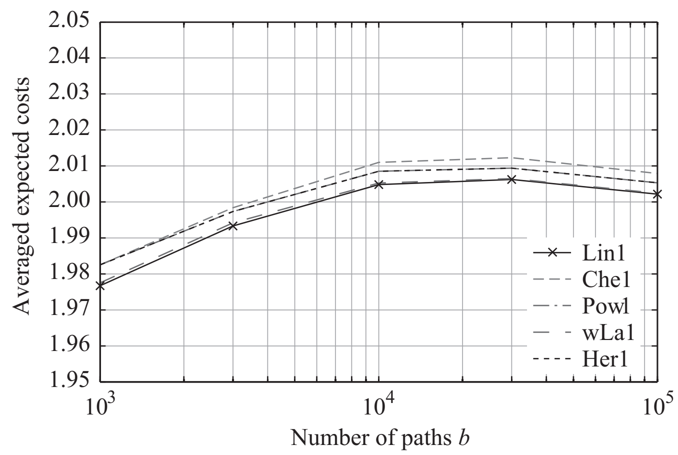

Figure 6.10: *Results of eLSM obtained with different types of basis functions (Linear, Chebyshev, Power, weighted Laguerre, and Hermite polynomials) for $K = 1$.*

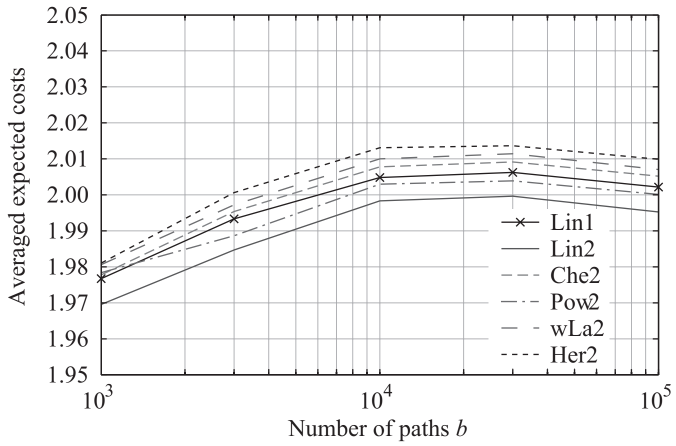

Figure 6.11: *Results of eLSM obtained with different types of basis functions (Linear, Chebyshev, Power, weighted Laguerre, and Hermite polynomials) for* $K = 2$.

Table 6.5: *Assumed initial conditions for decision situation B.*

Parameters	Values
Central pressures at $t = -2, -1, 0$	930, 930, 930 [hPa]
Translation speeds at $t = 0$	20[km/h]
Translation angles at $t = -1, 0$	22.5[°],0[°] (Northwards)
Position at $t = 0$	(129.25°E, 28.5°N)
SST at the location of the typhoon at $t = 0$	27.9 [°C]
Radius of max. wind speed, R_M	100 [km]
Location of the platform	(130.3°E, 31.25°N)

Table 6.6: *Assumed initial conditions for decision situation C.*

Parameters	Values
Central pressures at $t = -2, -1, 0$	930, 930, 930 [hPa]
Translation speeds at $t = 0$	24[km/h]
Translation angles at $t = -1, 0$	0[°],0[°] (Northwards)
Position at $t = 0$	(128°E, 28.75°N)
SST at the location of the typhoon at $t = 0$	27.9 [°C]
Radius of max. wind speed, R_M	100 [km]
Location of the platform	(130.3°E, 31.25°N)

are equivalent for both methods since the same seed is used for their simulation in order to be able to compare the results. In order to verify the results of the eLSM method, different numbers of partitions in the NI and of MCS in the cMCM are used.

For the NI, in Situation A the sample space of each error term has $P = 3$ and $P = 5$ partitions (i.e. 3 and 5 representative values) at each time step; whereas in the other two situations $P = 7$ partitions are computational feasible since the considered time frame is shorter. Equivalent, for the cMCM $C = 27, 125, 343$ independent simulations are evaluated at each time step. In the case of Situation A and $C = 27$, the cMCM is applied 100 times to estimate the average value and the corresponding COV; in Situations B and C this is done also for $C = 125$. In general, in all cases where it was possible to simulate 100 realizations the COV is presented in the brackets in Table 6.7.

In Table 6.7 it can be seen that the results of the NI method are not sufficiently exact to verify the results of the extended LSM nor the eLSMs method or to give evidence on the bias. The number of partitions is too low. Therefore cMCM is taken as a benchmark. First of all, it is found that in all three exemplary situations the cMCM, the LSM method with additional MCS and eLSM method come to the same optimal decision; that is in Situation A decision $a_0^{(1)}$ and in Situations B and C decision $a_0^{(0)}$. Thus, it can be concluded that both types of the LSM method are applicable for these decision situations. Note that in the later situations the typhoon is so close to the platform that in average it is too late to initiate decision $a_0^{(1)}$. Nevertheless, these situations are chosen to investigate whether the eLSM method is able to obtain correct result or not; i.e. the correct expected consequence for the decision to postpone $a_0^{(0)}$ at the initial time step.

In the following, only the expected consequences for the decision $a_0^{(0)}$ are of interest. By comparing the expected consequences obtained with the cMCM to the eLSM method, it can be seen that the COV value for the cMCM is over ten times larger, which results in a larger confidence interval (CI). The 100 realizations of the estimates $\hat{c}_0^{\circ}(\mathbf{x}_0)$ appear to follow approximately a normal distribution with mean $\bar{c}_{0,\mathrm{cMCM}}$ ($\bar{c}_{0,\mathrm{eLSM}}$) and standard deviation $\bar{s}_{0,\mathrm{cMCM}}$ ($\bar{s}_{0,\mathrm{eLSM}}$) the 95%-CI is estimated by

$$\left[\bar{c}_{0,\cdot} - 1.96\frac{\bar{s}_{0,\cdot}}{\sqrt{100}}, \bar{c}_{0,\cdot} + 1.96\frac{\bar{s}_{0,\cdot}}{\sqrt{100}}\right] \tag{6.20}$$

In Situation A the average value for $C = 27$ is $\bar{c}_{0,\mathrm{cMCM}} = 1.9549$ with 95%-CI [1.8818,2.0280], whereas that of the eLSM is $\bar{c}_{0,\mathrm{eLSM}} = 2.0021$ with 95%-CI [1.9995,2.0047]. In Situation B, the average of the cMCM ($C = 125$) is $\bar{c}_{0,\mathrm{cMCM}} = 3.0137$ and the 95%-CI [2.9700,3.0604], that of the eLSM is

Table 6.7: *Results obtained by the aforementioned methods: numerical integration (NI), cMCM, LSM with $M = 1$ additional MCS and eLSM (for the latter two methods $b = 100'000$). The table presents the estimates of the conditional expected costs at time $t = 0$ for the three decision alternatives. The numbers in brackets represent the estimated COV. The decision minimizing the expected cost is highlighted by bold letters.*

Example	NI			cMCM			LSM	eLSM
A	$P = 3$	$P = 5$	$P = 7$	$C = 27$	$C = 125$	$C = 343$	$M = 1$	
$\hat{c}_0^\circ(\mathbf{x}_0)$	**1.6107**	1.8402	-	1.9549 (0.1908)	2.1192	-	1.9652 (0.0067)	2.0021 (0.0066)
$\hat{l}_0^\circ(a_0^{(1)}, \mathbf{x}_0)$	1.6790	**1.8127**	-	**1.8443**	**1.8571**	-	**1.8437**	**1.8437**
$\hat{l}_0^\circ(a_0^{(2)}, \mathbf{x}_0)$	1.6312	1.8649	-	1.9883	2.1884	-	2.0147	2.0147
B								
$\hat{c}_0^\circ(\mathbf{x}_0)$	**3.0137**	**2.9737**	**2.9434**	**3.0215** (0.1902)	**3.0137** (0.0791)	**3.0117**	**2.9862** (0.0053)	**3.0049** (0.0052)
$\hat{l}_0^\circ(a_0^{(1)}, \mathbf{x}_0)$	3.1111	3.1151	3.0834	3.1342	3.1029	3.1767	3.0957	3.0957
$\hat{l}_0^\circ(a_0^{(2)}, \mathbf{x}_0)$	3.0479	2.9759	2.9461	3.0262	3.1395	3.0132	3.0052	3.0052
C								
$\hat{c}_0^\circ(\mathbf{x}_0)$	**2.1829**	**2.0821**	**2.0825**	**2.1113** (0.2135)	**2.1548** (0.0966)	**2.2799**	**2.1240** (0.0060)	**2.1389** (0.0060)
$\hat{l}_0^\circ(a_0^{(1)}, \mathbf{x}_0)$	2.3827	2.3110	2.3002	2.3001	2.3310	2.4269	2.3008	2.3008
$\hat{l}_0^\circ(a_0^{(2)}, \mathbf{x}_0)$	2.1897	2.0852	2.0855	2.1149	2.2376	2.2831	2.1391	2.1391

$\bar{c}_{0,\text{eLSM}} = 3.0049$ with a 95%-CI [3.0018,3.0080]. Finally in situation C, one gets an average value $\bar{c}_{0,\text{cMCM}} = 2.1548$ for the cMCM ($C = 125$) and the 95%-CI [2.1140,2.1956] and for the eLSM $\bar{c}_{0,\text{eLSM}} = 2.1389$ with [2.1364,2.1414].

6.2.6 Illustration of application

The application of the eLSM method in case of a typhoon event is presented in Figures 6.12 and 6.13. Figure 6.12 illustrates the transition of typhoon Bart (199918) in 1999. Typhoon Bart is taken as a reference event to apply the DSS with the settings of the aforementioned example. The initial values for the illustrative example with typhoon Bart are computed with data provided by JMA; these are the initial location, translation direction, wind speed and central pressure. The best track of the typhoon is presented in Figure 6.12. The crosses of the best track represent the time steps $t = 0, 1, ..., 9$ at which new information becomes available. Using the typhoon model introduced above with the data from JMA, it is calculated that between time steps $t = 7$ and $t = 8$ the 10-minute sustained wind speed u at the surface of the location of the platform exceeds the threshold u_c $(= 38[m/s])$. The time intervals, within which the wind threshold was exceeded, are illustrated by the dashed line of the best track in Figure 6.12.

The computations with the eLSM method are performed at time steps $t = 0, 1, ..., 9$ in order to obtain for each decision the associated expected costs and thus the optimal decisions. The development of the expected costs for each decision alternative is presented in Figure 6.13. In the figure it is seen that the optimal decision at time steps $t = 0$ until $t = 4$ is $a_0^{(0)}$, whereas at time $t = 5$ it is $a_0^{(1)}$. Assuming that the evacuation takes 12 hours, starting the evacuation at time $t = 5$ implies that the platform is evacuated at time step $t = 7$, which is the last time step before the typhoon strikes the platform according to the illustration in Figure 6.12.

Both figures illustrate how the DSS can be applied for real-time decisions in the face of a natural hazard event.

6.2.7 Discussion

The evaluation of the results shows that the eLSM method is a promising algorithm to solve the real-time decision problem considered. However, it cannot be concluded that it will work efficiently in all problems. Examples in which the algorithm may fail are decision situations where the hazard index is characterized by underlying random processes that have jump distributions or where the consequence model is more complex. The case where

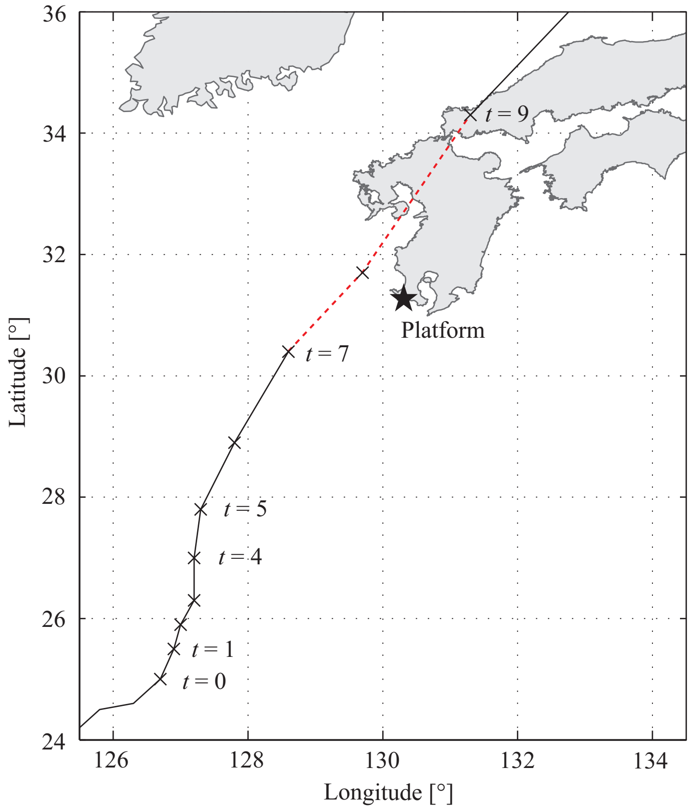

Figure 6.12: *Best track of Typhoon Bart (19918) in 1999. The crosses illustrate the time steps at which the eLSM method is performed to obtain the optimal decisions.*

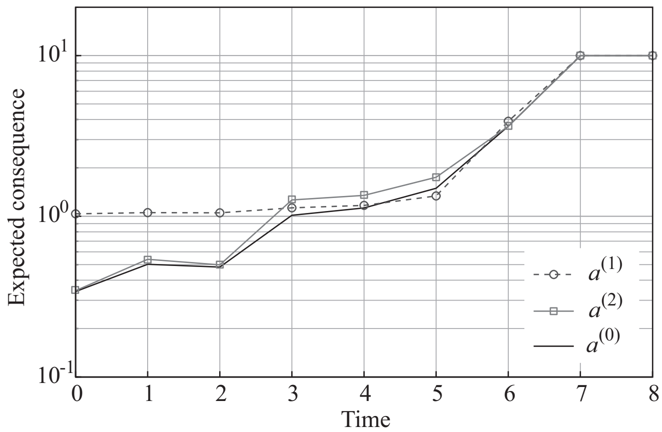

Figure 6.13: *The time series of the estimated expected consequences of the three decision alternatives corresponding to the best track of Typhoon Bart, illustrated in Figure 6.12. The estimates are calculated with the eLSM method and* $b = 10^5$.

the underlying random processes are characterized by jumps may be solved through continuous approximations or further embedding of the idea of the LSM/eLSM method. The later mentioned problem is usually the case in practice when for instance the consequence model is considered to be uncertain or life safety is an issue.

From the results in Situation A it seems that, compared to the result of the cMCM, the eLSM method is biased high; whereas in Situation B and C one observes that it is biased low. The observations in Situation A may seem contradictory to the observation presented in Figure 6.9 In the figure, the estimated costs computed with the extended LSM method with additional MCS converge from below to the estimated costs of the eLSM method. However, it can be assumed that the cMCM underestimates the true value as the calculations in Situation A are performed with only $C = 27$ simulations at each time step. Furthermore, comparing the 95%-CI of the cMCM estimate with the 95%-CI of the eLSM method, it is seen that the former encloses the latter. The observation of a bias is in accordance with those made in American option pricing and noted in Section 4.3.

Chapter 7

Conclusions & Outlook

7.1 Conclusions

The present thesis proposes an algorithm for real-time decision optimization in the face of emerging natural hazard events. The efficiency of the algorithm is evaluated with two examples. Thereby, it is demonstrated that the proposed algorithm is efficient enough to be utilized in real-time decision optimizations in practice. A decision support system is introduced enclosing the proposed algorithm. The applicability and the use of the decision support system are evaluated also with the two examples, which successfully demonstrates its usefulness to practical applications.

In the following the originality of the thesis and the limitations of the proposed algorithm are accounted for. Thereafter, the main findings are summarized, which is followed by the conclusions.

Originality of the work

The originality of the work is summarized as:

1. The introduction of the Least Squares Monte Carlo method (LSM method), originally proposed for pricing American options, to the real-time decision problems in the face of emerging natural hazard events.
2. The development of an efficient algorithm by adapting and extending the original LSM method to the considered real-time optimization problems.
3. The development of a general decision support system enclosing the proposed algorithm.

Main findings in the thesis

A detailed literature survey on existing algorithms that are used for real-time decision optimization revealed that there is no efficient algorithm available in practice so far. The thesis first addressed this lack.

The real-time decision framework was compared to the framework for American option pricing problems. Thereby, it was found that both frameworks have common characteristics and follow a similar mathematical formulation. These common characteristics are:

- Decisions have to be made fast in accordance with information available.
- The decisions can be made any time in a predefined finite time frame.
- The decisions are affected by random phenomena characterized by Markov processes.
- At each decision phase the conditional expected utility related to the decision "wait" (i.e. to postpone a terminal decision) is compared to the conditional expected utility related to a terminal decision.

Both frameworks need to overcome the following computational problem: the exponential increase in combinations of possible states that need to be considered. In the research field of American option pricing, many algorithms have been proposed that tackle this problem. After a detailed survey of these methods, the LSM method was found to be a promising method to take as the basis.

In devising the LSM method for the application to the real-time decision problems in consideration, several differences were found that need to be addressed. These are summarized in the following:

- In American option pricing the underlying random processes are in general characterized by stationary, first-order Markov processes; this is not necessarily the case for the real-time decision problems in consideration. In order to apply the algorithm of the LSM method to cases of higher-order, non-stationary Markov processes, the basis functions, which are used in the least squares method, use all the states necessary to fully characterize the conditional expected value over the state space.
- In American option pricing the value of the option is known when it is exercised; this is in general not the case for the real-time decision problems in consideration. Therefore, at first additional Monte Carlo

simulations are introduced (extended LSM method), which are substituted later by additional approximations using least squares method (enhanced LSM (eLSM) method).

- In American option pricing, the set of terminal decisions comprises only two alternatives: to execute or not to execute the option; whereas in the considered real-time decision problems more decision alternatives can be relevant. This can be treated by calculating the expected values for all terminal decisions alternatives and selecting the optimal one.

The LSM method combined with the three enhancements constitutes the proposed algorithm.

The proposed algorithm is illustrated by means of two examples. The first considers a situation where a decision maker has to decide whether to evacuate people from a village in the face of an avalanche hazard. The second considers a situation where a decision maker has to decide whether to shutdown an offshore platform in the face of an emerging typhoon. It is found that the proposed algorithm can be applied to both considered types of real-time decision problems. An advantage of the proposed algorithm is that the computational time is reduced significantly (by a factor up to 100) compared to the two standard methods; i.e., Monte Carlo simulations and numerical integration. Further, the proposed algorithm turns out to be flexible and robust with respect to the choice of parameters and type of basis functions used in the least squares method.

It was also found that the proposed algorithm is applicable to other types of real-time decision problems in engineering. In this thesis the application is demonstrated for decisions in quality control in Annex A.

Limitations

Although the proposed algorithm and decision support system are proved to be flexible and robust with respect to the implementation, there are limitations in their applications.

First of all, the proposed algorithm is not applicable, as it is now, to real-time decision problems in which the characteristics of the underlying random processes change after applying a risk reducing measure. For instance, in maintenance decision problems the underlying random processes represent the physical state of the considered structure, and its state changes after the repair works. In this type of decision problems, additional simulations are required reflecting the effects of the repair works in estimating the expected consequences corresponding to the chosen repair works.

Furthermore, the proposed algorithm as formulated now is limited to random processes that are characterized by known probability distribution functions. However, they are in general not known and the probabilistic distribution family as well as the distribution parameters are estimated with a limited amount of historical data; being subject to epistemic uncertainties.

Conclusions from the findings

It is demonstrated that the proposed algorithm can lead to significant improvements in the optimization of the real-time decisions in the face of emerging natural hazard events.

Due to the reduction of computational time, the optimal decision can be made faster. In case the optimal decision is to commence a risk reducing measure, this implies that more time is available to complete the measure successfully.

Thus it is concluded that the real-time decision framework together with the proposed algorithm is a useful tool to support decision makers in the face of emerging natural hazard events and will contribute to the reduction of adverse consequences.

7.1.1 Outlook

The application of the real-time decision framework requires probabilistic models that describe the physical development of the underlying random processes as well as their inherent variability. This thesis considered two examples in which only the natural hazard model includes random processes that describe the transition and the intensity of the natural hazard. However, in practice, the performance of the engineering systems is not deterministic, which is also true for the consequences. The application of probabilistic models to describe the behavior of the resistance and consequences within the framework is a future research task. Potential methods to implement the probabilistic characteristics are additional Monte Carlo simulations or by using expected values that are calculated in advance.

The considered examples include only monetary values as consequence measure. However, the framework may be required to be extended to include casualties as part of consequences. The consideration of both monetary losses and live losses may require to formulate the optimization problems as constrained optimization problems with marginal life-saving costs as a constraint. Alternatively, the value of life could be monetized by using the human compensation cost concept, which however may pose ethical problems. The extension of the framework in this direction remains as a future research topic.

The integration of physical hazard models, such as meso-scale weather forecast models, into the framework is a challenging topic. The extension in this direction is promising for improving the performance of the decision support system, since the weather forecast models have been and are being sophisticated and the computational capacities, required for running the models, are being advanced drastically.

The presented framework is “static” in the sense that the parameters used in the models are treated as deterministic values; i.e. parameters have no epistemic uncertainties. This implies that new information that may improve the model accuracies cannot be utilized to update the models when running; the models themselves need to be exchanged. The extension in this direction remains one of the future tasks, where the Bayesian statistical framework provides the basis.

In cases where the number of Monte Carlo simulations is restricted due to computational time constraints, variance reduction methods may be useful to improve the performance of the algorithm. These methods have been demonstrated to lead to large improvements in the applications to the pricing of American options; however, these methods need to be chosen carefully. Therefore, the development of general instructions on the choice of appropriate variance reduction methods is useful.

Finally but not least, possibilities of the applications of the developed decision support system to other types of engineering decision problems that share common characteristics such as, among others, structural health monitoring should be investigated.

Annex A

Numerical example with Matlab code

This chapter consists of two section. In the first section some numerical results are presented for the avalanche example introduced in Section 6.1 using the eLSM method. In the second section the corresponding Matlab code is provided.

A.1 Numerical example

The example introduced in Section 6.1 is the basis for the numerical example investigated in this section. In order to illustrate the steps of the eLSM method the following parameters are chosen: $b = 10$ and $n = 6$. Note that the time horizon applied in the numerical example is less than it is assumed in Section 6.1. This is done to shorten the procedure and from the simulations it could be seen that no significant event occurs after the sixth time step. Further, linear functions are used as basis functions in the least squares method. In the following, the steps of the eLSM method are given as introduced in Section 6.1 supported by numerical results.

As described in Section 6.1 the first step is to generate 10 independent realizations of $\{\mathbf{Y}_t\}_{t=1}^{6}$ and $\mathbf{S}^i = (S_0^i, S_1^i, ..., S_6^i)$, $i = 1, 2, ..., 10$, where $S_t^i = S_t^i(\underline{\mathbf{y}}_t^i)$ and $\mathbf{y}_t^i = (j_t^i, x_t^i)$. For the purpose of this example only the crude Monte Carlo method is applied. The realizations $\mathbf{y}_1^i, \mathbf{y}_2^i, ..., \mathbf{y}_6^i$ are simulated according to the probability density functions in Equations (6.1) and (6.4); the paths are denoted by $\mathbf{y}^i = (\mathbf{y}_{-1}^i, \mathbf{y}_0^i, ..., \mathbf{y}_6^i)$ with the following initial values: $\mathbf{y}_{-1}^i = \mathbf{y}_{-1} = (0, 0)$, $\mathbf{y}_0^i = \mathbf{y}_0 = (0, 0)$. The realization of the Monte Carlo simulation using predefined seeds (see the Matlab code in Annex A.2) are given in Table A.1 for $i = 1, 2, \ldots, 10$.

Table A.1: *Realizations of the random processes J_t, X_t and S_t obtained with crude Monte Carlo simulations. Ten paths are generated over the time interval $[0, 6]$. The realizations of the total additional snow amount S_t that exceed the critical threshold 800[mm] are highlighted in bold.*

		Time step						
	Path	$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$
J_t	1	0	1	1	1	1	1	1
	2	0	1	1	1	1	1	0
	3	0	1	1	1	1	1	0
	4	0	1	1	1	1	1	1
	5	0	1	1	1	1	1	1
	6	0	1	1	1	1	1	0
	7	0	1	1	1	1	1	0
	8	0	1	1	1	1	1	0
	9	0	1	1	1	1	1	0
	10	0	1	1	1	1	1	1
X_t	1	0.00	77.37	375.28	395.64	73.95	49.76	12.54
	2	0.00	21.35	116.45	117.73	176.57	89.27	0.00
	3	0.00	21.90	93.86	54.00	81.11	259.19	0.00
	4	0.00	33.69	309.51	77.68	37.19	92.31	54.76
	5	0.00	30.63	58.24	18.64	111.99	13.74	18.43
	6	0.00	2.25	10.68	111.94	147.50	53.88	0.00
	7	0.00	9.67	25.83	125.24	66.16	13.67	0.00
	8	0.00	97.98	286.18	271.26	315.40	81.16	0.00
	9	0.00	167.81	105.90	181.37	3.32	16.87	0.00
	10	0.00	163.46	137.33	153.62	93.64	150.21	87.66
S_t	1	0.00	77.37	452.65	**848.29**	**922.24**	**972.00**	**984.54**
	2	0.00	21.35	137.80	255.53	432.10	521.37	521.37
	3	0.00	21.90	115.76	169.76	250.86	510.05	510.05
	4	0.00	33.69	343.21	420.89	458.07	550.39	605.14
	5	0.00	30.63	88.87	107.51	219.49	233.23	251.66
	6	0.00	2.25	12.93	124.87	272.37	326.25	326.25
	7	0.00	9.67	35.50	160.75	226.91	240.58	240.58
	8	0.00	97.98	384.15	655.42	**970.81**	**1051.97**	**1051.97**
	9	0.00	167.81	273.70	455.07	458.39	475.27	475.27
	10	0.00	163.46	300.79	454.41	548.04	698.26	785.91

The second step as it is described in Section 6.1 is applied in parallel to the backward induction. This is described in the subsequent steps. For the purpose of simplicity this step was introduced separately in the theoretical description of the algorithm. In the following the eLSM algorithm is demonstrated as implemented in Annex A.2.

The subsequent steps are going backward in time, which is referred to as backward induction. Starting at time $n = 6$, for each path i, $i = 1, 2, \ldots, 10$, the value of $q_6^i(\underline{\mathbf{y}}_6^i)$ is set equal to $h_6^i(\underline{\mathbf{y}}_6^i) = \min\{u_6^i(a_6^{(1)}, \mathbf{z}^i), u_6^i(a_6^{(2)}, \mathbf{z}^i)\}$, where the function $u_t(a_t, \mathbf{z})$ denotes the utility (or consequence) function related to the chosen decision alternative a_t and the state of nature $\mathbf{z}$ as described in Section 4.5 Part 2. There is no value for the CVF as at the time horizon the decision to postpone is not available.

Moving to time $t = n - 1 = 5$, the values of $c_5(\underline{\mathbf{y}}_5)$, $l_5(a_5^{(1)}, \underline{\mathbf{y}}_5)$ as well as $l_5(a_5^{(2)}, \underline{\mathbf{y}}_5)$ are estimated using the least squares method with linear functions. The applied realizations are listed in Table A.2. Using linear functions one obtains, for the different decision alternatives at time $t = 5$, the following approximations of the SVF and the CVF representing the expected consequences conditional on the information available

$$\begin{aligned}
\hat{l}_5(a_5^{(1)}, \underline{\mathbf{Y}}_5) &= -3.1089 - 0.0158X_5 + 0.0062X_4 + 0.0117S_5 \\
\hat{l}_5(a_5^{(2)}, \underline{\mathbf{Y}}_5) &= -4.5654 - 0.0176X_5 + 0.0069X_4 + 0.0130S_5 \\
\hat{c}_5(\underline{\mathbf{Y}}_5) &= -4.5654 - 0.0176X_5 + 0.0069X_4 + 0.0130S_5
\end{aligned}$$

Note that the equations for the decisions $a^{(0)}$ and $a^{(2)}$ are equivalent. This is due to the fact that the following equation holds for all paths i: $q_6^i(\underline{\mathbf{y}}_6^i) = h_6^i(\underline{\mathbf{y}}_6^i) = \hat{l}_6^i(a_6^{(2)}, \underline{\mathbf{y}}_6^i)$.

Then, for each path i the value of $q_5 = q_5(\underline{\mathbf{y}}_5)$ is determined by

$$q_5^i = \begin{cases} u_5^{*,i}, \text{if } \hat{h}_5^i < \hat{c}_5^i \\ q_6^i, \ \text{otherwise} \end{cases} \tag{A.1}$$

where $u_5^{*,i} = u_5^i(a_5^*, \mathbf{z}^i)$ denotes the observed future consequence in path i for the optimal terminal decision a_5^*. Note that in the present example the realizations of $l_5(a_5^{(1)}, \underline{\mathbf{y}}_5)$, $l_5(a_5^{(2)}, \underline{\mathbf{y}}_5)$ and $c_5(\underline{\mathbf{y}}_5)$ are bounded by the upper threshold $C_D = 10$ and the following lower limits: (1) for $l_t(a_t^{(1)}, \cdot)$ it is the cost of evacuation $C_{Ev} = 1$ and (2) for $l_t(a_t^{(2)}, \cdot)$ and $c_t(\cdot)$ it is Zero. The results are presented in Table A.3.

Moving another time step back to time step $t = 4$ the same procedure is repeated. The realizations applied in the least squares method are presented

Table A.2: *Realizations of the consequences $u_6(a_6^{(1)}, \mathbf{Z})$ and $u_6(a_6^{(2)}, \mathbf{Z})$ at time $t = 6$ for the corresponding decision alternatives $a^{(1)}$ and $a^{(2)}$ as well as the realizations J_5, X_5, X_4 and S_5 of the random processes that are applied in the least squares method in time step $t = n - 1 = 5$.*

Path	$u_6(a_6^{(1)}, \mathbf{Z})$	$u_6(a_6^{(2)}, \mathbf{Z})$	$q_6(\underline{\mathbf{y}}_6)$	J_5	X_5	X_4	S_5
1	10.00	10.00	10.00	1.00	49.76	73.95	972.00
2	1.00	0.00	0.00	1.00	89.27	176.57	521.37
3	1.00	0.00	0.00	1.00	259.19	81.11	510.05
4	1.00	0.00	0.00	1.00	92.31	37.19	550.39
5	1.00	0.00	0.00	1.00	13.74	111.99	233.23
6	1.00	0.00	0.00	1.00	53.88	147.50	326.25
7	1.00	0.00	0.00	1.00	13.67	66.16	240.58
8	10.00	10.00	10.00	1.00	81.16	315.40	1051.97
9	1.00	0.00	0.00	1.00	16.87	3.32	475.27
10	1.00	0.00	0.00	1.00	150.21	93.64	698.26

Table A.3: *For each path the following estimated values of $l_5(a_5^{(1)}, \underline{\mathbf{y}}_5)$, $l_5(a_5^{(2)}, \underline{\mathbf{y}}_5)$ and $c_5(\underline{\mathbf{y}}_5)$ are obtained using the least squares method. With these values the MEU $q_5(\underline{\mathbf{y}}_5)$ is then determined at time step $t = n - 1 = 5$.*

Path	$l_5(a_5^{(1)}, \underline{\mathbf{y}}_5)$	$l_5(a_5^{(2)}, \underline{\mathbf{y}}_5)$	$c_5(\underline{\mathbf{y}}_5)$	$q_5(\underline{\mathbf{y}}_5)$
1	10.00	10.00	10.00	10.00
2	2.67	1.85	1.85	0.00
3	1.00	0.00	0.00	0.00
4	2.09	1.21	1.21	0.00
5	1.00	0.00	0.00	0.00
6	1.00	0.00	0.00	0.00
7	1.00	0.00	0.00	0.00
8	10.00	10.00	10.00	10.00
9	2.20	1.33	1.33	0.00
10	3.25	2.51	2.51	0.00

Table A.4: *Realizations of* $u_4(a_4^{(1)}, \mathbf{Z})$, $u_4(a_4^{(2)}, \mathbf{Z}$, $q_5(\underline{\mathbf{Y}}_5)$, J_4, X_4, X_3 *and* S_4 *that are applied in the least squares method at time step* $t = 4$.

Path	$u_4(a_4^{(1)}, \mathbf{Z})$	$u_4(a_4^{(2)}), \mathbf{Z})$	$q_5(\underline{\mathbf{Y}}_5)$	J_4	X_4	X_3	S_4
1	10.00	10.00	10.00	1.00	73.95	395.64	922.24
2	1.00	0.00	0.00	1.00	176.57	117.73	432.10
3	1.00	0.00	0.00	1.00	81.11	54.00	250.86
4	1.00	0.00	0.00	1.00	37.19	77.68	458.07
5	1.00	0.00	0.00	1.00	111.99	18.64	219.49
6	1.00	0.00	0.00	1.00	147.50	111.94	272.37
7	1.00	0.00	0.00	1.00	66.16	125.24	226.91
8	10.00	10.00	10.00	1.00	315.40	271.26	970.81
9	1.00	0.00	0.00	1.00	3.32	181.37	458.39
10	1.00	0.00	0.00	1.00	93.64	153.62	548.04

in Table A.4. Using these realizations and the linear functions one obtains, for the three decision alternatives, the following approximations

$$\begin{aligned}
\hat{l}_4(a_4^{(1)}, \underline{\mathbf{Y}}_4) &= -3.3270 + 0.0107X_4 + 0.0172X_3 + 0.0049S_4 \\
\hat{l}_4(a_4^{(2)}, \underline{\mathbf{Y}}_4) &= \hat{c}_4(\underline{\mathbf{Y}}_5) \\
&= -4.8078 + 0.0119X_4 + 0.0191X_3 + 0.0055S_4
\end{aligned}$$

The equations for the decisions $a^{(0)}$ and $a^{(2)}$ are again equivalent which follows from the same argumentation as given above.

This backward procedure is applied until $t = 1$. The realizations for the least squares method at time step $t = 1$ are given in Table A.5.

With the linear functions one obtains, at time $t = 1$ for the different decision alternatives, the following equations

$$\begin{aligned}
\hat{l}_1(a_1^{(1)}, \underline{\mathbf{Y}}_1) &= 1.6580 + 0.0039X_1 \\
\hat{l}_1(a_1^{(2)}, \underline{\mathbf{Y}}_1) &= 1.0869 + 0.0146X_1
\end{aligned}$$

Again in this example the estimates for decision $a^{(2)}$ and $a^{(0)}$ are equivalent. Applying these function for each path the values q_1^i, $i = 1, 2, \ldots, 10$, are determined; see Table A.6.

Execute Step 6 as it is described in Section 4.5 to determine the optimal decision at time $t = 0$: first compute the expected consequences for each decision alternative, then compare $\hat{h}_0$ and $\hat{c}_0$; in case $\hat{h}_0 < \hat{c}_0$ it is optimal to make a terminal decision, otherwise it is optimal to continue and wait until

Table A.5: *Realizations of $u_1(a_1^{(1)}, \mathbf{Z})$, $u_1(a_1^{(2)}, \mathbf{Z})$, $q_2(\underline{\mathbf{Y}}_2)$, J_1, X_1, X_0 and S_1 that are applied in the least squares method in time step $t = 1$.*

Path	$u_1(a_1^{(1)}, \mathbf{Z})$	$u_1(a_1^{(2)}, \mathbf{Z})$	$q_2(\underline{\mathbf{Y}}_2)$	J_1	X_1	S_1
1	10.00	10.00	10.00	1.00	77.37	77.37
2	1.00	0.00	0.00	1.00	21.35	21.35
3	1.00	0.00	0.00	1.00	21.90	21.90
4	1.00	0.00	0.00	1.00	33.69	33.69
5	1.00	0.00	0.00	1.00	30.63	30.63
6	1.00	0.00	0.00	1.00	2.25	2.25
7	1.00	0.00	0.00	1.00	9.67	9.67
8	1.00	10.00	10.00	1.00	97.98	97.98
9	1.00	0.00	0.00	1.00	167.81	167.81
10	1.00	0.00	0.00	1.00	163.46	163.46

Table A.6: *Expected consequences associated to each decision alternative as in Table A.3 for time step $t = 1$.*

Path	$l_1(a_1^{(1)}, \underline{\mathbf{y}}_1)$	$l_1(a_1^{(2)}, \underline{\mathbf{y}}_1)$	$c_1(\underline{\mathbf{y}}_1)$	$q_1(\underline{\mathbf{y}}_1)$
1	1.96	2.22	2.22	10.00
2	1.74	1.40	1.40	0.00
3	1.74	1.41	1.41	0.00
4	1.79	1.58	1.58	0.00
5	1.78	1.53	1.53	0.00
6	1.67	1.12	1.12	0.00
7	1.70	1.23	1.23	0.00
8	2.04	2.52	2.52	1.00
9	2.31	3.53	3.53	1.00
10	2.29	3.47	3.47	1.00

the next time step when new information becomes available:

$$\hat{c}_0 = \frac{1}{10}\sum_{i=1}^{10} q_1^i = 1.3 \tag{A.2}$$

$$\hat{h}_0 = \min\{\hat{l}_0^{(1)}, \hat{l}_0^{(2)}\} \tag{A.3}$$

$$= \min\{1.0, 2.0\} = 1.0 \tag{A.4}$$

The estimation of the expected consequences related to the terminal decisions at time $t = 0$ is obtained using the realizations presented in Table A.1; i.e.

$$\hat{l}_0^{(1)} = \frac{1}{10}\sum_{i=1}^{10} u_0(a_0^{(1)}, \mathbf{Z}) = 1.0 \tag{A.5}$$

$$\hat{l}_0^{(2)} = \frac{1}{10}\sum_{i=1}^{10} u_0(a_0^{(2)}, \mathbf{Z}) = 2.0 \tag{A.6}$$

With this low number of realizations the optimal decision would be to evacuate. However, it can be seen that such a low number of realizations is not reliable.

A.2 Matlab code corresponding to the numerical example

The code used to obtain the numerical results presented in the previous section is given in the following.

```
%% Batch for the avalanche example
% by Annett Anders

close all
clear;
clc;

%% parameter of the model
a0 = 4.5;
a1 = 0.26;
a2 = 0.1;
a3 = 0.5;
a4 = 0.05;
a5 = -0.2;
a = [a0, a1, a2, a3, a4, a5];
```

```
b0 = 1.95;
b1 = -0.2;
b2 = 0.25;
b3 = - 0.04;
b = [b0, b1, b2, b3];

c0 = 0.5;
c1 = 0.15;
c2 = 0.3;
c = [c0, c1, c2];

% Shape parameter:
r = 1.5;

% Factor for snow density:
Ft = 10;

% Number of simulations:
Sim = 10;

% Set predefined seeds:
seedsVal = [123456, 234567];

% Run the calculation:
result = avalancheEx_diss(Sim,a,b,c,r,Ft,seedsVal);
save('Result_Avalanche_Ex',result)


%% LSM method applied to the avalanche example
% by Annett Anders

function out = avalancheEx_diss(Sim,a,b,c,r,Ft,seedsVal)
tic
n = 8;  % time horizon
sThreshold = 800;   % snow threshold

%% Initialization
y = zeros(Sim,n);   % intensity process
j = zeros(Sim,n);   % occurence process
S = zeros(Sim,n);   % total snow = sum(y)
EV = zeros(Sim,n);  % expected Value (Cost) matrix
EF = zeros(Sim,n);  % auxiliary matrix

%% Consequence model
CostD = 10; % damage cost, if no evacuation has been done or too late
CostE = 1;  % cost for the evacuation

%% Random variables
% Uniform random variable for occurence process
```

```
s1 = RandStream.create('mrg32k3a','Seed',seedsVal(1));
u = rand(s1,Sim,n-2);

% gamma distributed random variable for intensity process
%with shape parameter r and scale parameter 1
s2 = RandStream('mrg32k3a','Seed',seedsVal(2));
RandStream.setDefaultStream(s2);
GamRV = randg(r,[Sim,n-2]);

i = 2;
while i < n
    i = i+1;
    t = i-2;    % time; initial time is zero
    % Parameter for the link function in the occurence model
    mu = a(1) + a(2) * j(:,i-1) + a(3) * j(:,i-2) ...
            + a(4) * log(y(:,i-1)+c(2)) + a(5) * log(y(:,i-2)+c(3)) ...
            + a(6) *(t).^2;
    % Probability that it will be raining/snowing in the next time step
    probRain = exp(mu)./(1+exp(mu));
    % Scale parameter of the Gamma distribution for the intensity model
    mean = exp(b(1) + b(2) * j(:,i-1) ...
            + b(3) * log(y(:,i-1)+c(1))  ...
            + b(4) * (t).^2);
    % Realizations of random processes J, Y and S
    j(:,i) = logical(u(:,t)<=probRain);
    y(:,i) = Ft * mean/r .* GamRV(:,t) .* j(:,i);
    S(:,i) = S(:,i-1) + y(:,i);
end

% Cost matrix (including damage costs when threshold is exceeded)
C = CostD * logical(S>sThreshold);
TS = logical(sum(C,2)>0);
% Probability that an avalanche occurs within the time frame:
prob2failNoE = sum(TS)/Sim;
% Expected cost for a^(2) [doing nothing] at time 0:
CostDa2 = prob2failNoE * CostD;
% Probability that an avalanche occurs within the first 2 time steps:
prob2failE = sum(logical(sum(C(:,1:4),2)>0))/Sim;
% Expected cost for a^(1) [evacuation] at time 0:
CostEa1 = prob2failE * CostD + (prob2failNoE - prob2failE)* CostE ...
    + (1 - prob2failNoE) * CostE;

EV = C;
% Initialize the matrix EF where the expected cost are stored
% when making the optimal decision:
EF(:,n) = C(:,n);
% Initialize an auxiliary cost matrix to estimate
%the expected cost related to decision a^(1)
EVevM = EV + CostE.*logical(C==0);
```

```
% under the assumption that after n nothing significant occurs
EVevM(:,end+1) = EVevM(:,end);

%% Backward induction
for i = n-1:-1:4
    %% Regression matrix entries for the linear Regression
    %%  first-order of basis function:
    regrmat = [ones(Sim,1),y(:,i),y(:,i-1), S(:,i)];

    idx = C(:,i) == CostD;

    %% Linear regression
    % (using matlab-function pinv() to obtain the inverse)
    % 1: Least squares method to obtain estimate for l(a^(2), y):
    RegMInv = pinv(regrmat);
    YregNoE = C(:,n);
    parN = RegMInv * YregNoE;
    EVne = regrmat * parN;
    % after the approximation, the expected costs are bounded to
    % the maximum damage cost and to the minimum cost of zero
    EVne(idx,1) = CostD;
    EVne(EVne<0,1) = 0;

    % 2: Least squares method to obtain estimate for l(a^(1),y)
     YregEv = EVevM(:,i+2);
     parE = RegMInv*YregEv;
     EVev = regrmat * parE;
     % after the approximation, the expected costs are bounded to the
    % maximum damage cost and to the minimum cost of evacuation [CostE]
     EVev(idx,1) = CostD;
     EVev(EVev<1,1) = CostE;

    % 3: Least squares method to obtain estimate for c(y)
    Yreg = EV(:,i+1);
    par = RegMInv*Yreg;
    EVp = regrmat * par;
    % after the approximation, the expected costs are bounded to the
    % maximum damage cost and to the minimum cost of zero
    EVp(idx,1) = CostD;
    EVp(EVp<0,1) = 0;

    % decision for evacuation only if expected cost strictly lower
    indEvac = EVev(:,1) < min(EVp,EVne(:,1));

    EV(indEvac,i) = YregEv(indEvac,1);
    EF(indEvac,i) = YregEv(indEvac,1);
    EV(indEvac==0,i) = EV(indEvac==0,i+1);
    EF(indEvac,i+1:end) = 0;
end
```

```
%% For the case i=3
%% Regression matrix entries for the linear Regression
%%  first-order of basis function:
regrmat = [ones(Sim,1),y(:,3)]; % because y(:,i-1)=0 and S(:,i)=y(:i)
idx = C(:,3) == CostD;

%% Linear regression
% (using matlab-function pinv() to obtain the inverse)
% 1: Least squares method to obtain estimate for l(a^(2), y):
RegMInv = pinv(regrmat);
YregNoE = C(:,n);
parN = RegMInv * YregNoE;
EVne = regrmat * parN;
% after the approximation, the expected costs are bounded to the
% maximum damage cost and to the minimum cost of zero
EVne(idx,1) = CostD;
EVne(EVne<0,1) = 0;

% 2: Least squares method to obtain estimate for l(a^(1),y)
YregEv = EVevM(:,5);
parE = RegMInv*YregEv;
EVev = regrmat * parE;
% after the approximation, the expected costs are bounded to the
% maximum damage cost and to the minimum cost of evacuation [CostE]
EVev(idx,1) = CostD;
EVev(EVev<0,1) = CostE;

% 3: Least squares method to obtain estimate for c(y)
Yreg = EV(:,4);
par = RegMInv*Yreg;
EVp = regrmat * par;
% after the approximation, the expected costs are bounded to the
% maximum damage cost and to the minimum cost of zero
EVp(idx,1) = CostD;
EVp(EVp<0,1) = 0;
%end i = 3

% decision for evacuation only if expected cost strictly lower
indEvac = EVev(:,1) < min(EVp,EVne(:,1));

EV(indEvac,3) = YregEv(indEvac,1);
EF(indEvac,3) = YregEv(indEvac,1);
EV(indEvac==0,3) = EV(indEvac==0,3+1);
EF(indEvac,3+1:end) = 0;

%% result of the expected cost when postponing the decision
resultp = sum(sum(EF))/Sim;
```

```
%% Expected cost corresponding to the terminal decisions
Hev0 = CostEa1;
Hne0 = CostDa2;

time = toc;

out = [resultp, Hev0, Hne0, time];
%% end function
```

Annex B

DeGroot examples

The following two examples are published in Nishijima & Anders (2012). The efficiency and advantages of the proposed optimization scheme are demonstrated with two numerical examples on the sequential sampling in the context of quality control of manufactured product. Note however that under this framework variable decision rules or competing decision rules are possible. In the examples, the updated probability distributions given the information $\underline{e}_t$ can be obtained analytically. However, in general this is not the case. In such cases, Markov Chain Monte Carlo simulations to simulate a realization(s) from the updated probably distributions may be useful; the implementation of which is addressed as a future task.

B.1 Example 1: Sequential sampling from the Bernoulli distribution

B.1.1 Decision problem

A manufactured product is designed, and the performance of the product line is to be controlled. For simplicity, assume that the probability p that a product is good quality is to be either 1/3 or 2/3. If $p = 2/3$, the product line is satisfactory; otherwise unsatisfactory. It is to be judged whether the product line is satisfactory. The decision maker has the option to perform inspections before her judgment. The maximum number of inspections is assumed to be n. The outcomes of the inspections are the sequence of random samples $Y_1, Y_2, \ldots, Y_n$, which independently follow an identical Bernoulli distribution with a given parameter $p = P[Y_i = 0] = 1 - P[Y_i = 1]$ (0: good quality, 1: not good quality). The cost for inspecting one sample is assumed to be $C = 1$. Given that the design was made and the product line was built, the penalty is imposed if and only if the judgment is incorrect, which is assumed to be $C_P = 20$. The prior distribution of p is assumed to be $\xi = P[p = 1/3] = 1 - P[p = 2/3]$, $0 \leq \xi \leq 1$. The decision shall be made for a given value of ξ whether the first inspection should be performed, or make the judgment without any inspection. In the following the case of $n = 2$ is considered, the analytical solution to which is available in DeGroot (1970).

B.1.2 Application of the proposed scheme and result

The underlying random sequence Y_t in this decision problem is the outcomes from the inspections, each of which follows the Bernoulli distributions of the parameter p, which in turn is uncertain and is characterized by $\xi = P[p = 1/3] = 1 - P[p = 2/3]$. The information E_t is equal to Y_t; i.e. the state of the underlying random sequence is deterministically known to the decision maker without uncertainty. The decision alternatives are $a^{(0)}$ (continue sampling), $a^{(1)}$ (terminate sampling and judge $p = 1/3$) and $a^{(2)}$ (terminate sampling and judge $p = 2/3$) at each decision time; hence, $A_t^{(c)} = \{a^{(0)}\}$, $A_t^{(s)} = \{a^{(1)}, a^{(2)}\}$. Decisions at a decision time are possible only if $a^{(0)}$ is chosen at all the earlier decision times.

The steps to apply the proposed scheme are explained for one value of ξ. These steps are repeated in order to obtain the set of the solutions for different values of ξ, which are shown in Figure B.1. Note, however, such repetitions are not necessary in practice, since in a practical situation a single value of ξ is assigned based on the decision maker's degree of belief, for which the optimal decision is to be identified.

Step 1: The first step is to simulate the realizations of p, i.e. p^i, $i = 1, 2, \ldots, b$. For each p^i, the realizations of the underlying random sequence (Y_1, Y_2) are simulated, i.e. (y_1^i, y_2^i), $i = 1, 2, \ldots, b$. In this example, $\underline{e}_t^i = \underline{y}_t^i$, $i = 1, 2, \ldots, b$, $t = 1, 2$.

Step 2: The expected cost for $a^{(1)}$ at time $t = 2$ is calculated for each realization (e_1^i, e_2^i). For this, first the probability of $P[p = 1/3]$ is updated with the realization (e_1^i, e_2^i) by the Bayes' theorem. Based on the updated probability the expected cost for $a^{(1)}$ is calculated. The expected cost for $a^{(2)}$ is calculated in the same manner. By comparing these two values, the optimal decision at time t is obtained for each $\underline{e}_2^i = (e_1^i, e_2^i)$.

Step 3: The maximized expected utilities, which are defined as the negative of the expected costs, $q_2(\underline{a}_1, \underline{e}_2^i)$, are obtained for each realization i.

Step 4: The expected cost for $a^{(0)}$ (continuing sampling) at time $t = 1$ is assumed to be approximated by $r_{1,0} + r_{1,1}e_1$. The coefficients $r_{1,0}$, $r_{1,1}$

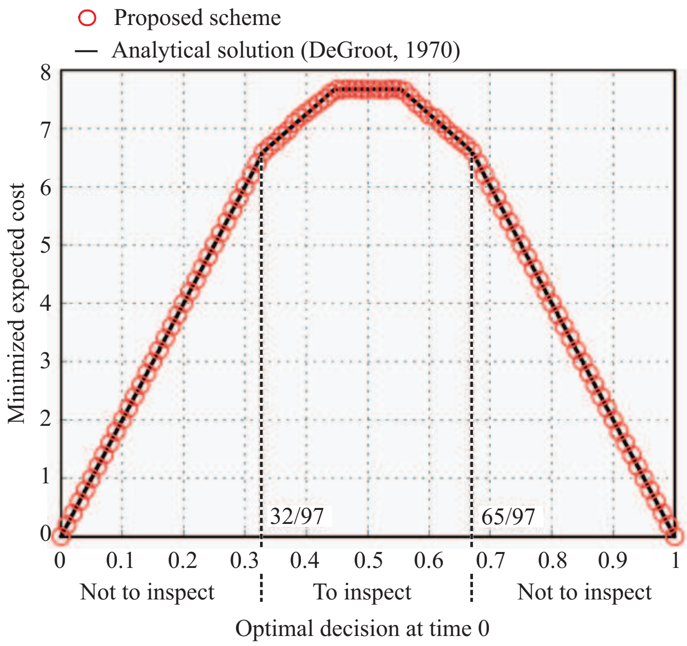

Figure B.1: *Optimal decisions in Example 1.*

are estimated with the set of points $(e_1^i, q_2(\underline{a}_1, \underline{e}_2^i))$ by the least square method. Then, the expected cost for $a^{(0)}$ at time $t = 1$ is obtained for each realization i. Note in this example, the functional form $r_{1,0}+r_{1,1}e_1$ can precisely represent the expected cost for $a^{(0)}$, since e_1 takes only two values; therefore, a function with two coefficients is flexible enough (Step 4). The expected costs for $a^{(1)}$ and $a^{(2)}$ at $t = 1$ are calculated in the same manner as $t = 2$ for each realization i. The minimum of the expected costs for $a^{(0)}$, $a^{(1)}$ and $a^{(2)}$ is calculated for each realization i, whose negative values are the maximized expected utilities at time $t = 1$.

Step 5: The average of these minimum expected costs for all the realizations i is the estimate of the expected cost for $a^{(0)}$ at time $t = 0$. By comparing this with the expected costs for $a^{(1)}$ and $a^{(2)}$, the optimal decision at time $t = 0$ is obtained.

The optimal decisions for different values of ξ are shown in Figure B.1. It is seen that the proposed scheme performs satisfactorily.

B.2 Example 2: Sequential sampling from the normal distribution

B.2.1 Decision problem

A manufactured product is designed, and the performance of the product line is to be controlled. The quality of the product is measured through an indicator Y and the indicator follows the normal distribution with unknown mean W and known precision r (= inverse of variance). The decision maker has the option to perform inspections before her judgment. The maximum number of inspections is assumed to be n. The outcomes of the inspections are the sequence of random samples $Y_1, Y_2, \ldots, Y_n$, which follows the identical distributions as Y. The random samples are observable without uncertainty to the decision maker; hence, the information $E_t = Y_t$. The decision maker has to judge whether the mean of Y is above w_0 or not. The penalty of misjudgment is proportional to the difference between the true mean value w and w_0; i.e. $L = |w - w_0|$. The cost for one inspection is C.

B.2.2 Application of the proposed scheme and result

The decision alternatives are $a^{(0)}$ (continue sampling), $a^{(1)}$ (terminate sampling and judge $w \leq w_0$) and $a^{(2)}$ (terminate sampling and judge $w > w_0$) at each decision time; $A_t^{(c)} = \{a^{(0)}\}$, $A_t^{(s)} = \{a^{(1)}, a^{(2)}\}$. Decisions at a decision time are possible only if $a^{(0)}$ is chosen at all the earlier decision times. In this example, the expected cost for continuing sampling at time t is found to be a function only of the average m_t of the realizations of $\underline{e}_t = (e_1, e_2, \ldots, e_t)$. Here, a functional form $r_{t,0} + r_{t,1}m_t + r_{t,2}m_t^2 + r_{t,3}m_t^3$ is assumed for the least squares estimation, where $m_t = \sum_{j=1}^{t} e_j/t$. Note that other functional forms are tested and the results are found to be insensitive to the choice of functional forms. The optimal decisions are computed and shown for different values of the mean μ and precision τ (inverse of variance) of the unknown mean W, for the case where $n = 10$, $w_0 = 1$ and $C = 0.2$, see Figure B.2. The optimal decisions obtained by the proposed scheme are indicated with the symbols. The optimal decision bounds for the case where n is infinite are analytically obtained by DeGroot (1970) for a subset of μ and τ: At the left side of the left line in the figure the optimal decision is $a^{(0)}$, at the right side of the right line the optimal decision is $a^{(1)}$ for $\mu \leq 1$ and $a^{(2)}$ for $\mu > 1$, and between the two lines, optimal decisions are not obtained. Given that $n = 10$ is sufficient large, these two results are comparable. As can be seen in Figure B.2, the optimal decisions obtained by the proposed scheme cor-

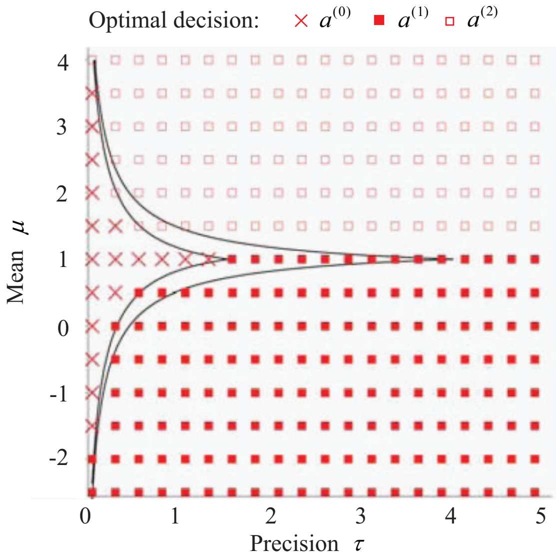

Figure B.2: *Optimal decisions in Example 2.*

responds to the optimal decisions obtained by DeGroot (1970); furthermore, the proposed scheme can identify the optimal decisions in the domain where the optimal decisions are not obtained by DeGroot (1970).

Annex C

Conference articles

Adaption of option pricing algorithm to real time decision optimization in the face of emerging natural hazards

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Adaption of option pricing algorithm to real time decision optimization in the face of emerging natural hazards

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ABSTRACT: The present paper proposes an approach for the optimization of real time decision problems in the face of emerging natural hazards. It takes basis in the Least Squares Monte Carlo method (the LSM method) originally developed for option pricing in financial mathematics. In the present paper, first the decision problems considered in the paper are described. Then, the fundamental idea underlying the LSM method is introduced. Thereafter, extensions are presented, which are required for the application to the real time decision problems in consideration. The application of the LSM method and these extensions constitute the proposed approach. The performance of the proposed approach is investigated with an example; decision in regard to shut-down of the operation of a platform in the event of an approaching typhoon. The numerical result shows clear advantages of the proposed approach. Finally, possible applications to other engineering decision problems of pre-posterior type are briefly discussed.

1 INTRODUCTION

1.1 *Background*

Formal decision analysis in engineering often requires that a sequence of decisions has to be jointly optimized; i.e. the decision at each decision phase must be optimized based on the information available up to the current decision phase as well as on the consideration of all possible outcomes and decisions undertaken in the future. Important examples of such decision analysis include inspection planning of deteriorating structures (see e.g. Straub (2004)), quality control of manufactured products (e.g. Nishijima & Faber (2007)), and decision support for real time decisions in the face of emerging natural hazards (Nishijima et al. (2008), (2009)).

For the formulation of this type of decision problems, the pre-posterior decision framework provides the philosophical basis (Raiffa & Schlaifer (1961)). It has been applied, among others, in civil engineering for the examples mentioned above. The framework is general and the formulation of decision problems based on the framework is straightforward; however, the analytical solutions to the decision problems are available only in limited cases (see e.g. Chapter 12 in DeGroot (1970)). Quite often, the solutions are not available even numerically without any approximations to the problems. This is due to the large number of combinations of the decision alternatives and the realizations of random phenomena at all decision phases, which must be considered in the optimization. Thus, the original decision problems are often simplified; e.g. by reducing the number of decision phases and/or decision alternatives at each decision phase; otherwise, with coarse discretization of the sample space of the random phenomena. However, the adaptation of appropriate simplifications often requires trial-and-errors, and the validation of the simplifications is often difficult. On the other hand, the coarse discretization of the sample space may result in significant errors. The lack of algorithms for the solutions to the decision problems without such approximations significantly undermines the possibility to apply the pre-posterior decision framework in practice.

1.2 *Objective and focus*

The present paper proposes an approach for solving pre-posterior decision problems without the approximations mentioned above. The proposed approach takes basis in a method originally developed for option pricing in financial mathematics; the Least Squares Monte Carlo method (LSM method) proposed by Longstaff & Schwartz (2001).

The example examined in Nishijima et al. (2009) is re-examined; however, the main focus in the present paper is the solution to the decision problem, whereas Nishijima et al. (2009) focus on the formulation of the decision framework and the solution to the decision problem and its efficiency are not examined in detail.

Although the present paper focuses on the application to the real time decision optimization in the face of emerging natural hazards, the fundamental idea underlying the proposed approach in the paper can be applied to broader decision problems of the pre-posterior type.

1.3 *Structure of the paper*

In the next section, the characteristics of the decision problem considered in the present paper and its formulation are provided. Thereafter, options and option pricing in the context of finance are briefly introduced. Then, the fundamental idea underlying the LSM method is explained without going into the detail of the theory. Subsequently, identifying the relevant differences between the engineering decision problems and option pricing, the ideas for extensions are presented. The LSM method together with the extensions constitute the proposed approach of the present paper. Finally, the implementation of the proposed approach is provided with a numerical example. Discussion and conclusion follow.

2 FRAMEWORK FOR REAL TIME DECISIONS

This section introduces the framework for real time decision optimization in the face of emerging natural hazards, which is proposed in Nishijima et al. (2008), (2009).

2.1 *Description of decision problem*

The decision situation considered here is described by the following characteristics: (a) The hazard process evolves relatively slow and allows for reactive decision making; (b) various types of information can be obtained prior to the impact of the hazard, which can be utilized to predict its severity; (c) the decision making is subject to uncertainties, part of which might be reduced at a cost; (d) decision makers have options for risk reduction activities which may be commenced at any time, supported by the information available. Here, the typical problem arises that "waiting" will imply the reduction of uncertainty but it might also reduce available time for initiating the risk reduction activities; (e) and on top of all, the decisions must be made fast, in near-real time. The decision problems characterized as above is referred to as *real time* decision problems.

One of the important characteristics of the real time decision problems is that the decision maker has an option to postpone the decision for risk reduction measures, for the purpose to reduce the uncertainty concerning the decision problem. In general, there are two types of ways to reduce the uncertainty; i.e. reductions of aleatory uncertainty and epistemic uncertainty. The aleatory uncertainty is reduced by "waiting". For instance, the possibility that a typhoon hits an engineering facility becomes more evident as time goes by. Namely, by postponing the decision the probability that the decision maker makes a suboptimal decision can be reduced; but, in turn, the probability increases that risk reduction activities are undertaken too late if they are necessary. The epistemic uncertainty is reduced by collecting more information to update the probability model representing random phenomena underlying the decision problem. However, such measures for collecting more information are worth undertaking only if the corresponding cost is smaller than the expected value of the additional costs arising from the suboptimal decision.

Note that whereas both types of the uncertainty reduction are relevant in general, only the former type of the uncertainty reduction is considered in the present paper for simplicity and without loss of the essence of the ideas in the proposed approach.

2.2 *Conditional probability representation*

Denote by Z the random variable of relevance to the consequences in a decision problem; e.g. the maximum wind speed during a storm event. Let $\boldsymbol{Y} = (Y_0, Y_1, \ldots, Y_n)$ be a sequence of random variables at different discrete time steps $t = 0, 1, \ldots, n$ that characterize the phenomena underlying the decision problem and are required to calculate the probabilistic characteristics of Z. n is the number of points in time at which observations can be made. Prior to or at the last time step n, a terminal decision (the definition is provided later) must be made. Denote by $\mathbf{E} = (E_0, E_1, \ldots E_n)$ a sequence of random variables representing the observed information at the respective times. The observed information can be utilized to reduce the uncertainty associated with the future states of $\mathbf{Y}$ and in turn with the hazard index Z. Note that the variables Z, Y_t and E_t, $t = 0, 1, \ldots, n$, can be scalar or vector.

The illustrative relationship between these variables is shown in Figure 1 for the case of a second-order Markov model. Each node represents a variable and each directed edge link represents the probabilistic dependency between the connected variables. For instance, the edge link directed from the node Y_0 to the node E_0 represents that the random variable E_0 is characterized by the conditional probability $P[E_0|Y_0]$. When more than two edge links are directed to a node, it signifies that the random variable represented by the node is characterized by the conditional probability on the variables represented by the nodes from which the edge links are directed. When all the conditional probabilities corresponding to the directed edge links and the (unconditional) probabilities for the nodes to which no edge link is directed are given, conditional probabilities of any variable in the graph

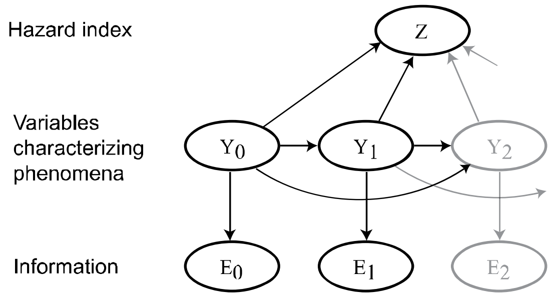

Figure 1. Conditional probabilistic model representation in the case of a second-order Markov model (after Nishijma et al. (2009)).

can be calculated. Hence, the probabilistic characteristics of the random phenomena underlying the decision problem can be completely defined.

2.3 *Formulation of decision optimization*

A decision maker has a set of m terminal decision alternatives, $A = \{a^{(1)}, a^{(2)}, \ldots, a^{(m)}\}$ out of which one alternative has to be chosen at latest by the time n, in response to the information E_t that becomes available at each point in time $t = 0, 1, \ldots, n$. Without loss of generality, it is assumed that the decision is made only at one of the discrete times $t = 0, 1, \ldots, n$; the decisions that actions should be commenced between the consecutive discrete times can be included in the set A, if relevant. In addition to the terminal decision alternatives, the decision maker has the option to postpone the terminal decision at times $t = 0, 1, \ldots, n-1$. This optional alternative is denoted by $a^{(0)}$.

The decision optimization problem is then formulated as the minimization of the expected cost, or more generally, the maximization of the expected utility by identifying the optimal action out of the set of the decision alternatives $A^{(0)} \equiv A \cup \{a^{(0)}\}$ at each respective time as a function of the information available up to that time. Here, it should be emphasized that in order to identify the optimal decision at a certain time, all possible decisions and information available at the future times must be considered.

Taking basis in the concept of the pre-posterior decision framework (Raiffa & Schlaifer, 1961), the optimal decision $a_t^*(e_0, e_1, \ldots, e_t)$ at time t given the information $e_0, e_1, \ldots, e_t$ is identified as the decision alternative which maximizes the conditional expected utility $E[U_t(Z, a)|e_0, \ldots, e_t]$, $a \in A^{(0)}$:

$$E[U_t(Z,a_t^*(e_0,\ldots,e_t)) \mid e_0,\ldots,e_t] = \begin{cases} \max\limits_{i=0,1,\ldots,m} E[U_t(Z,a^{(i)}) \mid e_0,\ldots,e_t], \text{for } t=0,1,\ldots,n-1 \\ \max\limits_{i=1,2,\ldots,m} E[U_t(Z,a^{(i)}) \mid e_0,\ldots,e_t], \text{for } t=n \end{cases} \quad (1)$$

where for $t = 0, 1, \ldots, n-1$

$$E[U_t(Z,a^{(0)}) \mid e_0,\ldots,e_t] = \int E[U_{t+1}(Z,a^*(e_0,\ldots,e_{t+1})) \mid e_0,\ldots,e_{t+1}] \cdot f(e_{t+1} \mid e_0,\ldots,e_t)de_{t+1} \quad (2)$$

$U_t(z, a)$ denotes the utility given the realization z of the hazard index Z and the decision alternative a at time t, which itself should be interpreted as the expected value if uncertainties are associated with the assessment of the utility. $f(e_{t+1} \mid e_0, \ldots, e_t)$ is the conditional probability density that e_{t+1} is observed at time $t + 1$ given the information $e_0, e_1, \ldots, e_t$. The probability density can be calculated using conditional probability models described above.

3 ALGORITHMS FOR THE OPTIMIZATION

3.1 *American option*

From a mathematical point of view, the decision problem formulated above is analogous to the problems of the *American option* pricing in the field of finance. Note that an *option* is a financial instrument on the contract for buying or selling an asset, e.g. stock, at a pre-defined execution price. An American option is an option that can be exercised at any time before or at the maturity date of the option. In order to identify the appropriate price of an American option, the expected (discounted) benefits gained by executing the option and by not executing (i.e. postponing the execution) must be compared; the maximum value of these two benefits is regarded as the price of the option. Therein, in order to assess the expected benefit gained by not executing the option, the prices of the option (i.e. its expected benefits) at future times must be known; thus, a backward induction similar in the decision problem formulated above (Equations 1 and 2) is required.

3.2 *Pricing of American option*

This subsection provides the formulation for the pricing of an American option.

Let $\mathbf{Y}(s)$ denote the first-order Markov process for the price of an underlying asset at time s, on which an American option is defined. In the case where $\mathbf{Y}(s)$ is a continuous process in time, it is approximated by discretization and written as $\mathbf{Y}(s_t)$, $t = 0, 1, \ldots, n$. For simplicity, hereafter $\mathbf{Y}(s_t)$ is abbreviated by $\mathbf{Y}_t$. Note that the state of the process $\mathbf{Y}_t$ can be scalar or vector, and discrete or continuous. In the following, the state of the process $\mathbf{Y}_t$ is assumed to be continuous. Similar discussions hold for the other case. The probabilistic characteristics of the first-order Markov process $\mathbf{Y}_t$ is then characterized by the transition probability density $f(\mathbf{y}_{t+1} \mid \mathbf{y}_t;\, t)$ from $\mathbf{Y}_t = \mathbf{y}_{t+1}$ to $\mathbf{Y}_{t+1} = \mathbf{y}_{t+1}$, $t = 0, 1, \ldots, n-1$, with the initial known condition $\mathbf{Y}_0 = \mathbf{y}_0$.

Let denoted by $c(t,\mathbf{y}_t)$ and $h(t, \mathbf{y}_t)$ the *continuing value function* and the *stopping value function* respectively for the American option. The continuing value function $c(t,\mathbf{y}_t)$ represents the expected benefit gained by not executing the option (which corresponds to choose the option $a^{(0)}$ in the formulation in Section 2.3) at time t, given the state of the process $\mathbf{Y}_t = \mathbf{y}_t$. The stopping value function $h(t, \mathbf{y}_t)$ represents the benefit gained by executing the option (which corresponds to choose one of the terminal decisions in the formulation in Section 2.3) at time t, given $\mathbf{Y}_t = \mathbf{y}_t$. Normally, the value of $h(t, \mathbf{y}_t)$ is assumed to be analytically known for all t given $\mathbf{y}_t$; it is calculated by comparing the execution price and the price $\mathbf{y}_t$ of the underlying asset at time t.

Analogous to Equation 1, the benefit $Q(t, \mathbf{y}_t)$ of the American option with any given state $\mathbf{Y}_t = \mathbf{y}_t$ is then written as:

$$Q(t,\mathbf{y}_t) = \begin{cases} h(t,\mathbf{y}_t), & t = n \\ \max\{h(t,\mathbf{y}_t),\, c(t,\mathbf{y}_t)\}, & t = 0,1,\ldots,n-1 \end{cases} \quad (3)$$

and the continuing value function can be expressed by:

$$c(t,\mathbf{y}_t) = E[Q(t+1,\mathbf{Y}_{t+1}) \mid \mathbf{Y}_t = \mathbf{y}_t] \quad (4)$$

With this setting, the pricing of American option is formulated as: To identify the maximized expected benefit $Q(0, \mathbf{y}_0)$under the random process $\mathbf{Y}_t$ with the known initial state $\mathbf{Y}_0 = \mathbf{y}_0$.

3.3 *Least squares Monte Carlo method*

The main technical problem of the option pricing formulated above is the evaluation of the expectation in Equation 4. In principle, for any given state $\mathbf{Y}_t = \mathbf{y}_t$ at time t ($\leq n-1$) the expected value $E[Q(t+1, \mathbf{Y}_{t+1})|\mathbf{Y}_t = \mathbf{y}_t]$ can be estimated by Monte Carlo simulations of the underlying process. However, in order to estimate the conditional expected value above, the conditional expected values $E[Q(t+2, \mathbf{Y}_{t+2})|\mathbf{Y}_{t+1} = \mathbf{y}_{t+1}]$ for the individual realizations $\mathbf{y}_{t+1}$ at time $t+1$, which are simulated by Monte Carlo simulation starting from the state $\mathbf{Y}_t = \mathbf{y}_t$, must be evaluated. This requires another set of simulations corresponding to each of the realizations $\mathbf{y}_{t+1}$. Consequently, the total number of the required simulations increases exponentially as a function of the number n, which usually is not computationally feasible. The Least Squares Monte Carlo method (LSM method) circumvents this, by employing least squares regressions.

In the LSM method, the continuing value functions $c(t,\mathbf{y}_t)$ are approximated with certain functions for all time steps, and these functions are estimated by regression utilizing a single set of realizations of $\mathbf{Y}_t$ simulated by Monte Carlo method. Before introducing how this is performed, it should be emphasized that the continuing value function at time t is a function only of $\mathbf{y}_t$, because the probabilistic characteristics of the underlying first-order Markov process at future times is fully characterized by the state $\mathbf{Y}_t = \mathbf{y}_t$ at time t. Therefore, under regular conditions (see Longstaff & Schwartz (2001) for detail), the continuing value function can be represented by an appropriate set of basis functions $L_k(\mathbf{y}_t)$, $k = 0, 1, \ldots$, with respect to the state $\mathbf{Y}_t = \mathbf{y}_t$ as:

$$c(t,\mathbf{y}_t) = \sum_{k=0}^{\infty} r_{t,k} L_k(\mathbf{y}_t) \quad (5)$$

with the constant coefficients $r_{t,k}$, $k = 0, 1, \ldots$ for $t = 0, 1, \ldots, n-1$. In the regressions, this is approximated as:

$$c(t,\mathbf{y}_t) \approx \sum_{k=0}^{K} r_{t,k} L_k(\mathbf{y}_t) \quad (6)$$

with the finite number K of the basis functions. The coefficients $r_{t,k}$ are estimated by regressions.

The first step in the LSM method is, by Monte Carlo simulations, to generate a set of b independent paths of the random process $\mathbf{Y}_t$ according to the transition density $f(\mathbf{y}_{t+1}|\mathbf{y}_t;t)$, $t = 0, 1, \ldots, n-1$, with the initial condition $\mathbf{Y}_0 = \mathbf{y}_0$. These realizations of the paths are denoted by $\mathbf{x}^i = (\mathbf{x}_0^i, \mathbf{x}_1^i, \ldots, \mathbf{x}_n^i)$, $i = 1, 2, \ldots, b$. Note that, $\mathbf{x}_0^i = \mathbf{y}_0$ for all paths.

The second step is to estimate the continuing value functions $c(t,\mathbf{y}_t)$ at times $t = 0, 1, \ldots, n-1$. This is performed backwards in time, since the continuing value functions are defined backward recursively. At time n, the option expires; hence, the realization of the benefit $Q(t,\mathbf{y}_t)$ for each individual path is calculated as $Q(n,\mathbf{x}_n^i) = h(n,\mathbf{x}_n^i)$ according to Equation 3. Then, moving to time $n-1$, relating each realization $Q(n,\mathbf{x}_n^i)$ to $\mathbf{x}_{n-1}^i$, the dataset $(\mathbf{x}_{n-1}^i, Q(n,\mathbf{x}_n^i))$ $i = 1, 2, \ldots, b$ is obtained. This dataset is utilized to approximate $c(n-1, \mathbf{y}_{n-1}) = E[Q(n,\mathbf{Y}_n)|\mathbf{Y}_{n-1} = \mathbf{y}_{n-1}]$ with Equation 6 by least squares regression, see Figure 2. The estimated continuing value function is denoted by $\hat{c}(n-1, \mathbf{y}_{n-1})$. Here, it should be emphasized that the individual realizations $Q(n,\mathbf{x}_n^i)$ corresponding to the different realizations $\mathbf{x}_{n-1}^i$ are "shared" to estimate $E[Q(n,\mathbf{Y}_n) \mid \mathbf{Y}_{n-1} = \mathbf{x}_{n-1}^j]$ ($j \neq i$), but also to interpolate and extrapolate the estimates over the support of $\mathbf{y}_{n-1}$ where the realizations of $\mathbf{y}_{n-1}$ are not available in the simulation.

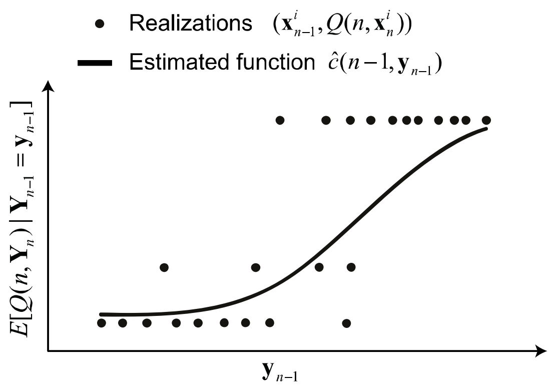

Figure 2. Illustration of the estimation of the continuing value function using the realizations $(\mathbf{x}^i_{n-1}, Q(n,\mathbf{x}^i_n))$.

Having obtained the approximation of the continuing value function $\hat{c}(n-1, \mathbf{y}_{n-1})$ for time $t = n-1$, the realizations of $Q(n-1, \mathbf{y}_{n-1})$, i.e. $Q(n-1, \mathbf{x}^i_{n-1})$, are calculated using Equation 3 with $\hat{c}(n-1, \mathbf{y}_{n-1})$ for $\mathbf{y}_{n-1} = \mathbf{x}^i_{n-1}$. Moving to time $n-2$, each realization $Q(n-1, \mathbf{x}^i_{n-1})$ is related to a state $\mathbf{x}^i_{n-2}$, such that the dataset $(\mathbf{x}^i_{n-2}, Q(n-1, \mathbf{x}^i_{n-1}))$ $i = 1, 2, \ldots, b$ is obtained. Then, in the same way as at time $t = n-1$, the continuing value function is estimated as $\hat{c}(n-2, \mathbf{y}_{n-2})$. This procedure is repeated until time $t = 1$, such that for all paths the maximum expected benefit $Q(1, \mathbf{x}^i_1)$ is obtained. Since the initial value of all paths equals $\mathbf{y}_0$ the estimate of the continuing value function $\hat{c}(0, \mathbf{y}_0)$ is obtained by the average of the realizations $Q(1, \mathbf{x}^i_1)$, $i = 1, 2, \ldots, b$ and finally $Q(0, \mathbf{y}_0)$ is obtained with Equation 3.

3.4 *Extensions for real time decision problems in engineering*

Real time decision problems in the face of emerging natural hazards and other pre-posterior types of engineering decision problems have several differences in the problem setting from the pricing of options described above. Among others, (1) the underlying random processes may not be a first-order Markov process, and (2) the stopping values, i.e., the expected utilities corresponding to terminal decisions, are not analytically known and often the evaluation of the stopping values requires Monte Carlo simulations.

Concerning the first difference, the LSM method can be straightforwardly extended; by approximating the continuing value function defined by Equation 5 with a set of basis functions whose arguments include all the states of the underlying process to the extent that these states can fully characterize the probabilistic characteristics of the process in the future. As for the second difference, it is found in the example in Section 4 that only relatively few numbers of simulations are required to estimate the stopping values. The reason for this is in principle the same as emphasized in the explanation of the use of least squares regressions for estimating the continuing value functions; information on the estimated stopping values in different paths is "shared", hence, precise estimates of the individual stopping values are not necessary. However, it should be mentioned that the estimated value of $Q(0, \mathbf{y}_0)$ in this way is biased high due to the convexity of the max-operator in Equation 3. In contrast, when a decision problem is formulated as the minimization of an expected cost, the estimate of $Q(0, \mathbf{y}_0)$ is biased low, as can be seen in the example below. The implementation of these extensions is explained along with the example.

4 EXAMPLE

4.1 *Problem setting*

A decision maker is faced to decide whether or not the operation of an offshore platform should be shut down in the emergence of a typhoon event. The possible decision alternatives are the terminal decisions of shut-down $a^{(1)}$, no shut-down $a^{(2)}$ and postponing the terminal decision $a^{(0)}$. When the decision maker chooses $a^{(0)}$, she/he can obtain further information on the state of the typhoon such as position, central pressure, translation speed and direction of the typhoon. The information is assumed to be provided by a meteorological agency once every six hours at no cost. It is assumed that the shut-down of the operation of the platform takes twelve hours after making the terminal decision $a^{(1)}$. In the decision problem considered here, it is assumed that the decision is terminated within 30 hours, and the time frame is discretized into five time intervals of six hours; i.e. there are six time steps where information becomes available and the decisions are made. This assumption seems reasonable, since the typhoon is very likely to pass through the area relevant for the platform until the 6th time step ($t = 5$), see Figure 3. The figure shows two possible transitions (indicated by dashed lines with circles) of the typhoon.

Since the time frame of the decision problem here is relatively short, discounting is not considered. In what follows, the models employed and further assumptions are explained.

4.2 *Probabilistic typhoon model*

The typhoon model developed by the group of Risk and Safety at ETH Zurich is employed for the modeling of the wind speed at the platform induced by the typhoon; see Graf et al. (2009). The typhoon model is composed of five components; occurrence model, transition model, wind

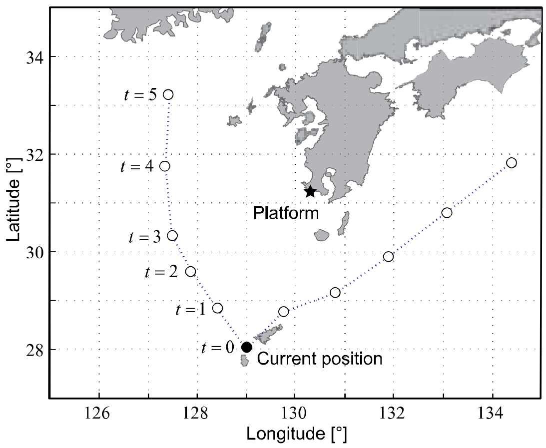

Figure 3. Illustration of the transition of the typhoon and the location of the platform (after Nishijima et al. (2009)).

field model, surface friction model and vulnerability model. For this example, only the transition model, the wind field model and the surface model are of relevance. In the following, a short summary of the transition model employed in the example is provided in order to show the probabilistic characteristics of the random process underlying the decision problem.

The transition model describes the transition of the state of typhoons probabilistically. It is assumed that the state of a typhoon is characterized by three parameters: V_t representing the translation speed [km/h]; Γ_t the translation angle [°] and $P_{C,t}$ the central pressure [hPa]. These parameters are modeled by the following components of a vector Markov process:

$$\ln(V_{t+1}) = (1+a_2)\ln(V_t) + a_1 + a_3\Gamma_t + \varepsilon_{V,t+1} \tag{7}$$

$$\Gamma_{t+1} = (1+b_3)\Gamma_t + b_1 + b_2V_t + b_4\Gamma_{t-1} + \varepsilon_{\Gamma,t+1} \tag{8}$$

$$P_{C,t+1} = c_1 + c_2P_{C,t} + c_3P_{C,t-1} + c_4P_{C,t-2} + c_5T_t + c_6(T_{t+1} - T_t) + \varepsilon_{P_C,t+1} \tag{9}$$

where T_t is the sea surface temperature (SST) at the location of the typhoon at time t. The coefficient vectors $\mathbf{a} = (a_1, a_2, a_3)^T$, $\mathbf{b} = (b_1, b_2, b_3, b_4)^T$ and $\mathbf{c} = (c_1, c_2, \ldots, c_6)^T$, as well as the distribution of the random terms $\varepsilon_{V,t+1}$, $\varepsilon_{\Gamma,t+1}$ (both are modeled by normal distributions with mean zero and standard deviations $\sigma_{V,t+1}$ and $\sigma_{\Gamma,t+1}$ respectively) and $\varepsilon_{P_C,t+1}$ (modeled by an empirical distribution) are estimated using historical data. Therein, in order to incorporate the spatial and temporal inhomogeneity of the probabilistic characteristics of the typhoon transition, these coefficients and the distributions are estimated for each individual grid area (5° latitude-by-5° longitude grids in the northwest Pacific) and for each month.

The hazard index, i.e. the 10-minute sustained wind speed u [m/s] at the platform in this example, is calculated by the wind field model and the surface friction model. Given the state of the typhoon together with the radius of maximum wind speed R_M and the roughness length z_0 of the location of the platform, the wind speed u is calculated deterministically.

Note here that the random process underlying the decision problem is expressed by $\mathbf{Y}_t = (V_t, \Gamma_t, P_{C,t})^T$ at times t, $t = 0, 1, \ldots, n$, and it is assumed that the precise state of the typhoon is known at each time, hence, the information $\mathbf{E}_t$ about the state of the typhoon is equal to $\mathbf{Y}_t$.

4.3 *Other assumptions*

The platform is assumed to be damaged if the 10-minute sustained wind speed u at surface of the location of the platform exceeds the threshold u_c (= 38[m/s]), while the platform is in operation. The expected damage cost C_D is equal to 10. The platform is assumed not to be damaged if the wind speed does not exceed the threshold, or if the operation of the platform is successfully shut down, i.e. not in operation when the wind speed exceeds the threshold. However, in the latter case the cost C_{PI} for production interruption is incurred. Here C_{PI} is set equal to 1. The summary of the assumed consequence model is shown in Table 1.

Three cases are possible in which the expected damage cost C_D is incurred; the first case is the case where the decision $\boldsymbol{a}^{(2)}$ is made and the wind speed exceeds the threshold u_c, the second case is where the decision $\boldsymbol{a}^{(1)}$ is made but the wind speed exceeds u_c before the shut-down is completed, and last case is the case where the decision $\boldsymbol{a}^{(0)}$ is made and the wind speed exceeds u_c before the next time a decision is made. No consequence occurs if and only if the decision $\boldsymbol{a}^{(2)}$ is made and the wind speed does not exceed the threshold. Remember that until time n, either action $\boldsymbol{a}^{(1)}$ or $\boldsymbol{a}^{(2)}$ has to be chosen. The expected cost C_{PI} for production interruption is incurred if the decision $\boldsymbol{a}^{(1)}$ is made and the shut-down is completed before the wind speed u exceeds the threshold (if it does), or if the decision $\boldsymbol{a}^{(1)}$ is made but the wind speed does not exceed the threshold.

Table 1. Conditions and associated costs postulated in the consequence model.

	Wind speed	
Platform	$u > u_c = 38$[m/s]	$u \le u_c = 38$[m/s]
In operation	$C_D = 41$	0
Not in operation	$C_{PI} = 1$	$C_{PI} = 1$

Table 2. Assumed initial conditions.

Central pressures at t = –2,–1,0	930, 930, 930 [hPa]
Translation speeds at $t = 0$	20 [km/h]
Translation angles at $t = -1, 0$	0[°],0[°] (Northwards)
Position at $t = 0$	(129°E, 28°N)
SST at the location of the typhoon at $t = 0$	27.9 [°C]
Radius of max. wind speed, R_M	100 [km]
Location of the platform	(130.3°E, 31.25°N)

In the example, it is assumed that R_M is constant and the current as well as the relevant previous states of the typhoon are known. The initial conditions assumed in the example are summarized in Table 2.

4.4 *Solution with the extended LSM method*

The first step is to simulate, by Monte Carlo simulation, b independent paths $\mathbf{x}^i = (\mathbf{x}_0^i, \mathbf{x}_1^i, ..., \mathbf{x}_5^i)$ with the initial values $\mathbf{y}_0 = (v_0, \gamma_0, p_{C,0})^T$, $\mathbf{y}_{-1} = (\gamma_{-1}, p_{C-1})^T$ and $\mathbf{y}_{-2} = p_{C,-2}$. Note here that the individual path in regard to the typhoon movement is first generated as the collection of the realizations of the translation speed V_t and angle Γ_t according to Equations 7 and 8; then, using these realizations together with the initial location of the typhoon, the locations of the typhoon at times $t = 0, 1, \ldots, 5$ are identified. Thus, the variable $\mathbf{Y}_t$ representing the state of the typhoon at time t and its realizations $\mathbf{x}_t^i$ can be (re-)composed, e.g. by the location (longitude and latitude) and the central pressure, instead of the translation speed and angle and the central pressure. Having simulated the paths, the 10-minute sustained wind speeds at 10-minute intervals are calculated by interpolating the states of the realized typhoons.

The second step is to estimate the function for the expected cost corresponding to the postponing of the terminal decision at each time. Note, this expected cost corresponds to the negative of the expected utility in Equation 1 or the continuing value in Equation 3. Since the underlying random process (i.e. the transition of the typhoon) is a third-order Markov process (first-order with respect to the movement and third-order with respect to the central pressure), this is a function of the typhoon states in the last three time steps. Thus, it is written as $g(t, \mathbf{x}_{t-1}, \mathbf{x}_{t-2}, \mathbf{x}_{t-3})$, which hereafter is called *continuing cost function*. The continuing cost function can in principle be approximated with a set of basis functions with respect to $\mathbf{x}_{t-1}, \mathbf{x}_{t-2}, \mathbf{x}_{t-3}$ as in Equation 6; however, it is anticipated that the function may be better represented with parameters, which themselves are the functions of $\mathbf{x}_{t-1}, \mathbf{x}_{t-2}, \mathbf{x}_{t-3}$ and are physically more meaningful. With trial-and-errors, in this example the translation speed v_{t-1}, the distance d_{t-1} between the location of the typhoon and the platform, and the central pressures $p_{c,t-1}$, $p_{c,t-2}$, $p_{c,t-3}$ are adopted. Consequently, the continuing cost function is assumed to be represented by $g(t\text{-}1, v_{t-1}, d_{t-1}, p_{c,t-1}, p_{c,t-2}, p_{c,t-3})$, and the function is approximated based on a set of basis functions as in Equation 6. Several sets of the basis functions are investigated and these performances are discussed in the next section together with the results.

At time $t = 5$, since the decision must be terminated, the expected costs corresponding to $a^{(1)}$ and $a^{(2)}$ (hereafter the minimum of these two costs is called *stopping cost*) are determined in the similar manner as in Equation 3 for the case $t = n$, and the realizations of the stopping cost for the individual paths are obtained. Using the set of the realizations of the stopping cost and the corresponding realizations of $(v_4, d_4, p_{c,4}, p_{c,3}, p_{c,2})$ for the individual paths, the continuing cost function $g(4, v_4, d_4, p_{c,4}, p_{c,3}, p_{c,2})$ is estimated by least squares regression. At time $t = 4$, using the estimated continuing cost function $\hat{g}'(4, v_4, d_4, p_{c,4}, p_{c,3}, p_{c,2})$ and the stopping costs, which are estimated by Monte Carlo simulation of the sample size M, the realizations of the continuing costs are obtained. Applying the regression, the continuing cost function $g'(3, v_3, d_3, p_{c,3}, p_{c,2}, p_{c,1})$ is estimated. This procedure is repeated for $t = 3,2,1$.

The last step is to compare the expected costs corresponding to $a^{(0)}$, $a^{(1)}$ and $a^{(2)}$ at time $t = 0$. The expected costs for $a^{(1)}$ and $a^{(2)}$ are estimated by sufficient numbers of Monte Carlo simulations, whereas the expected cost for $a^{(0)}$ is estimated as the average of the realizations of the minimized expected costs at time $t = 1$ over all the paths.

4.5 *Results*

Following the algorithm described above, the optimal decision at time $t = 0$ is identified. The estimated value of the expected cost for $a^{(0)}$ is 1.9881 (b = 200,000; M = 300), whereas the expected costs of the two terminal decisions $a^{(1)}$ and $a^{(2)}$ are estimated as 1.8614 and 2.0429 respectively. Thus, the optimal decision is identified as $a^{(1)}$; i.e., to evacuate.

In the optimization, various sets of basis functions (Linear, Chebyshev, Legendre, weighted Laguerre and Power polynomials) with different truncation order K are examined, and it is found that the numerical result shown above is insensitive to the choice of the basis functions and the order K. Thus, the result and the figures are shown for the case of the first-order polynomials.

The convergence of the numerical results with respect to the numbers b and M are shown in Figure 4. The figure above shows the convergence as a function of the number b of generated paths, where $M = 1$ is fixed. The figure below shows the

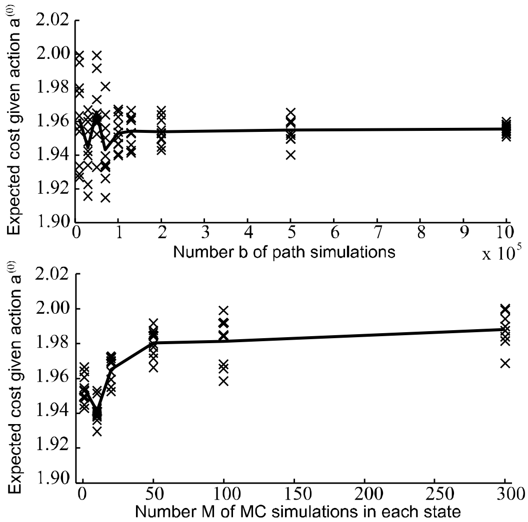

Figure 4. Two illustrations of the convergence of the estimated expected cost for $a^{(0)}$ at time $t = 0$: (top) as a function of the number b of generated paths ($M = 1$); (bottom) as a function of the number M of the Monte Carlo simulations ($b = 200{,}000$). The results of 10 independent trials are illustrated, and the bold line represents the average of these results.

convergence as a function of the number M of the simulations for estimating expected stopping costs for each typhoon state at each time, where $b = 200{,}000$ is fixed. As explained earlier this figure shows that the estimated values are biased low and converge as M increases.

It is found that the solution with a coarse discretization of the probability space, adopted in Nishijima et al. (2009), is biased; the possibility of which is mentioned in their paper.

5 DISCUSSION

The computational time required to obtain one solution ($b = 200{,}000$ and $M = 50$) in the example above is approximately two hours with a standard PC. Various variance reduction methods in Monte Carlo simulation such as importance sampling techniques and use of quasi-random sequences can be easily implemented into the proposed method, which facilitate further reduction of the computational time.

A larger number b of the Monte Carlo simulations for estimating the expected stopping costs may be required for the cases where the probability of the occurrence of the consequences is smaller. In such cases, the separate simulations for estimating the stopping costs for individual states of the typhoon as undertaken in the example may be computationally prohibitive; it totally requires $b \cdot M$ realizations of (part of) the typhoon paths. This may be circumvented by regressing the expected stopping costs using the b realizations of the typhoon path that are simulated at the first step in the LSM method; i.e. the idea described in Section 3.3 to estimate the continuing value functions $c(t, \mathbf{y}_t)$ is applied also for estimating the function for the relation between the expected stopping cost and the state of the typhoon. However, the dependency between these two estimated functions caused by using the same set of realizations and its consequences with respect to bias and convergence must be investigated.

Finally, the LSM method and its extensions investigated in the present paper may be applicable for broader pre-posterior type of engineering decision problems, where the reduction of epistemic uncertainty or both epistemic uncertainty and aleatory uncertainty play the role in motivating decision makers to "wait".

6 CONCLUSION

The present paper (1) introduces the Least Squares Monte Carlo method (LSM method) which is originally developed for option pricing, (2) points out that the method is useful to pre-posterior type of engineering decision problem, and (3) proposes extensions of the LSM method for the application to an engineering decision problem; i.e. real time decision making in the face of emerging natural hazards. The performance of the proposed approach is investigated with an example, and it is shown that it is applicable in practice from a computational point of view. Possible improvements and applications are briefly discussed.

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Enhanced least squares Monte Carlo method for real-time decision optimization for evolving natural hazards

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Enhanced least squares Monte Carlo method for real-time decision optimizations for evolving natural hazards

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ABSTRACT: The present paper aims at enhancing a solution approach proposed by Anders & Nishijima (2011) to real-time decision problems in civil engineering. The approach takes basis in the Least Squares Monte Carlo method (LSM) originally proposed by Longstaff & Schwartz (2001) for computing American option prices. In Anders & Nishijima (2011) the LSM is adapted for a real-time operational decision problem; however it is found that further improvement is required in regard to the computational efficiency, in order to facilitate it for practice. This is the focus in the present paper. The idea behind the improvement of the computational efficiency is to "best utilize" the least squares method; i.e. least squares method is applied for estimating the expected utility for terminal decisions, conditional on realizations of underlying random phenomena at respective times in a parametric way. The implementation and efficiency of the enhancement is shown with an example on evacuation in an avalanche risk situation.

1 INTRODUCTION

Real-time decision optimization has become an interesting and challenging topic with the progress of real-time information processing technology. Relevant applications in civil engineering include situations where operational decisions have to be made in response to real-time information on evolving natural hazard events. In these situations, all real-time information available can and should be best utilized to find the optimal decisions at respective times; taking into account not only possible future outcomes, but also opportunities to make decisions in future times. This type of decision problem is generally described within the framework of the preposterior/sequential decision analysis, see Nishijima et al. (2009); however, the development of efficient solution schemes to the formulated decision problems has remained a technical challenge.

An efficient solution scheme is proposed by Anders & Nishijima (2011), taking basis in the Least Squares Monte Carlo method (hereafter, abbreviated as LSM), which is developed originally by Longstaff & Schwartz (2001) for American option pricing. In Anders & Nishijima (2011) the original LSM is extended and applied to an example for a real-time operational decision problem for shut-down of the operation of a technical facility in the face of an approaching typhoon. However, due to multiple evaluations of the expected consequences for different possible future states of the typhoon by means of Monte Carlo simulation (MCS), the solution scheme becomes less efficient, if the computational time required for MCS becomes dominant. The present paper proposes an enhanced solution scheme, which overcomes this drawback.

The present paper is organized as follows. Section 2 formulates the real-time decision problems in consideration within the framework presented in Nishijima & Anders (2012). Section 3 provides a brief introduction to the extensions of the LSM. Thereafter, the proposed enhancement to the extended LSM is introduced. Section 4 presents an application example, which illustrates the performance of the enhanced LSM (eLSM). Section 5 concludes the presented work.

2 REAL-TIME DECISION FRAMEWORK

2.1 *Problem setting*

The decision situation considered in the present work is characterized by the following characteristics, see Nishijima et al. (2009): (a) The hazard process evolves relatively slowly and allows for reactive decision making; (b) information relevant to predict the severity of the evolving hazard event can be obtained prior to its impact; (c) the decision making is subject to uncertainties, part of which might be reduced at a cost; (d) decision makers have options for risk reducing activities which may be commenced at any time, supported by the information available up to the time. Here, "waiting" to commence the risk reducing measures implies the reduction of uncertainty but might also reduce available time to complete the risk reducing activities; (e) and on top of all, the decisions must be made fast, in near-real time. The decision makers are then required to make decisions whether they commence one of the risk reducing activities which at the same time terminates the decision process (hence, hereafter these are called terminal decisions) or they postpone making a terminal decision.

2.2 *Formulation of decision problem*

The decision problem characterized above can be formulated in accordance with Nishijima & Anders (2012). Denote by A_t the decision set consisting of possible decision alternatives at time t. Here, time is discretized. It is assumed that the decisions must be terminated before or at time n; hence, $t = \{0,1,2,...,n\}$. The decision set A_t generally depends on the decisions made before time t. If a decision maker decides to terminate the decision process, no decision alternative is available at later decision times. It is thus convenient to divide the decision set into two mutually exclusive subsets; i.e. $A_t = A_t^{(c)} \cup A_t^{(s)}$, $A_t^{(c)} \cap A_t^{(s)} = \varnothing$ where $A_t^{(c)}$ consists of one decision alternative $a_t^{(c)}$ "waiting" (i.e. $A_t^{(c)} = \{a_t^{(c)}\}$) and $A_t^{(s)}$ is the set consisting of risk reducing decisions available. Let $\mathbf{E}_t$ be a set of variables representing possible information available at time t on the states of the evolving natural hazard event in consideration.

Given that no terminal decision is made up to time t, the optimal decision a_t^* at time t is identified as the one that maximizes the expected utility at time t conditional on the collection of the information up to time t:

$$E[U_t(\mathbf{Z}, a_t^*) \mid \underline{\mathbf{e}}_t] = \begin{cases} \max\limits_{a_t \in A_t} E[U_t(\mathbf{Z}, a_t) \mid \underline{\mathbf{e}}_t], & \text{for } t = 0,1,...,n-1 \\ \max\limits_{a_t \in A_t^{(s)}} E[U_t(\mathbf{Z}, a_t) \mid \underline{\mathbf{e}}_t], & \text{for } t = n \end{cases} \tag{1}$$

where, for $t = 0,1,....,n-1$ and $a_t^{(c)}$,

$$E[U_t(\mathbf{Z}, a_t^{(c)}) \mid \underline{\mathbf{e}}_t] = \int E[U_{t+1}(\mathbf{Z}, a_{t+1}^*) \mid a_t^{(c)}, \underline{\mathbf{e}}_{t+1}] f(\mathbf{e}_{t+1} \mid, \underline{\mathbf{e}}_t) d\mathbf{e}_{t+1}. \tag{2}$$

Here, $U_t(\mathbf{z}, a_t)$ is the utility, which is a function of the decision alternative a_t and the realization $\mathbf{z}$ of the hazard index $\mathbf{Z}$ relevant for the decision problem. The hazard index $\mathbf{Z}$ is defined through the underlying random sequence $\{\mathbf{Y}_t\}_{t=0}^n$, representing the evolution of the natural hazard event. $\underline{\mathbf{e}}_t = (\mathbf{e}_0, \mathbf{e}_1, ..., \mathbf{e}_t)$ is the collection of the information available up to time t. Here, it is assumed that $\mathbf{y}_t = \mathbf{e}_t$, $(t = 0,1,...,n)$; namely, the state of the event relevant to the decision problem is known to the decision maker without uncertainty. Thus, the symbols $\mathbf{y}_t$ and $\mathbf{e}_t$ are utilized interchangeably in the following. $f_t(. \mid \mathbf{e}_t)$ is the conditional probability density/mass function of information $\mathbf{E}_{t+1}$ given $\mathbf{E}_t = \mathbf{e}_t$. From Equation 2 it is seen that for the decision $a_t^{(c)}$ at time t the optimization requires to know all optimal decisions at future times, $t+1, t+2, ..., n$; hence, backward induction is required. Equation 1 can be rewritten as:

$$q_t(\underline{\mathbf{e}}_t) = \begin{cases} \max\{h_t(\underline{\mathbf{e}}_t), c_t(\underline{\mathbf{e}}_t)\}, & \text{for } t = 0,1,...,n-1 \\ h_t(\underline{\mathbf{e}}_t), & \text{for } t = n. \end{cases} \tag{3}$$

Here,

$$q_t(\underline{\mathbf{e}}_t) = E[U_t(\mathbf{Z}, a_t^*) \mid \underline{\mathbf{e}}_t] \tag{4}$$

$$h_t(\underline{\mathbf{e}}_t) = \max_{a_t \in A_t^{(s)}} l_t(a_t, \underline{\mathbf{e}}_t) \tag{5}$$

$$l_t(a_t, \underline{\mathbf{e}}_t) = E[U_t(\mathbf{Z}, a_t) \mid \underline{\mathbf{e}}_t], \; a_t \in A_t^{(s)} \tag{6}$$

$$c_t(\underline{\mathbf{e}}_t) = E[q_{t+1}((\underline{\mathbf{e}}_t, \mathbf{E}_{t+1})) \mid \underline{\mathbf{e}}_t]. \tag{7}$$

The function $q_t(\underline{\mathbf{e}}_t)$, $t = 0,1,...,n$, is the maximized expected utility, hereafter abbreviated as MEU. The functions $h_t(\underline{\mathbf{e}}_t)$ and $c_t(\underline{\mathbf{e}}_t)$ are named stopping value function (SVF) and continuing value function (CVF), respectively. Note that, whereas the evaluation of the SVF is straightforward in the sense that it does not require backward induction, the evaluation of CVF requires backward induction. However, no matter how complex the structure of the decision optimization problem may seem, $c_t(\underline{\mathbf{e}}_t)$ is only a function of $\underline{\mathbf{e}}_t$. Furthermore, if the underlying random sequence $\{\mathbf{Y}_t\}_{t=0}^n$ follows s^{th}-order Markov sequence, $c_t(\underline{\mathbf{e}}_t)$ is a function effectively of the last s information, $\mathbf{e}_{t-s+1}, \mathbf{e}_{t-s+2}, ..., \mathbf{e}_t$.

3 ENHANCEMENT OF THE EXTENDED LSM

3.1 *Extended LSM*

The main technical challenge of the optimization problem formulated in Section 2.2 is the evaluation of the CVF. The CVF can in principle be evaluated by calculating the expected utility for each combination of all possible discretized future states and possible decision opportunities. However, in practice this is not computationally feasible, since the total number of the possible combinations increases exponentially as a function of the number n. The LSM circumvents this by employing the least squares method. The idea behind the LSM is that any regular function can be represented by a linear combination of an appropriate set of basis functions; therefore, the CVF is approximated as such, for details see Longstaff & Schwartz (2001). In the context of American option pricing, this means that if the price of a stock follows a first order Markov sequence, the price of its American option is a function only of the current stock price. Consequently the CVF is approximated as a superposition of basis functions whose argument is only the current stock price. The way on how this idea is implemented in the optimization is explained along with the extended version of the LSM (called extended LSM) in the following.

In Anders & Nishijima (2011), it is demonstrated that the idea behind the LSM can be applied for the case where the underlying random sequence follows an inhomogeneous higher-order Markov sequence. Therein, two extensions are made: (1) the assumptions on the underlying random sequence is relaxed from stationary first-order Markov sequence to non-stationary higher-order Markov sequence, and (2) the SVF is evaluated by MCS. Note that in many engineering applications the SVF cannot be evaluated analytically, unlike the case when executing American options. Moreover, the MCS in the second extension is computationally expensive and the computational effort increases proportional to n. In the following, the steps of the extended LSM are presented:

Step 1: A set of b independent realizations (paths) of the random sequence $\mathbf{Y}_t$ is generated by MCS according to the Markov transition density $f_t(\mathbf{y}_{t+1} \mid \underline{\mathbf{y}}_t)$, $t = 0,1,...,n-1$ with the initial condition $\mathbf{Y}_0 = \mathbf{y}_0$, where $\underline{\mathbf{y}}_t = (\mathbf{y}_0, \mathbf{y}_1, ..., \mathbf{y}_t)$. These paths are denoted by $\mathbf{y}^i = (\mathbf{y}_0^i, \mathbf{y}_1^i, ..., \mathbf{y}_n^i)$, $i = 1,2,...,b$, where $\mathbf{y}_0^i = \mathbf{y}_0$ for all paths, see Figure 1 (a).

Step 2: The SVF for all realizations $\{\mathbf{y}_t^i\}_{t=0}^n$, $i = 1,2,...,b$, are estimated by additional MCS.

Step 3: Starting at the time horizon n as illustrated in Figure 1 (a), for each path i the value of the MEU $q_n((\underline{\mathbf{y}}_{n-1}, \mathbf{Y}_n))$ is identified by equating $q_n(\underline{\mathbf{y}}_n^i) = h_n(\underline{\mathbf{y}}_n^i)$ according to Equation 3.

Step 4: Moving to time $n-1$ the CVF is approximated. This begins by relating each MEU $q_n(\underline{\mathbf{y}}_n^i)$ to $\underline{\mathbf{y}}_{n-1}^i$, to obtain the dataset $(\underline{\mathbf{y}}_{n-1}^i, q_n(\underline{\mathbf{y}}_n^i))$, $i = 1,2,...,b$, see the dots in Figure 1 (b). This dataset is utilized to approximate the CVF $c_{n-1}(\underline{\mathbf{y}}_{n-1})$ with the least squares method. The approximated CVF is illustrated by the curve in Figure 1 (b). See Nishijima & Anders (2012) for details. The approximated CVF is denoted by $\hat{c}_{n-1}(\underline{\mathbf{y}}_{n-1})$.

Step 5: Having obtained $\hat{c}_{n-1}(\underline{\mathbf{y}}_{n-1})$ for time $t=n-1$, the realizations of $q_{n-1}((\underline{\mathbf{y}}_{n-2},\mathbf{Y}_{n-1}))$, i.e. $q_{n-1}(\underline{\mathbf{y}}^i_{n-1})$, $i=1,2,...,b$, are determined as follows:

$$q_{n-1}(\underline{\mathbf{y}}^i_{n-1})=\begin{cases} h_{n-1}(\underline{\mathbf{y}}^i_{n-1}), \text{ if } h_{n-1}(\underline{\mathbf{y}}^i_{n-1})>\hat{c}_{n-1}(\underline{\mathbf{y}}^i_{n-1}) \\ q_n(\underline{\mathbf{y}}^i_n), \quad \text{otherwise.} \end{cases} \tag{8}$$

The procedure is repeated backwards in time until $t=1$, hence $q_1(\underline{\mathbf{y}}^i_1)$ is obtained for all paths.

Step 6: At $t=0$ the estimate $\hat{c}_0=\hat{c}_0(\mathbf{y}_0)$ is defined as the average of the realizations $q_1(\underline{\mathbf{y}}^i_1)$, $i=1,2,...,b$. Finally $q_0(\mathbf{y}_0)$ is obtained as the maximum of $\hat{c}_0(\mathbf{y}_0)$ and $h_0(\mathbf{y}_0)$. The optimal decision is the one that corresponds to the maximum.

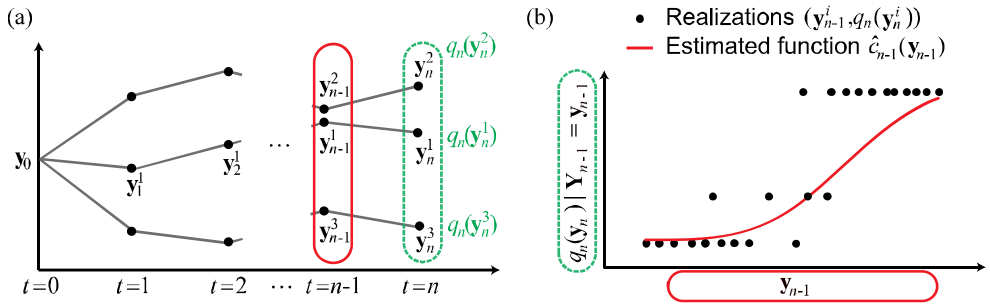

Figure 1. Illustration of (a) three paths of a underlying random sequence with corresponding values $q_n(\mathbf{y}^i_n)$ $(i=1,2,3)$ at time n and (b) the estimation of the CVF using the sets $(\mathbf{y}^i_{n-1}, q_n(\mathbf{y}^i_n))$.

3.2 *Enhancement of the extended LSM*

As seen in Section 3.1, additional MCS are required in Step 2 to estimate the SVF in the extended LSM. The enhanced LSM (eLSM) circumvents this by applying the least squares method for the estimation of the SVF. The general idea is explained in the following.

Analogous to Equation 5 the SVF $h_{t,\text{eLSM}}(\underline{\mathbf{y}}_t)$ of the eLSM is defined as maximum of the conditional expected utilities $l_{t,\text{eLSM}}(a^{(j)}_t,\underline{\mathbf{y}}_t)$ with respect to the terminal decisions $a^{(j)}_t \in A^{(s)}_t$. Here, the functions $l_{t,\text{eLSM}}(a^{(j)}_t,\underline{\mathbf{y}}_t)$ are estimated with the least squares method using the realizations $\{\underline{\mathbf{y}}^i_t\}^b_{i=1}$, similar to the estimation of the CVF described in Section 3.1; i.e. by linear combination of basis functions $\{L_{t,k}(\cdot)\}^K_{k=1}$ with unknown coefficients $r^{(j)}_{t,k}$

$$l_{t,\text{eLSM}}(a^{(j)}_t,\underline{\mathbf{y}}_t)\cong\sum_{k=1}^{K}L_{t,k}(\underline{\mathbf{y}}_t)r^{(j)}_{t,k}. \tag{9}$$

Therein the least squares method is utilized to estimate the coefficients $\mathbf{r}^{(j)}_t=(r^{(j)}_{t,1},r^{(j)}_{t,2},...,r^{(j)}_{t,K})^T$ by minimizing the sum of the squared distances between the observed realizations of the dependent variable $l_{t,\text{eLSM}}(a^{(j)}_t,\underline{\mathbf{y}}_t)$ in the dataset and their fitted values; in the matrix form this is expressed by

$$\mathbf{r}^{(j)}_t=\arg\min_{\mathbf{r}}\|\mathbf{u}^{(j)}_t-\mathbf{L}_t\mathbf{r}\|^2_2 \tag{10}$$

where $\|\cdot\|_2$ denotes the Euclidian norm, $\mathbf{L}_t$ is a $b\times K$ matrix consisting of values of basis functions $\{L_{t,k}(\cdot)\}^K_{k=1}$ which are functions of realizations of $\underline{\mathbf{y}}_t$ and $\mathbf{u}^{(j)}_t$ the $b\times 1$ vector of observed future utilities $u_t(\mathbf{z}^i,a^{(j)}_t)$, $i=1,2,...,b$, given the realization $\mathbf{z}^i$ of the hazard index related to the path $\underline{\mathbf{y}}^i_n$ and decision $a^{(j)}_t$ is made at time t. Note that $u_t(\mathbf{z}^i,a^{(j)}_t)$ is a realization of $l_{t,\text{eLSM}}(a^{(j)}_t,\underline{\mathbf{y}}^i_t)$. Furthermore, to avoid a bias introduced by the least squares estimation within the determination of the MEU, Equation 8 is changed to:

$$q_{t,\text{eLSM}}(\underline{\mathbf{y}}^i_t)=\begin{cases} u^*_t(\mathbf{z}^i,a^*_t), \quad \text{if } \hat{h}_{t,\text{eLSM}}(\underline{\mathbf{y}}^i_t)>\hat{c}_{t,\text{eLSM}}(\underline{\mathbf{y}}^i_t) \\ q_{t+1,\text{eLSM}}(\underline{\mathbf{y}}^i_{t+1}), \text{ otherwise} \end{cases} \tag{11}$$

where $u^*_t(\mathbf{z}^i,a^*_t)$ is the observed future utility of path i for the optimal terminal decision a^*_t.

4 EXAMPLE

The aim of this section is to demonstrate how the eLSM can be applied to an engineering decision problem and to compare its performance to that of the extended LSM. For this purpose, a decision situation of the evacuation of people in the face of an avalanche event is considered.

4.1 *Problem setting*

Consider a village located nearby a mountain slope having a critical angle for snow avalanches. Given prevailing winter conditions and critical snow heights, a decision has to be made whether to evacuate people from the village. Assume that the occurrence of a severe avalanche, causing significant damages to the village, depends only on the additional snow height S_t; i.e. S_t is the hazard index. Further, if S_t exceeds the threshold $\tilde{s}\ (=800\ [mm])$ a severe avalanche occurs. Weather forecast by a meteorological agency predicts that snowfall can occur within the next hours, which increases the likelihood of the occurrence of the avalanche. However, the duration and the intensity of the snowfall are uncertain. New information becomes available every 8 hours from the meteorological agency; i.e. the time interval between the subsequent decision phases is set to 8 hours ($dt = 8$). At each decision phase a decision is made according to information available. Three decision alternatives are assumed; i.e. to evacuate the people $a^{(1)}$, not to evacuate $a^{(2)}$, and to wait $a^{(c)}$. It is assumed that the evacuation takes 16 hours to complete.

4.2 *Consequence model*

The consequences are postulated as follows, see also Table 1: The consequence is equal to $C_{Ev} = 1$ in two cases: (1) when the evacuation has been initiated but the avalanche does not occur, and (2) when the evacuation is completed before the avalanche occurs. A consequence of $C_D = 10$ is incurred if the avalanche occurs and the people are not evacuated or the evacuation was initiated but not completed. No consequence is incurred only in the case when no evacuation is initiated and no avalanche occurs.

Table 1. Conditions and associated consequences postulated in the consequence model.

	Additional snow height in the time period $[0,t]$	
People	$S_t > \tilde{s} = 800[mm]$	$S_t \le \tilde{s} = 800[mm]$
Not evacuated	$C_D = 10$	0
Evacuated	$C_{Ev} = 1$	$C_{Ev} = 1$

4.3 *Probabilistic snowfall model*

A hypothetical probabilistic snowfall model is assumed, which is adapted from a rainfall model developed by Hyndman & Grunwald (2000). Let X_t denote the random sequence representing the amount of snowfall in the time period $(t - dt, t]$. Hereafter, this time period is denoted by $(t-1, t]$ (i.e. the time unit is $dt = 8$) and thus $\{X_t\}_{t=0}^{n}$ for simplicity. The distribution of X_t is a mixture comprising a discrete component concentrated at $x_t = 0$ and a continuous component for $x_t > 0$. The discrete component of X_t represents the non-occurrence of snowfall and is characterized by the Bernoulli sequence J_t, whose conditional probability function is:

$$\pi_t(\mathbf{y}_{t-1}, \mathbf{y}_{t-2}) = P(J_t = 1 \mid \mathbf{Y}_{t-1} = \mathbf{y}_{t-1}, \mathbf{Y}_{t-1} = \mathbf{y}_{t-2}) = l(\mu_t(\mathbf{y}_{t-1}, \mathbf{y}_{t-2})) \qquad (12)$$

where $\mathbf{Y}_t = (J_t, X_t)$ and $l(\cdot)$ denotes the logit function which is defined as $l(\mu) = \exp(\mu)/(1+\exp(\mu))$ if $\mu > 0$ and $l(\mu) = 0$ otherwise, and

$$\mu_t(\mathbf{y}_{t-1}, \mathbf{y}_{t-2}) = \alpha_0 + \alpha_1 j_{t-1} + \alpha_2 j_{t-2} + \alpha_3 \log(x_{t-1} + c_1) + \alpha_4 \log(x_{t-2} + c_2) + \alpha_5 t^2 . \qquad (13)$$

The continuous component of X_t is strictly positive and characterizes the intensity of the snowfall. If $J_t = 1$, X_t is described by the continuous conditional density $g_t(x \mid \mathbf{y}_{t-1})$, $x > 0$. $g_t(\cdot \mid \cdot)$ follows the Gamma distribution with shape parameter κ and mean $\nu_t(\mathbf{y}_{t-1})$, where

$$\log(\nu_t(\mathbf{y}_{t-1})) = \beta_0 + \beta_1 j_{t-1} + \beta_2 \log(x_{t-1} + c_3) + \beta_3 t^2 . \qquad (14)$$

Then the transition probability density function of X_t is defined as (see Figure 2):

$$f_t(x_t \mid \mathbf{y}_{t-1},\mathbf{y}_{t-2}) = (1-\pi_t(\mathbf{y}_{t-1},\mathbf{y}_{t-2}))\delta_0(x_t) + \pi_t(\mathbf{y}_{t-1},\mathbf{y}_{t-2}) g_t(x_t \mid \mathbf{y}_{t-1}) \tag{15}$$

where δ_0 is the Dirac delta function. The additional snow height is obtained by multiplying the snow intensity by the factor F_s, which accounts for the density of the snow; i.e.

$$S_t = S_t(\underline{\mathbf{y}}_t) = \sum_{s=0}^{t} F_s x_t 1_{\{j_s=1\}} = S_{t-1} + F_t x_t 1_{\{j_t=1\}}. \tag{16}$$

Hence, S_t (the hazard index) at time t is characterized by the index S_{t-1} at time $t-1$ and a stochastic process composed of a second- and a first-order Markov process (the second term in the rightmost equation). The values of the parameters of the model are summarized in Table 2. The time frame is set to three days; i.e. $n=9$.

Table 2. Parameters of the probabilistic snowfall model.

Parameter	Value	Parameter	Value
j_{-1}, j_0, S_0	$0,0,0$	$\mathbf{c}=(c_1,c_2,c_3)$	$(0.15,0.3,0.5)$
$\boldsymbol{\alpha}=(\alpha_0,\alpha_1,\ldots,\alpha_5)$	$(4.5,0.26,0.1,0.5,0.05,-0.2)$	κ	1.5
$\boldsymbol{\beta}=(\beta_0,\beta_1,\beta_2,\beta_3)$	$(1.95,-0.2,0.25,-0.04)$	F_s	10

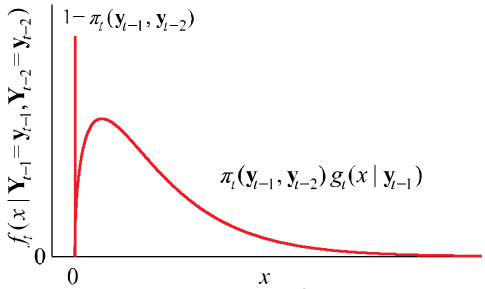

Figure 2. Illustration of $f_t(x \mid \mathbf{y}_{t-1},\mathbf{y}_{t-2})$.

4.4 *Solution with the eLSM*

Here, the MEU in Equation 3 is defined by the expected consequence; i.e. the minimum operator is used and the inequality sign of Equation 8 is turned. The steps in Section 3.1 are executed with the extended LSM and the eLSM to obtain the optimal decision.

Step 1: By MCS, generate b independent realizations of $\{\mathbf{Y}_t\}_{t=1}^{n}$ and $\mathbf{S}^i=(S_0^i,S_1^i,\ldots,S_n^i)$, $i=1,2,\ldots,b$, where $S_t^i=S_t^i(\underline{\mathbf{y}}_t^i)$ and $\mathbf{y}_t^i=(j_t^i,x_t^i)$. The realizations $\mathbf{y}_1^i,\mathbf{y}_2^i,\ldots,\mathbf{y}_n^i$ are simulated according to the probability density functions in Equations 12 and 15; the paths are denoted by $\mathbf{y}^i=(\mathbf{y}_{-1}^i,\mathbf{y}_0^i,\ldots,\mathbf{y}_n^i)$, where $\mathbf{y}_{-1}^i=\mathbf{y}_{-1}$, $\mathbf{y}_0^i=\mathbf{y}_0$ and $i=1,2,\ldots,b$.

Step 2: For each $\mathbf{y}_t^i$ the value $h_t^i=h_t(\mathbf{y}_t^i,\mathbf{y}_{t-1}^i)$ of the SVF is estimated. At time $n=9$ the consequence related to each realization and decision is assumed to be known; i.e. either s_n^i exceeds the threshold $\tilde{s}$ or not, thus $h_{n,\mathrm{MC}}^i=h_{n,\mathrm{eLSM}}^i$ for all i. Further, for $t=1,2,\ldots,n-1$
(1) with the extended LSM: Simulation of additional M paths $\mathbf{y}_n^{i,m}=(\mathbf{y}_{-1}^i,\ldots,\mathbf{y}_t^i,\mathbf{y}_{t+1}^{i,m},\ldots,\mathbf{y}_n^{i,m})$, $m=1,2,\ldots,M$, for which the observed consequences $u_t(\mathbf{s}^m,a_t^{(j)})$, $j=1,2$, are determined. Here $\mathbf{s}^m$ is the realization of the additional snow height related to the path realization $\underline{\mathbf{y}}_n^{i,m}$. Define $\hat{l}_{t,\mathrm{MC}}(a_t^{(j)},\mathbf{y}_t^i,\mathbf{y}_{t-1}^i)=\sum_{m=1}^{M} u_t(\mathbf{s}^m,a_t^{(j)})/M$, then

$$\hat{h}_{t,\mathrm{MC}}^i = \min\{\hat{l}_{t,\mathrm{MC}}(a_t^{(1)},\mathbf{y}_t^i,\mathbf{y}_{t-1}^i),\hat{l}_{t,\mathrm{MC}}(a_t^{(2)},\mathbf{y}_t^i,\mathbf{y}_{t-1}^i)\} \tag{17}$$

(2) with the eLSM as explained in Section 3.2: Define

$$\hat{h}_{t,\mathrm{eLSM}}^i = \min\{\hat{l}_{t,\mathrm{eLSM}}(a_t^{(1)},\mathbf{y}_t^i,\mathbf{y}_{t-1}^i),\hat{l}_{t,\mathrm{eLSM}}(a_t^{(2)},\mathbf{y}_t^i,\mathbf{y}_{t-1}^i)\} \tag{18}$$

where $\hat{l}_{t,\mathrm{eLSM}}(a_t^{(j)},\mathbf{y}_t^i,\mathbf{y}_{t-1}^i)=\mathbf{L}_t^i\cdot\mathbf{r}_t^{(j)}$, $j=1,2$. The vector $\mathbf{r}_t^{(j)}$ of the coefficients related to $a_t^{(j)}$ is computed by Equation 10. $\mathbf{L}_t^i$ denotes the i^{th} row of matrix $\mathbf{L}_t$; $\mathbf{L}_t$ consists of values of basis functions with arguments $\mathbf{y}_t$, $\mathbf{y}_{t-1}$ and S_t; e.g. for 1^{st} order linear basis functions

$$\mathbf{L}_t = \begin{bmatrix} 1 & x_t^1 & x_{t-1}^1 & s_t^1 \\ 1 & x_t^2 & x_{t-1}^2 & s_t^2 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_t^b & x_{t-1}^b & s_t^b \end{bmatrix}. \tag{19}$$

For $t = 0$ set $\hat{l}_0^{(j)} = \hat{l}_{0,\mathrm{MC}}(a_0^{(j)}, \mathbf{y}_0, \mathbf{y}_{-1}) = \hat{l}_{0,\mathrm{eLSM}}(a_0^{(j)}, \mathbf{y}_0, \mathbf{y}_{-1}) = \sum_{i=1}^{b} u_0(\mathbf{s}^i, a_0^{(j)}) / b$, $j = 1, 2$.

Step 3: Starting at time n, for both LSM approaches, the values of $q_{n,\mathrm{MC}}^i = q_{n,\mathrm{MC}}(\mathbf{y}_n^i, \mathbf{y}_{n-1}^i)$ and $q_{n,\mathrm{eLSM}}^i$ are set equal to $h_{n,\mathrm{MC}}^i$ and $h_{n,\mathrm{eLSM}}^i$ respectively, for all i.

Step 4: Moving to time $n-1$ the values of $c_{n-1}(\mathbf{y}_{n-1}, \mathbf{y}_{n-2})$ are similarly estimated for both approaches using the least squares method as described in Section 3.1.

Step 5: Then, for each path i determine the values of $q_{n-1}(\mathbf{y}_{n-1}, \mathbf{y}_{n-2})$:
(1) for the extended LSM with the estimate $\hat{h}_{t,\mathrm{MC}}^i$ obtained by means of MCS:

$$q_{n-1,\mathrm{MC}}^i = \begin{cases} \hat{h}_{n-1,\mathrm{MC}}^i, & \text{if } \hat{h}_{n-1,\mathrm{MC}}^i < \hat{c}_{n-1,\mathrm{MC}}^i \\ q_{n,\mathrm{MC}}^i, & \text{otherwise} \end{cases} \tag{20}$$

(2) for eLSM with the estimate $\hat{h}_{t,\mathrm{eLSM}}^i$ obtained by means of the least squares method:

$$q_{n-1,\mathrm{eLSM}}^i = \begin{cases} u_{n-1}^{*,i}, & \text{if } \hat{h}_{n-1,\mathrm{eLSM}}^i < \hat{c}_{n-1,\mathrm{eLSM}}^i \\ q_{n,\mathrm{eLSM}}^i, & \text{otherwise} \end{cases} \tag{21}$$

where $u_{n-1}^{*,i}$ denotes the observed future consequence in path i for the optimal terminal decision a_{n-1}^*. As in Section 3.1, moving another time step back the same procedure is repeated. This is continued until time $t = 1$ and for each path $q_{1,\mathrm{MC}}^i$ and $q_{1,\mathrm{eLSM}}^i$ are determined.

Step 6: Execute Step 6 of Section 3.1.

4.5 *Results*

To evaluate the performance of the eLSM compared to the extended LSM, both methods are applied to solve the decision problem of the example. The optimal decision at the initial time is obtained by estimating the expected consequences for the three decisions alternatives. Various types and degrees of basis functions are implemented; e.g. linear, Legendre and Chebyshev polynomials. Applying these basis functions, it is found that the results do not significantly differ. Thus, only the results obtained with linear basis functions are presented.

Figure 3 illustrates the findings for different parameter settings of the LSM. Therein, Figure 3 (a) shows for increasing number b of paths, $b = \{10^2, 3\cdot10^2, 10^3, 3\cdot10^3, 10^4, 3\cdot10^4, 10^5\}$, the convergence of the consequence estimates for the three decisions. For each b the estimates are calculated by the average of 100 computations of the indicated method. To be able to compare the results 100 different yet fixed sets of random numbers are used to generate the paths in Step 1. Hence, the estimates for the terminal decisions are identical for all methods; they are presented by solid lines with circles. The following results are obtained for $b = 10^5$: $\hat{l}_0^{(1)} = 1.0192$, $\hat{l}_0^{(2)} = 0.8969$ and e.g. $\hat{c}_{0,\mathrm{eLSM}} = 0.8055$ with the eLSM. The optimal decision is $a_0^{(c)}$ which is independent of the type of LSM; see Figure 3 (a). Further, the figure shows that the estimate $\hat{c}_0$ obtained by the extended LSM with $M = 10$ is biased. Therefore it is not considered in Figure 3 (b) which illustrates the convergence rate in terms of the coefficient of variation (COV) of the estimates $\hat{c}_0$ as a function of the computational time [sec]. The figure shows a significant improvement with the eLSM in terms of computational time; a reduction by the factor of 100.

An application of the proposed approach in practice is presented in Figure 4. Figure 4 (a) illustrates a hypothetical time series of the additional snow height $\{S_t\}_{t=0}^6$ where the threshold $\tilde{s}$ is exceeded within the time interval $(3, 4]$. Applying the eLSM subsequently for each time step it is found that the optimal decision at time $t = 0$ is $a^{(c)}$ whereas at time $t = 1$ it is found to be $a^{(1)}$ given that the snow height at time $t = 1$ in the figure is realized.

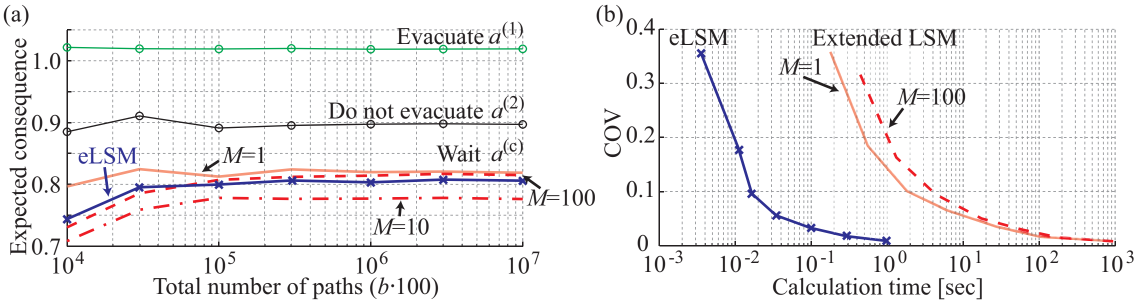

Figure 3. Comparison of the results of the extended LSM (with various numbers M of additional MCS) and eLSM. (a) Convergence of the average expected consequences with increasing total number of paths. (b) Illustration of the decreasing COV of $\hat{c}_0$ related to the increasing calculation time for one LSM computation as the number b of paths increases.

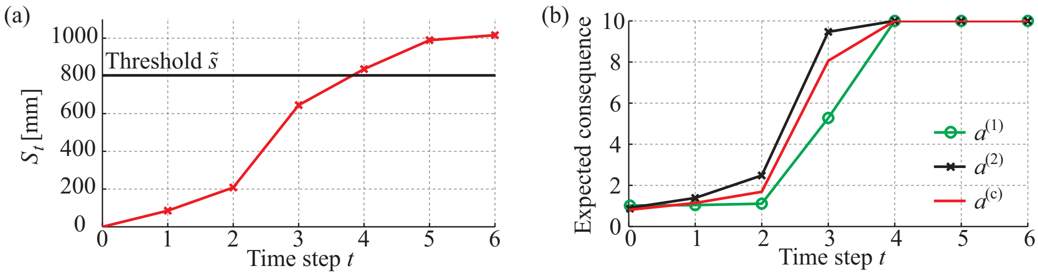

Figure 4. Illustration of (a) a hypothetical time series of S_t and (b) the corresponding time series of the estimated expected consequence of the three decision alternatives calculated with the eLSM and $b = 10^5$.

5 CONCLUSION

The present paper proposes an enhancement of the extended LSM in the context of real-time operational decision problems for evacuation in the face of emerging natural hazards. The proposed approach (eLSM) is applied to an example and it is found that the eLSM significantly improves the computational efficiency; by the factor up to 100.

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List of Abbreviations

ASCE	American Society of Civil Engineers
CI	Confidence Interval
cMCM	crude Monte Carlo Method
COV	Coefficient Of Variation
CVF	Continuing Value Function
DSS	Decision Support System
eLSM method	enhanced Least Squares Monte Carlo method
EM-DAT	Emergency Disasters Database
EWS	Early Warning System
FEMA	Federal Emergency Management Agency
FORM	First-Order Reliability Method
GARCH model	Generalized Autoregressive Conditional Heteroscedasticity Model
HAZUS	Hazards U.S. (freely distributed geographic information system-based natural hazard loss estimation software package; developed by FEMA)
HURREVAC	Hurricane Evacuation (restricted-use computer program funded by FEMA and USACE)
JCSS	Joint Committee on Structural Safety
JMA	Japan Meteorological Agency
LSM method	Least Squares Monte Carlo method

MCS	Monte Carlo simulation
MEU	Maximized Expected Utility
NHC	National Hurricane Center
NI	Numerical Integration
OED	Oxford English Dictionary
PRA	Probabilistic Risk Assessment
SLF	Swiss federal Institute for snow and avalanche research
SORM	Second-Order Reliability Method
SST	Sea Surface Temperature
SVF	Stopping Value Function
TC	Tropical Cyclone
UNEP	United Nations Environment Programme
UNISDR	United Nations International Strategy for Disaster Reduction
WCDR	World Conference on Disaster Reduction
WMO	World Meteorological Organization

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Engineering structures are designed to resist a certain range of intensities of natural hazards like tropical storms, floods or avalanches, but not their entire range. In cases they are likely to fail in a hazard event, risk reducing measures are undertaken to minimize possible consequences. Whether to commence such measures needs to be decided in real-time as information becomes available to avoid unnecessary losses. The present thesis aims at developing a framework to support real-time decision making in emerging natural hazard events. The framework supports decision makers in the process of solving the decision optimization of the choice and commencement of risk reducing measures in real-time.

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